On the Wave-Particle Duality of the Photons and the Matter-Photon Particle Mixture Model

Boling Xi[a], Shisheng Wang[b],*

[a]*ASML Holding, De Run 6501, 5504, Veldhoven, the Netherlands*
[b]*Karlsruhe Institute of Technology (KIT), Institute for Neutron Physics and Reactor Technology (INR), Postfach 3640, D-76021 Karlsruhe, Germany*

Abstract

Relativistic quantum mechanics and many experimental results and observations show that electron-positron annihilation can produce photons; photon-matter and photon-photon interaction can also create electrons and positrons. The photons have no rest mass, e.g. relative to Lab frame, and always travel at the speed of light in the vacuum. In this paper, a rotational moving electric dipole model with negative and positive charges was proposed for photons. In a vacuum, the electric dipoles are moving in a twisted helical motion around their propagation axis (their center of mass). Photon particles are traveling at the speed of light along the propagation axis. This pair of negative and positive charged particles exerts both electrostatic and magnetic force on each other. Both forces are attractive and act as a centripetal force to keep the electric dipole in a continuous helical motion around the axis of rotation. With this model, the wave-particle duality of photons can be described simultaneously. The space is filled by matter (having rest mass relative to Lab frame) and photons (without rest mass); it is a multicomponent mixture fluid. In "vacuum", though there is no rest matter particle, but there are still photon gas particles, the total energy-momentum tensor should include rest matter particles and photon gas particles.

Keywords: photon; wave-particle duality; electric dipole; matter-photon mixture; matter- photon energy-momentum tensor, vacuum energy

*Second Author**,** E-mail: *Shisheng.Wang@kit.edu*

https://doi.org/10.32388/NE8DRJ

1 Introduction

The prediction of the positron by the Dirac equation was a significant achievement in physics[1]. It indicates that a positron is the antiparticle of the electron with the same mass but opposite charge. During the studying of the behavior of cosmic ray[2-5], Anderson observed the positively charged electron (hence the positron), as predicted by the Dirac equation. Hereafter, he produced the proof that is more conclusive by shooting gamma rays produced by the natural radioactive nuclide into other materials, resulting in the creation of electron-positron pairs, and confirmed by other experiments[6,7]. Electron-positron pairs are simultaneously ejected from the material[8], and the experimental results indicate a common point of emitting origin of two particles. In Feynman's[9,10] diagram, an electron and a positron can annihilate by emitting photons. Today the Na-22 positron source (β^+ decay emitting positrons) is widely applied as a non-destructive spectroscopy technique to study voids and defects in solids, where the positrons collide with electrons to become energetic photons.

The photon-matter interaction shows the Thomson and Compton scattering phenomena within the mid-energy range. Blackett[11] showed electron-position pairs could be created out of light particles for photon-matter-interaction with high energy. A detailed and historical overview for electron-positron pair production by photons can refer to Hubbell's paper and the literatures inside[12].

For photons with high energies, the photon splitting phenomena by photon-photon interaction can be observed[13-18]. In the study of positron production of light-by-light scattering with high energy by Burk et al.[19], electron-positron pairs are created by a collision between the high-energy photons. These results are astounding evidence that inelastic light-by-light scattering involving only real photons can produce electron-positron pairs.

The PAMELA satellite[20] observed an excess of positrons in the atmosphere during thunderstorms, indicating that a significant amount of positrons was produced.

The Fermi Gamma-ray Space Telescope by NASA has detected beams of streams of electrons and positrons, produced above thunderstorms on Earth. The result was presented by researchers in January 2011 at the 217th American Astronomical Society Meeting of US[21]. The results affirmed that when gamma rays pass near the nuclei of atoms, they could turn their energy into two particles: an electron-positron pair.

In July 2021, the STAR Collaboration team[22] has shown the first direct evidence that two particles of light, or photons, crash into one another and produce an electron and its counterpart, a positron, the experimental result is consistent with the prediction by the Breit-Wheeler process[23]: the positron-electron pair is created from the direct collision of light with light.

Relativistic quantum mechanics figure out that photons are constantly splitting into pairs of oppositely charged particles (negative and positive charges), which can re-annihilate into the photons. The above-mentioned experimental results show that not only electron-positron annihilation can produce photons, but photon-photon interaction can also create electrons and positrons.

The above-mentioned observations with high energy gamma rays imply the gamma-ray is composed of electrons and positrons. It is mainstream known that photon (such as gamma rays) has no electrical charge. Indeed, if the spatial resolution of the measurement is not higher enough, finer details of the object being measured will be lost, or rather to say, are not distinguishable. As a result, an electron-positron pair (a tiny electric dipole) will be thought of as a single particle. It appears electroneutral, if the measuring distance is far large compared to the size of an electric dipole. Besides, Photons do not have a rest mass (and hence no rest frame, at least for the Lab frame), and they always travel at the speed of light in the vacuum, the most common way to weigh the mass (relatively at rest) of a photon is not possible in the Lab frame. The Maxwell equations show that photons are electromagnetic waves that propagate through vacuum at the speed of light; however, they have energy and momentum (Planck-Einstein relation, $E=hv$), namely, they exhibit also particle properties. The photon has properties of both a wave and a particle. How do the photons act as both a wave and a particle all the time simultaneously? Based on the above experimental results and observations, in this paper, we propose a rotational moving model with negative and positive charges for photons — a rotating moving electric dipole model. The negatively and positively charged pair is travelling helically twisted around its propagation axis (direction).

This paper is organized as follows. Section 2 will focus on the wave properties of photons, while Section 3 covers the kinetic energy of moving photon particles. Section 4 will discuss the moving dipole interaction and Lorentz force. Finally, some typical values for representative wave frequencies will be provided in the discussion. The mathematical details related to these topics will be included in Appendix A and B.

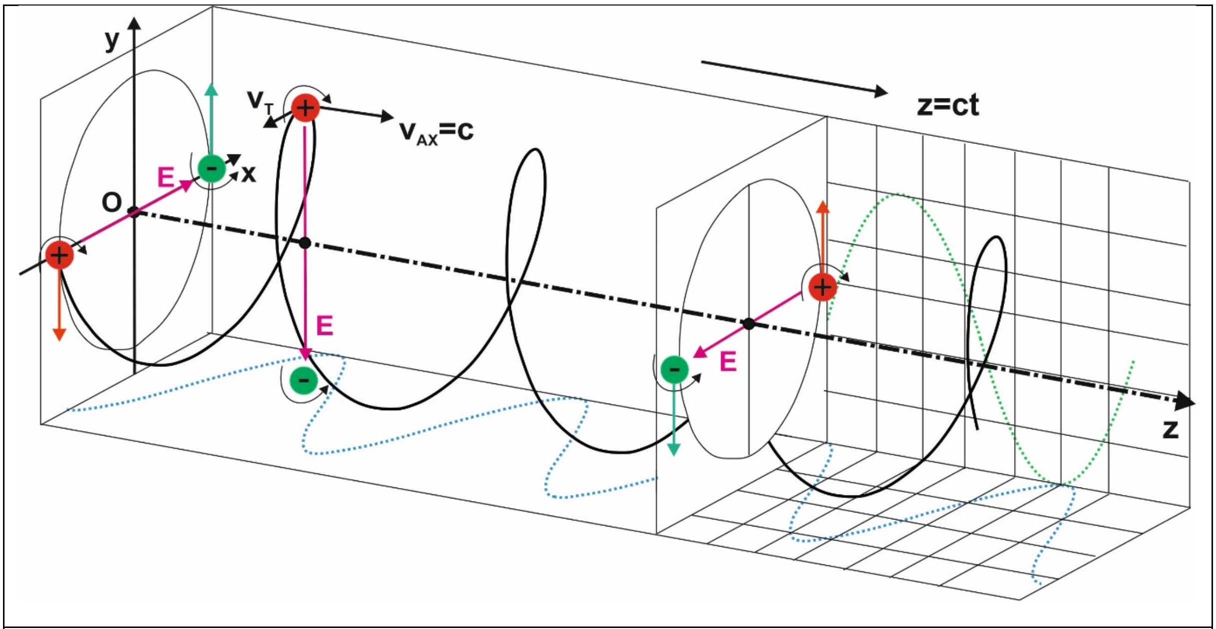
2 Helical Motion of Electric Dipoles and Different Wave Perspectives

In this section, we will discuss the wave-like behaviors of the rotational electric dipole, which is propagating in space at the speed of light. The electric dipole can be represented as electrical vectors pointing from the positive to the negative charge.

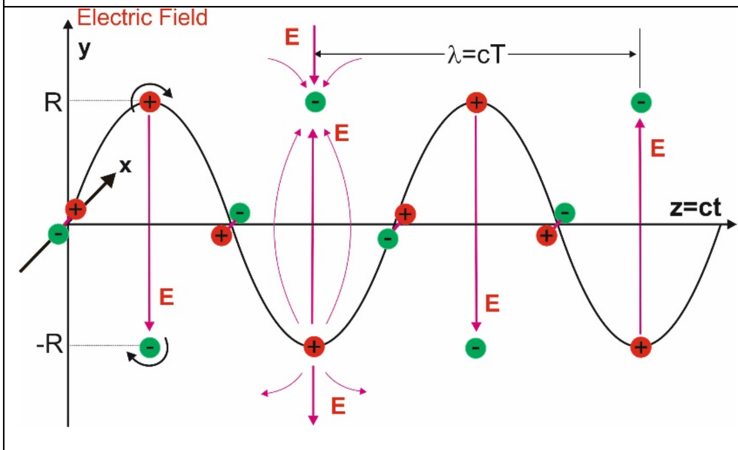
It is generally known that photons (electromagnetic waves) emitted from common sources (e.g. sunlight, light bulbs, etc.) are massive and randomly polarized, or rather to say, their propagation and polarization directions are randomly oriented and may propagate equally in all directions. Electromagnetic waves could be circularly polarized if their electrical field vectors are in two planes perpendicular to each other, equal in amplitude, but have a phase difference of $\pi/2$. One example is synchrotron radiation, where highly energetic charged particles move in a magnetic field and emit polarized electromagnetic waves (light). For simplicity of discussion, in this paper, we will discuss a simply organized light wave model — an isolated monochromatic circularly polarized wave that propagates in a vacuum. We will discuss several spatial perspectives of this type of wave, namely its electrical and magnetic field vectors.

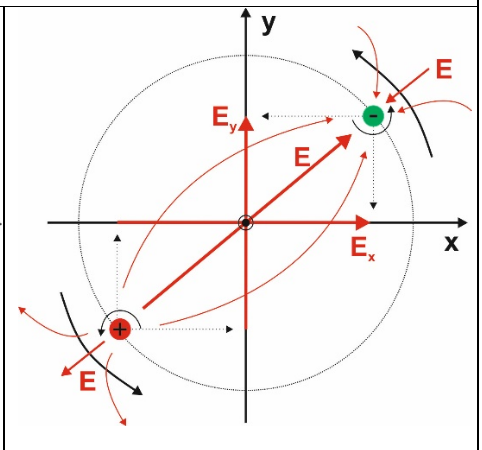
Fig. 1 gives the uniform helical motion of the negative-positive charged particle pairs in vacuum space. To avoid being messy, only the trajectory of the positively charged particle is shown in this figure.

For the sake of discussion, the rotating plane is assigned to be the X-Y plane, and the negative-positive charged particle pairs travel along the Z-direction at a speed of light, c, thus, z=ct. The coordinate origin is O, namely, this is a right-handed circularly polarized light, as shown in Fig. 1(a).



(a)





(b) (c)

Fig. 1. Uniform helical motion of electric dipoles

If the trajectories of these electric dipoles (and hence the electromagnetic field vectors) are projected onto different planes or directions (subspace), we will have different spatial perspectives of the motion (as well as the electrical and magnetic field vectors).

- If the motion is projected onto the Y-Z (or X-Z) plane, it exhibits a transverse electric wave (TE) profile along the direction of wave propagation (the time axis is ct, which serves also as the Z-axis). It can be read from the picture that the distance between the adjacent electric dipoles, along the propagation direction, is 1/4 wavelength (one-quarter of the wavelength), but perpendicular to each other, as shown in Fig. 1(b). The electrical vectors can be expressed as a traveling plane wave (i.e. if we only have interest and measure the electromagnetic field. Fig. 1(b) only shows some electric field lines for one dipole, they are curved and extend from the positive to the negative charge):

$$E_y(z,t) = R\sin(kz - \omega t), \tag{1a}$$

$$E_x(z,t) = R\cos(kz - \omega t). \tag{1b}$$

- If the motion is projected onto the X-Y plane (and hence, the motion along the Z-direction is suppressed), the electric dipole displays a circular motion around the rotation axis (a circularly polarized light). As confirmed experimentally by Allen et al.[24], the photon will show a well-defined orbital angular momentum (OAM). If this circular motion is further projected onto the X-axis or Y-axis, we will observe two simple harmonic oscillations (SHO) of the electrical vector around the origin – one along the X-axis, and another one along the Y-axis, as is shown in Fig. 1(c). (One circularly moving vector can be decomposed into two SHO vectors, which have a phase difference of π/2), for example.

$$E_y(t) = R\cos(\omega t), \tag{2a}$$

$$E_x(t) = R\sin(\omega t). \tag{2b}$$

- If the motion is projected onto the Z(ct)-axis, it merely becomes a 1-D linear translational motion with a constant speed of light, c.

$$z = ct. \tag{3}$$

Moreover, if the photon rotational plane (XY-plane) has an angle with the projected (observed) plane, we will get other perspectives (such as elliptical projection) of the electromagnetic wave vectors, depending on the projection directions[25, 26].

It can be recognized, with the projection procedure; we can get different spatial perspectives of the motion and the electromagnetic wave vectors, and measure it. However, we may lose some other information (and hence, the wave properties), for details see Appendix A.

3 The Kinetic Energy (Lagrangian) of Photon Particle

In this section, we focus on photon's particle properties, namely its energy, and momentum.

It should be stressed here that photons have no rest mass, and hence no rest frame (i.e. relative to the Lab frame). In fact, photons are always moving along the left and right light cones in the Z-ct coordinate in the Lab frame, see Fig. A1 for details. It is not possible to weigh the mass of a photon. An electric dipole as a whole will appear quasi electroneutral, if the measurement distance is far bigger than the size of the dipole – a so-called point dipole, (the potential falls very fast, as the distance increases from electric dipole, the effects of positive and negative charges nullify each other). The usual method to determine the ratio of the charge to its mass, e/m, from the curvature of the path of the charged particle (e.g. an electron) in a magnetic field is also impossible. However, according to the mass-energy equivalence principle, we can define a scalar parameter as a photon's "kinetic" mass in Lab frame (we call it momentum and energy carrier), namely a photon's energy equivalent mass, similar to the mass definition for other particles, e.g. for the mass definition of electrons.

3.1 The Total Kinetic Energy

The total kinetic energy of a photon particle can be broken up into two terms. The first is the kinetic energy of the center of mass, and the second is the kinetic energy of the particle relative to its center of mass.

The photon motion has two types of angular momentum, spin angular momentum (SAM) and orbital angular momentum (OAM)[27-30].

Thus, the particle motion can be decomposed into three parts:

- uniform translational motion along the Z-direction;
- rotational motion around the Z-axis (orbital), and
- spinning motion around the center of mass of the particle.

The total kinetic energy (translational, orbital, and spinning) of a helical moving particle is

$$E_{tot} = \frac{1}{2}m\vec{v}_{axi}^2 + \frac{1}{2}m\vec{v}_{tang}^2 + \frac{1}{2}\vec{\Omega}_{CM}^T\bar{\bar{I}}_{spin}\vec{\Omega}_{CM}, \tag{4}$$

The first two terms are the kinetic energy of the center of mass; the last term is the object's spinning kinetic energy relative to its center of mass. $\bar{\bar{I}}_{spin}$ is the moment of inertia of the spin motion, it is a tensor (i.e. a matrix). $\vec{\Omega}_{CM}$ is the spin angular frequency vector, and $\vec{\Omega}_{CM}^T$ is its transpose (row vector). Because of the symmetric property of the inertia tensor, the last term can be re-written as follows:

$$E_{spin} = \frac{1}{2}\vec{L}_{spin} \cdot \vec{\Omega}_{CM}, \tag{5}$$

where $\vec{L}_{spin}$ is the spin angular momentum around particle's center of mass.

The orbital kinetic energy is:

$$E_{orbital} = \frac{1}{2}m\vec{v}_{tang}^2 = \frac{1}{2}mR^2\vec{\omega} \cdot \vec{\omega}. \tag{6}$$

where $\vec{\omega}$ is the angular velocity vector of the orbital motion, it is perpendicular to the orbital (rotational) plane. Recalling the angular momentum definition, the kinetic energy of the orbital motion can be re-written as,

$$E_{orbital} = \frac{1}{2}\vec{L}_{axi} \cdot \vec{\omega}, \tag{7}$$

where $\vec{L}_{axi}$ is the orbital angular momentum of the photon particle, pointing to its propagation direction (z-axis), it is defined as:

$$\vec{L}_{axi} = mR^2\vec{\omega}. \tag{8}$$

To keep it to be simple, hereafter we omit the vector notation for orbital angular velocity and momentum, thus, the total kinetic energy of the photon particle is

$$E_{tot} = \frac{1}{2}mc^2 + \frac{1}{2}L_{axi} \cdot \omega + \frac{1}{2}\vec{L}_{spin} \cdot \vec{\Omega}_{CM}. \tag{9}$$

It can be re-written as:

$$E_{tot} = \frac{1}{2}\left(\frac{mc^2}{\omega} + L_{axi} + \frac{\vec{L}_{spin} \cdot \vec{\Omega}_{CM}}{\omega}\right) \cdot \omega. \tag{10}$$

3.2 Translational and Rotational Energy Partition

The total kinetic energy is the sum of translational, orbital, and spinning energy:

$$E_{tot} = E_{translation} + E_{orbital} + E_{spin}. \tag{11}$$

Translational kinetic energy thereof is (the propagation velocity is c):

$$E_{translation} = \frac{1}{2}mc^2. \tag{12}$$

According to Einstein's mass-energy equivalence principle, the total energy is defined as

$$E_{tot} = mc^2. \tag{13}$$

The 1D linear momentum projected onto the propagation direction (along Z-axis) is

$$p = \pm mc. \tag{14}$$

The positive and negative sign depends on the light propagation directions. This definition can ensure the energy-momentum vector to be a null vector (null cone) in the Minkowski space for photon particle motion (for details see Appendix A),

$$P^\alpha = \begin{bmatrix} \frac{E_{tot}}{c} \\ p \end{bmatrix} = \begin{bmatrix} mc \\ \pm mc \end{bmatrix} = \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix}(mc). \tag{15}$$

Provided we make a Lorentz boost along the Z-direction, the following relation is always true (the Lorentzian inner product is zero along the light cone surface), observed by different inertial frames (also by the observer in the Lab frame), regardless of their relative velocities of different observers:

$$\left(\frac{E_{tot}}{c}\right)^2 - p^2 = 0. \tag{16}$$

Accordingly, the sum of the orbital and spinning energy, thus, is:

$$E_{orbital} + E_{spin} = \frac{1}{2}mc^2. \quad (17)$$

Actually, the left and right light cones are eigenvectors of the Lorentz transformation, for details, please refer to Appendix A.

3.3 The Planck Constant and Equivalent Angular Momentum

Photons exist as moving particles (at least for the observer in the Lab frame). The Planck-Einstein relation says that the energy of photons depends on their frequency. It is directly proportional to the frequency.

$$E_{tot} = \hbar\omega \quad or \quad \omega = \frac{E_{tot}}{\hbar} \quad (18)$$

where $\hbar$ is the reduced Planck constant, and ω is the angular frequency of a photon wave.

Combining eq. (10), (13), and (18), we can get the reduced Planck constant expression as:

$$\hbar = L_{axi} + \frac{\vec{L}_{spin} \cdot \vec{\Omega}_{CM}}{\omega}. \quad (19)$$

Here, we can define an equivalent angular momentum, perpendicular to the rotational plane, as:

$$\hbar = L_{equ} = L_{axi} + \frac{\vec{L}_{spin} \cdot \vec{\Omega}_{CM}}{\omega}. \quad (20)$$

From the above equation, we can recognize that the reduced Planck constant is equal to an equivalent angular momentum of the photon particle motion, including the orbital and spin angular momentum components.

Finally, from equations (13), (18), and (20), we can get the photon particle mass expression as (observed in the Lab frame):

$$m = \frac{\hbar\omega}{c^2} = \frac{L_{equ}\omega}{c^2}. \tag{21}$$

This is the energy equivalent mass definition for photons, measured in the Lab frame, if we define the observed angular frequency in the Lab frame as ω.

3.4. De Broglie Relation for Photons — Duality Property

With the definitions of eq. (13) and (14), we have

$$p = \frac{E_{tot}}{c}. \tag{22}$$

Substituting (18) into (22), hence,

$$p = \frac{\hbar\omega}{c}. \tag{23}$$

Recalling the dispersion relation for plane wave in vacuum, and the equation of the reduced Planck constant with the equivalent angular momentum definition, eq. (20), finally, we get the De Broglie relation for photons:

$$p = \hbar k = L_{equ}k \quad or \quad k = \frac{p}{L_{equ}} \tag{24}$$

Substituting the eq. (18) and (24) into the eq. (1a), in a vacuum, we can get a plane wave equation for the photon, i.e. a wave function, propagating in the positive Z-direction, as:

$$\psi(t,\vec{r}) = R\exp\left[-i\left(\frac{(E_{tot}t - pz)}{L_{equ}}\right)\right] = R\exp\left[-i\left(\frac{mc}{L_{equ}}\right)(ct - z)\right]. \tag{25}$$

where the minus sign in front of the imaginary unit, i, represents clockwise rotational motion, if observed from positive Z-direction.

It can be shown that the Klein-Gordon equation for photon particles can be expressed as:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\psi = 0. \tag{26}$$

Mathematically it stands for a homogeneous wave transport equation without any source.

Schrödinger equation for this type of rotational motion, expressed by the eq. (25), in the matter rest frame (or we call it matter co-moving frame) reads:

$$\left[i\hbar\frac{\partial}{\partial t} + \left(\frac{\hbar^2}{2m}\right)\frac{\partial^2}{\partial z^2} + \frac{3}{2}(mc^2)\right]\psi = 0. \tag{27}$$

As shown by eq. (20), here the reduced Planck constant is equal to the equivalent angular momentum of the photon particle rotational motion.

4 The Dipole Interaction and Lorentz Force

4.1 The Geometrical Arrangement of the Electric Dipoles

In the following discussion, the gravitational force (and hence the gravitational energy) is neglected, since it is too small compared with electromagnetic force, as discussed in section 3, the Lagrangian has only the kinetic energy.

It is assumed that the electric dipoles are arranged in such an orientation that every electric dipoles feel a net-zero electrostatic force, exerted by its adjacent pairs through electrostatic interaction. Since the opposite-signed charges are at the same distance as the like-signed charges, as shown in Fig. 2, each charged particle of the electric dipole B feels a net-zero electrostatic force exerted by the A and C pairs. Because of the geometrical symmetries and motion symmetries of this arrangement, the electric dipole B feels also a net-zero magnetic force exerted by A and C pairs. (The direction of magnetic field produced by adjacent charge is perpendicular to the plane containing the line from source charge to field

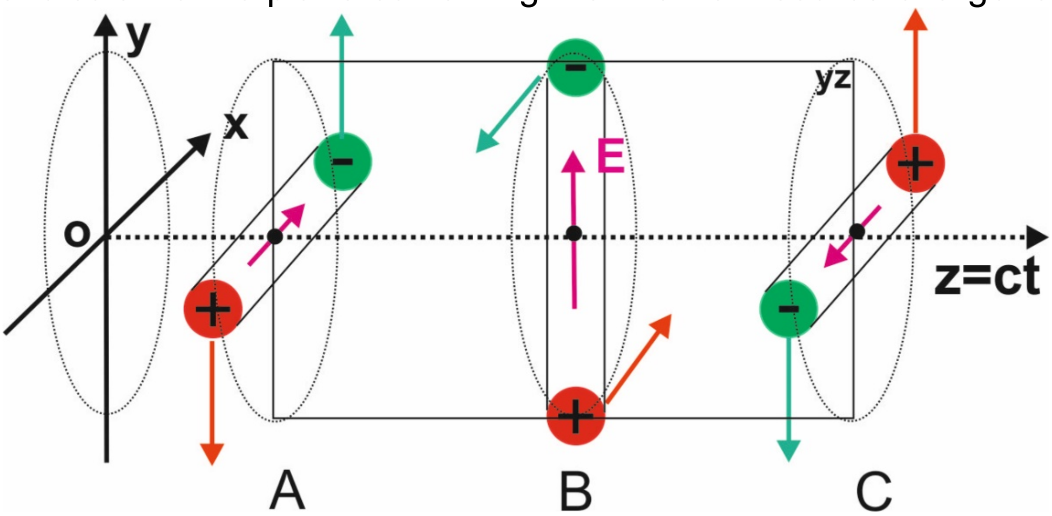


Fig.2 The electric dipole geometrical arrangements in space

point and the charge velocity vector, the exerted forces by adjacent charges balance out). This arrangement is reasonable, thus to keep each pair feeling a net-zero force exerted by the neighbor pairs. In fact, electric dipole B is in the center plane of A and C pairs (YZ-plane), where the electrical potential is zero, produced by A and C pairs.

Thus, we will discuss the interaction (force) between the negatively and positively charged particles in one pair in the following section.

4.2 The Lorentz Force as Centripetal Force

As an approximation, we ignore the magnetic field produced by the spinning motion of the charged particles (to neglect the spin-orbit interaction effect).

Along the Z-direction, negative and positive particles have no relative motion; the electron-positron pair exerts a mutual electrostatic attractive force through electrostatic interaction with each other. Fig 3. (a) shows this mutual electrostatic attractive force.

The electric field strength at a point only depends on the inverse square of the distance to the charge. e.g., suppose the positively charged particle, +q, is located at $(x, y, z) = (0, -R, ct)$, and the negatively charged particle, -q, is located at $(x, y, z) = (0, R, ct)$, see Fig. 3(a).

The electric field strength at point $(0, R, ct)$, produced by the positively charged particle is:

$$\vec{\boldsymbol{E}}_{q_+}(\hat{\boldsymbol{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{q_+}{d^2}\widehat{\boldsymbol{d}} = \frac{1}{16\pi\varepsilon_0}\cdot\frac{q_+}{R^2}\hat{\boldsymbol{r}} \tag{28}$$

The attractive electrostatic force on the negatively charged particle, due to the electric field produced by the positively charged particle, thus, is:

$$\vec{\boldsymbol{F}}_{q_-} = (-q)\cdot\vec{\boldsymbol{E}}_{q_+}(\hat{\boldsymbol{r}}). \tag{29}$$

This force vector points to the negative Y-direction (the rotational axis, namely, their common center of mass).

However, in the XY-plane, the negative and positive charged pair have relative motion with each other, with a constant speed of a tangent velocity relative to the rotational axis. Due to their relative motions, each charged particle appears to create a magnetic field around itself, which can be explained by special relativity and the electromagnetic field tensor. The induced magnetic field is perpendicular to the XY-plane, namely, perpendicular to the orbital rotating plane, or rather, parallel to the propagation direction of Z (ct). This induced magnetic field direction can

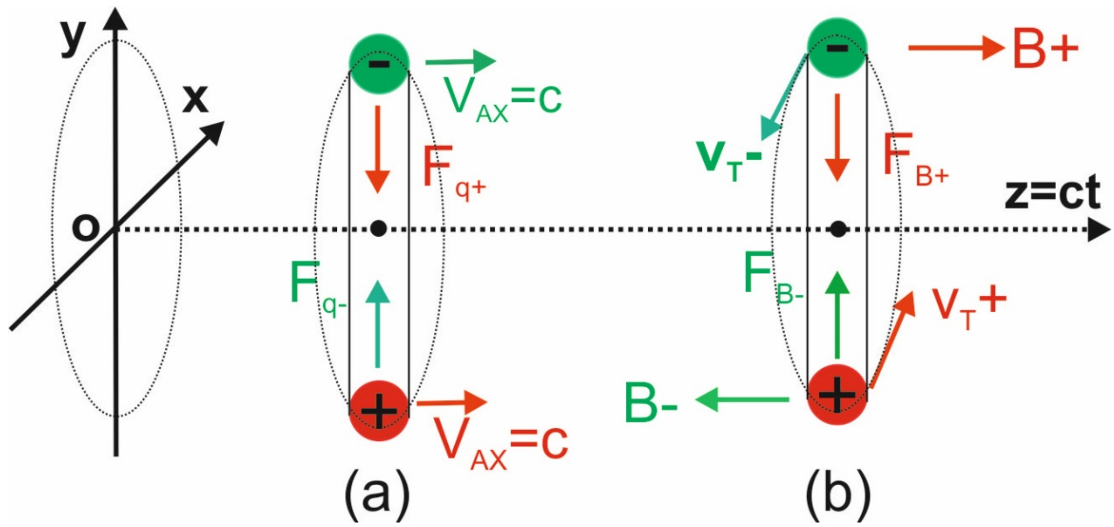


Fig. 3 (a) Mutual electrostatic attractive force, and (b) magnetic attractive force.

be read in Fig. 3(b). As a result, each particle feels a magnetic force exerted by its corresponding partner. With the help of the "Right-Hand-Rule", we can know the direction of this mutual magnetic force is also an attractive force, as shown in Fig 3(b). The magnetic field, in the space at the location of the negatively charged particle (-q), produced by the moving positive charged particle is given by the Biot-Savart law,

$$\vec{\boldsymbol{B}}_{q_+}(\hat{\boldsymbol{r}}) = \frac{\mu_0 q}{4\pi}\left(\frac{\vec{\boldsymbol{v}}_{tang} \times \widehat{\boldsymbol{d}}}{d^2}\right) = \frac{q}{16\pi\varepsilon_0 c^2}\left(\frac{\vec{\boldsymbol{v}}_{tang} \times \hat{\boldsymbol{r}}}{R^2}\right). \quad (30)$$

Using the Lorentz force law, the force felt by the negatively charged particle (or exerted by the positively charged particle) is

$$\vec{\boldsymbol{F}}_{q\_tot} = (-q)\left(\vec{\boldsymbol{E}}_{q_+} + \vec{\boldsymbol{v}}_{tang} \times \vec{\boldsymbol{B}}_{q_+}\right). \tag{31}$$

Note, here the negatively charged particle moving velocity relative to its position is $\vec{v}_{tang}$.

The first term is the electrostatic force, and the second term represents the magnetic force.

5 "Vacuum" is not Vacuum and the Multicomponent Mixture in Space

The space is filled with different type of particles (e.g. electrically charged or electrically neutral), with rest mass (having co-moving frame) or without rest mass (without co-moving frame). For simplification, in this paper, we consider only electrically neutral charged rest matter (hereafter we call it matter) and without rest mass, (the representative is the photon or neutrino particle, actually, if an electric dipole pair is regarded as a whole particle it can be called electrically neutral, e.g. the rotational radius is very tiny). For that, we introduce a matter (and a photon) particle distribution function in space, which is defined to be

$$H(t,x) = \begin{cases} 1 & if\ matter\ exists\ at\ position\ x\ and\ at\ time\ t \\ 0 & otherwise \end{cases} \tag{32}$$

As an approximation, we assume here matter and photon particles do not occupy a same spatial position at same time.

Then we can define a scalar field of mass, composited by matter mass and photon mass.

$$m(t,x) = H(t,m_m)m_m + H(t,m_f)m_f. \tag{33}$$

where, m_m represents rest mass and m_f is the photon mass.

If we assume the photon gas and matter as a homogenous fluid mixture, according to the mass-energy equivalence principle, the energy field can be expressed as

$$E(t,x) = mc^2 = \left[H(t,m_m)m_m + H(t,m_f)m_f\right]c^2. \tag{34}$$

Following this mass and energy field definition, the total energy-momentum tensor is sum of the rest matter and photon gas energy-momentum tensors:

$$T_{total}^{\alpha\beta} = H(t,m_m)T_m^{\alpha\beta} + H(t,m_f)T_f^{\alpha\beta}. \tag{35}$$

If we assume the energy-momentum tensor for rest mass as dust particles, the matter particle energy-momentum tensor reads:

$$T_m^{\alpha\beta} = P_m^{\alpha} \otimes U_m^{\beta}. \tag{36}$$

and the photon particle energy-momentum tensor is:

$$T_f^{\alpha\beta} = P_f^{\alpha} \otimes U_f^{\beta}. \tag{37}$$

where the P_m^{α} and P_f^{α} are 4-momentum vectors for matter and photon particles, respectively, and the U_m^{β} is the 4-velocity of matter particle, U_f^{β} is 4-velocity of photon gas.

The local formulation of the conservations of the energy-momentum for matter fluid and photon gas mixture then reads

$$\partial_\alpha T^{\alpha\beta} = \alpha_m \partial_\alpha T_m^{\alpha\beta} + (1 - \alpha_m)\partial_\alpha T_f^{\alpha\beta} = 0. \tag{38}$$

where α_m is the matter volume fraction. If we deal with $T^{\alpha\beta}$ as one mathematical object, this is the assumption of the homogenous flow model, as if they mix very well and there is only one phase.

In this mixture fluid model, according the above assumption, in "vacuum", though there is not rest matter particle, but there are still photon gas. The above conservation equation thus degrades to

$$\partial_\alpha T^{\alpha\beta} = \partial_\alpha T_f^{\alpha\beta} = 0. \tag{39}$$

This photon gas energy-momentum tensor can be regarded as dark energy in the absence of matter particles in space. In this tensor, we consider only the matter particles (with rest mass) and photon particles (without rest mass); in fact, the total energy-momentum tensor in space should also include other type of effects, e.g. include other electrically charged particles, it is out of the scope of this paper.

Appendix A. Motion Projection and Light Cone

A helix running particle around the Z(ct)-axis has a parametrization position vector (here the spinning motion is ignored) of:

$$\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} Rsin(\omega t) \\ Rcos(\omega t) \\ ct \end{bmatrix}, \tag{A1}$$

where R is the rotational radius, and ω is the angular frequency.

If this motion is projected onto the XY-plane, we observe (measure) the photon motion perpendicular to the propagation direction (z-direction), it will give:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = P_{xy}\vec{r} = \begin{bmatrix} Rsin(\omega t) \\ Rcos(\omega t) \\ 0 \end{bmatrix}. \tag{A2}$$

where the projection matrix is

$$P_{xy} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A3}$$

The projected motion will appear as a circularly polarized light (the electric dipole forms a rotational electrical vector), and the circularly polarized light will emerge with orbital angular momentums (OAMs)[24-30,32-36]. As indicated by Shen *et al*.[29], analogous to the hydrodynamic vortices, an optical vortex will appear as an isolated dark spot in the center.

Furthermore, if this motion is projected onto X-axis, it will give

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = P_x P_{xy} \vec{r} = \begin{bmatrix} Rsin(\omega t) \\ 0 \\ 0 \end{bmatrix}. \quad \text{(A4)}$$

where

$$P_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad \text{(A5)}$$

It will appear as a simple harmonic oscillation (SHO) along the X-axis.

If the helix running position vector is projected onto the X-Z(ct) plane, it gives

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = P_{xz} \vec{r} = \begin{bmatrix} Rsin(\omega t) \\ 0 \\ ct \end{bmatrix}, \quad \text{(A6)}$$

where the projection matrix is

$$P_{xz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{(A7)}$$

This motion can be expressed as a traveling-plane wave of an electric field along the ct-direction:

$$E_y(z,t) = R \cdot cos[k(z - ct)] = R \cdot cos\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right], \quad \text{(A8a)}$$

$$E_x(z,t) = R \cdot sin[k(z - ct)] = R \cdot sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]. \quad \text{(A8b)}$$

or as shown by eq. (25), It can be expressed as a particle wave function.

In this circumstance, the projected motion appears as linearly polarized light, traveling in the z-direction (the electric dipole forms a transverse electric vector (or magnetic vector) in $\hat{x}\ (or\ \hat{y}\ )$ direction); it is easy to see that linearly polarized light cannot show orbital angular momentum (OAM), or rather to say, on this spatial projection perspective, we cannot measure out the OAM.

Furthermore, the projection onto the Z-axis will give the following results

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = P_z \vec{r} = \begin{bmatrix} 0 \\ 0 \\ ct \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ (\lambda f)t \end{bmatrix}, \qquad \text{(A9)}$$

where the projection matrix is

$$P_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad \text{(A10)}$$

From above discussion, it can be recognized that if we simply regard (namely, projected onto the Z(ct)-axis) the photon particle as a 1-D linear motion, a piece of significant information about the particle motion is therefore lost, i.e., we completely lost the circulation motion information in the XY-plane.

We can also see from the above project matrices, mathematically, they are not invertible, if we project the motion onto the subspaces. Accompanied by the projection procedure is the loss of information, even the loss of valuable information. In other word, from the projected information to re-construct the original motion is impossible.

Fig. A1 shows the ct-Z diagram of the projected photon motion, in the picture; the projected 1-D photon motion trajectory is the left and right light cones.

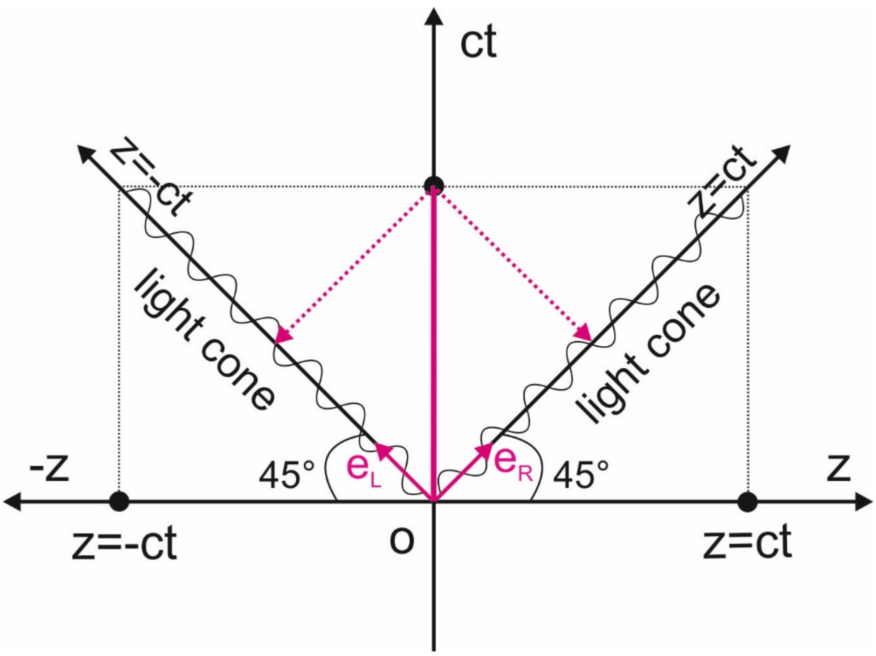


Fig. A1. The helical motion is projected onto the Z-coordinate (1-D); the moving information on the circular motion in the X-Y plane is lost. In the ct-Z coordinate system, the 1-D light travels along the left and right light cones.

The photon travels along the light cone surface in Minkowski spacetime. The world line is just the light cone surface. The left and right light cones can be expressed as two vectors in the Lab frame; they are orthogonal to each other.

$$\vec{e}_L = \begin{bmatrix} cT \\ -\lambda \end{bmatrix}; \quad \vec{e}_R = \begin{bmatrix} cT \\ \lambda \end{bmatrix}. \tag{A11}$$

where T and λ are the wave period and wavelength, respectively.

Recalling the relation of the wave phase dispersion for a plane wave in a vacuum:

$$\lambda = \pm cT. \tag{A12}$$

where the positive and negative sign represent the right and left light cone. The wavelength and period is a linear function, or rather to say, the wavelength and frequency function is a hyperbola:

$$\lambda = \frac{c}{f}. \tag{A13}$$

It is a null vector in the Minkowski space, to be specific:

$$\lambda^2 - (cT)^2 = 0. \tag{A14}$$

The left and right light cones are eigenvectors of Lorentz transformation[37]. Taking a Lorentz boost, for example, along the Z-direction, the transformed wavelength and period have the following relation

$$\begin{bmatrix} cT' \\ \lambda' \end{bmatrix} = \begin{bmatrix} \gamma & \pm\gamma\beta \\ \pm\gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} cT \\ \lambda \end{bmatrix}. \tag{A15}$$

where γ is the Lorentz factor and β is the ratio of the relative velocity of two observers, v to c, and v<c. The positive and negative sign depends on the relative velocity directions.

By applying the relation of the wave phase dispersion of eq. (A12), thus, we have

$$\begin{bmatrix} cT' \\ \lambda' \end{bmatrix} = (\gamma \pm \gamma\beta) \begin{bmatrix} 1 \\ 1 \end{bmatrix} cT = (\gamma \pm \gamma\beta) \begin{bmatrix} 1 \\ 1 \end{bmatrix} \lambda. \tag{A16}$$

In fact, the Lorentz transformation for light cone vectors is a scale transformation[37]. The coefficients (scale factors) of the above vector are the eigenvalues of the Lorentz transformation,

$$\epsilon_{1,2} = \gamma(1 \pm \beta) = \sqrt{\frac{1 \pm \beta}{1 \mp \beta}} \quad with \quad \epsilon_1 \cdot \epsilon_2 = 1. \qquad \text{(A17)}$$

After the Lorentz boost, the left and right light cones still form null (Eigen-) vectors in the Minkowski space (or Lorentz transformation maps light cones onto light cones[37,38]), $(\epsilon_1 \cdot \epsilon_2 = \gamma^2(1 - \beta^2) = 1)$,

$$\lambda'^2 - (cT')^2 = 0 \quad or \quad \lambda' = \pm cT'. \qquad \text{(A18)}$$

where the positive sign means the wave traveling to the left cone and the negative sign traveling to the right cone. In fact, after the transformation, the wavelength and frequency function is still a hyperbola.

By default, we observe the light wave propagation in the Lab frame. In this reference frame, if we apply the light speed as the reference velocity to define the photon linear translational energy and momentum for left and right light cone (keeping in mind, we lost the information about the rotational motion in the XY-plane, and simply think it as a 1D motion along the Z-direction),

$$E_{L\_tran} = \frac{1}{2} m(-c)^2 \quad and \quad P_L = -mc, \qquad \text{(A19a)}$$

$$E_{R\_tran} = \frac{1}{2} mc^2 \quad and \quad P_R = mc. \qquad \text{(A19b)}$$

If we use this linear translational energy and momentum to construct vectors, to ensure that they are null vectors (travelling along the light cone surface) for the left and right cone in the Minkowski space, the possibility is

$$\begin{bmatrix} \frac{2E_{L\_tran}}{c} \\ P_L \end{bmatrix} = \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)(mc) \quad and \quad \begin{bmatrix} \frac{2E_{R\_tran}}{c} \\ P_R \end{bmatrix} = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)(mc). \qquad \text{(A20)}$$

Comparing eq. (15), (16) and (A20), we have the following relation between the total energy and the linear translational energy for the left or right cone,

$$E_{tot} = 2E_{L\_tran} = 2E_{R\_tran} = mc^2. \qquad \text{(A21)}$$

According to this definition, the total energy and linear momentum can only form a null vector in the Minkowski space, in such a way as to ensure it is a Lorentz invariance:

$$\eta_{\alpha\beta}\left(\begin{bmatrix} \frac{E_{tot}}{c} \\ p \end{bmatrix}, \begin{bmatrix} \frac{E_{tot}}{c} \\ p \end{bmatrix}\right) = 0. \qquad \text{(A22)}$$

That is exactly the eq. (16), where $\eta_{\alpha\beta}$ is the Lorentzian metric tensor:

$$\eta_{\alpha\beta} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \qquad \text{(A23)}$$

According to the special relativity theory, eq. (A22) is valid for the light cone (null cone). With this energy definition or partition, the sum of the orbital and spinning energy, i.e. the eq. (17), equals the linear translational energy.

Thus, we have the energy for photon particle, corresponding to a specific frequency of ω, observed in Lab frame:

$$E_{translation} = \frac{1}{2} L_{equ}\omega = \frac{1}{2}\hbar\omega, \tag{A24a}$$

$$E_{orbital} + E_{spin} = \frac{1}{2} L_{equ}\omega = \frac{1}{2}\hbar\omega, \tag{A24b}$$

$$E_{tot} = L_{equ}\omega = \hbar\omega. \tag{A24c}$$

The 1-D linear momentum along the Z-direction is expressed by the eq. (23) and (24), namely,

$$p = \pm\frac{1}{c} L_{equ}\omega = \pm\frac{1}{c}\hbar\omega = \pm\frac{E_{tot}}{c}. \tag{A25}$$

Appendix B. Tangent Velocity and Radius

From eqs. (28) – (31), we have the Lorentz force

$$F_L = \left(1 + \frac{v_{tang}^2}{c^2}\right)\frac{q^2}{16\pi\varepsilon_0 R^2}. \tag{B1}$$

With the tangential velocity and rotational radius relation of

$$\vec{v}_{tang} = \vec{\omega} \times \vec{R}, \tag{B2}$$

thus, we have

$$F_L = \left(1 + \frac{R^2\omega^2}{c^2}\right)\frac{q^2}{16\pi\varepsilon_0 R^2}. \tag{B3}$$

This force serves as the centripetal force to keep the dipole rotation. The tangent velocity can be very high, thereby relativistic effect cannot be neglected; this force can be written in terms of the orbital angular velocity, so the centripetal force can be expressed as

$$F_c = \gamma m \frac{v_{tang}^2}{R} = \gamma m R\omega^2 = \frac{mR\omega^2}{\sqrt{1 - \frac{v_{tang}^2}{c^2}}} = \frac{mR\omega^2}{\sqrt{1 - \frac{R^2\omega^2}{c^2}}}. \quad \text{(B4)}$$

The eq. (B3) is equal to (B4), hence,

$$\frac{mR\omega^2}{\sqrt{1 - \frac{R^2\omega^2}{c^2}}} = \left(1 + \frac{R^2\omega^2}{c^2}\right) \frac{q^2}{16\pi\varepsilon_0 R^2}. \quad \text{(B5)}$$

According to the mass energy equivalence definition, the mass is

$$m = \frac{\hbar\omega}{c^2}. \quad \text{(B6)}$$

With a bit of algebra manipulation, finally we get

$$(\hbar\omega^3)R^3 - \left(\sqrt{1 - \frac{R^2\omega^2}{c^2}}\right)\left(\frac{q^2\omega^2}{16\pi\varepsilon_0}\right)R^2 - \left(\sqrt{1 - \frac{R^2\omega^2}{c^2}}\right)\frac{q^2c^2}{16\pi\varepsilon_0} = 0. \quad \text{(B7)}$$

Given the angular frequency, we can estimate the rotational radius using eq. (B7). With the estimated radius and eq. (B2), we can get the tangent velocity. Furthermore, the orbital angular momentum of a photon particle can be got by eq. (8). Finally, we can estimate the orbital energy and spin energy using the eq. (6) and (17), respectively.

For the calculation of the radius and tangential velocity for different angular frequencies, we assume a same charge value of $q = 1.6022 \times 10^{-19}\ (C)$ despite of the frequencies and do not consider the spin-orbit interaction.

With the rotational moving electric dipole model, we can estimate a number of values for different photon wave frequencies. Here we choose some typical electromagnetic wave frequencies, from the normal radio wave range, EUV to a typical gamma-ray range.

For the calculation of the total energy, we apply the Einstein-Planck relation of eq. (18). The Planck constant is $h = 6.62607 \times 10^{-34}\ (J \cdot m)$. The estimation of the energy equivalent mass is based on the light speed of $c = 2.99792458 \times 10^{8}\ (m/s)$. The vacuum permittivity and permeability values are $\varepsilon_0 = 8.854188 \times 10^{-12}\ (F \cdot m^{-1})$ and $\mu_0 = 4\pi \times 10^{-7}\ (H \cdot m^{-1})$, repectivily.

Table B1. Some photon values for different wave frequency

Wave type	Frequency (Hz)	Total Energy (keV)	Mass (kg)	Radius (nm)	E_orbital (KeV)	E_spin (Kev)	Lorentz force (N)
Radio	$3.0 \cdot 10^{5}$	$1.24 \cdot 10^{-12}$	$2.21 \cdot 10^{-45}$	$1.948 \cdot 10^{10}$	$9.31 \cdot 10^{-15}$	$6.11 \cdot 10^{-13}$	$1.54 \cdot 10^{-31}$
Microwave	$2.45 \cdot 10^{9}$	$1.01 \cdot 10^{-8}$	$1.81 \cdot 10^{-41}$	$2.39 \cdot 10^{6}$	$7.60 \cdot 10^{-11}$	$4.99 \cdot 10^{-9}$	$1.03 \cdot 10^{-23}$
Infrared	$3.0 \cdot 10^{13}$	$1.24 \cdot 10^{-4}$	$2.21 \cdot 10^{-37}$	194.8	$9.31 \cdot 10^{-7}$	$6.11 \cdot 10^{-5}$	$1.54 \cdot 10^{-15}$
Visible light	$6.0 \cdot 10^{14}$	$2.48 \cdot 10^{-3}$	$4.42 \cdot 10^{-36}$	9.74	$1.86 \cdot 10^{-5}$	$1.22 \cdot 10^{-3}$	$6.17 \cdot 10^{-13}$
EUV	$2.2 \cdot 10^{16}$	0.092	$1.64 \cdot 10^{-34}$	0.263	$6.89 \cdot 10^{-4}$	$4.53 \cdot 10^{-2}$	$8.46 \cdot 10^{-10}$
X-ray	$3.0 \cdot 10^{18}$	12.41	$2.21 \cdot 10^{-32}$	$1.948 \cdot 10^{-3}$	0.0931	6.111	$1.54 \cdot 10^{-5}$
Gamma-ray	$3.0 \cdot 10^{20}$	1241	$2.21 \cdot 10^{-30}$	$1.948 \cdot 10^{-5}$	9.3081	611.1	0.154

The calculated tangent velocities for different wave frequencies are the same. It gives $v_{tang} = 3.67198 \times\ 10^{7}\ (m/s)$. The ratio of tangential velocity to light propagation speed of c is $\beta_T = v_{tang}/c = 0.122484$. The calculated orbital angular momentum (OAM) of a photon particle is $L_{axi} = 1.58234 \times\ 10^{-36}\ (J \cdot s)$. The equivalent angular momentum is equal to the reduced Planck constant, $L_{equ} = \hbar$. Since there are no experimental values for the spinning angular velocity, SAM cannot be directly estimated. The spin energy is much higher than the orbital energy. The calculated ratio of wavelength to the radius, $\lambda/R = 51.298$, without considering the spin-orbit interaction[27-30], see Fig. 1(b).

In order to compare different photons with an electron clearly, here we give also out the electron values. For electron, we use the CODATA recommended values for reference[31].

Table B2 Reference values of electron

	unit	Value
Mass (m_e)	kg	$9.1094 \cdot 10^{-31}$
Unified atomic mass in u	u	$5.4858 \cdot 10^{-4}$
Mass energy equivalent ($m_e c^2$)	J	$8.1871 \cdot 10^{-14}$
Mass energy equivalent in keV	KeV	511.7
Charge ($-e$)	C	$1.6022 \cdot 10^{-19}$
Charge to mass quotient ($-e/m_e$)	C kg^{-1}	$1.7588 \cdot 10^{11}$

References

1. P.A.M. Dirac, "The Quantum Theory of the Electron," *Proceedings of the Royal Society of London,* Series A, Vol. 117, No. 778, 610-624, Feb. (1928).

2. C. D. Anderson, "Energies of cosmic-ray particles," *Physical Review*. 41, 405-421 (1932). [DOI: 10.1103/Physrev.41.405].

3. C. D. Anderson, "The positive electron," *Physical Review*. 43, 491-494 (1933). [DOI: 10.1103/Physrev.43.491].

4. C. D. Anderson, "Cosmic-ray positive and negative electrons," *Physical Review*. 44, 406-416 (1933). [DOI: 10.1103/Physrev.44.406].

5. C. D. Anderson, “The Positron,” *Nature*, March 3, 313-316 (1934).

6. C. D. Anderson, S. H. Neddermeyer, “Positrons from gamma-rays,” *Physical Review*. 43, 1034 (1933). [DOI: 10.1103/Physrev.43.1034].

7. S. H. Neddermeyer and C. D. Anderson, “Energy Spectra of Positrons Ejected by Artificially Stimulated Radioactive Substances,” *Phys. Rev.* 45, 498, April (1934). [DOI: https://doi.org/10.1103/PhysRev.45.498].

8. C. D. Anderson, “The Apparent Existence of Easily Deflectable Positives,” *Science*, 09. SEP. 238-239 (1932).

9. R.P. Feynman, “The theory of positrons,” *Physical Review,* 76 (6), 749-759 (1949). [DOI:https://doi.org/10.1103/PhysRev.76.749].

10. R. P. Feynman, “Space-time approach to quantum electrodynamics,” *Physical Review* 76, 769–789 (1949).

11. https://www.nobelprize.org/prizes/physics/1948/blackett/facts/

12. J.H. Hubbell, “Electron-positron pair production by photons: A historical overview”, *Radiation Physics and Chemistry*, 75, 614-623, (2006), [DOI: 10.1016/j.radphyschem.2005.10.008].

13. S. L. Adler, “Photon splitting and photon dispersion in a strong magnetic field,” *Annals of Physics,* Vol. 67, Issue 2, 599-647, Oct. (1971).

14. D. d’Enterria and G. G. da Silveira, “Observing Light-by-Light Scattering at the Large Hadron Collider,” *Phys. Rev. Lett.* 111, 080405 –22 August 2013; *Erratum Phys. Rev. Lett.* 116, 129901 (2016).

15. D. d’Enterria and G. G. da Silveira, “Erratum: Observing Light-by-Light Scattering at the Large Hadron Collider,” *Phys. Rev. Lett.* 116, 129901 (2016).

*16. CERN Courier, “*ATLAS spots light-by-light scattering,” https://cerncourier.com/a/atlas-spots-light-by-light-scattering”, 11 Nov (2016), Retrieved on 18, March 2023.

17. M. Bregant, et. al. (PVLAS Collaboration), “Limits on low energy photon-photon scattering from an experiment on magnetic vacuum birefringence,” *Phys. Rev. D,* 78, 032006 –12 August (2008).

18. The CMS Collaboration, “Evidence for light-by-light scattering and searches for axion-like particles in ultra-peripheral PbPb collisions at $\sqrt{s_{\mathrm{NN}}} = 5.02\ \mathrm{TeV}$,” *Physics Letters B,* Vol. 797, 134826;10 October (2019), [DOI: 10.1016/j.physletb.2019.134826].

19. D. L. Burke, et al. “Positron Production in Multiphoton Light-by-Light Scattering,” *Phys. Rev. Lett.* Vol 79. No 9, 1626-1629, 1 September (1997).

20. I. Cholis, et al. “The PAMELA positron excess from annihilations into a light boson,” *Journal of Cosmology and Astroparticle Physics,* Vol. 2009, 007, Dec. (2009).

21. J. McEnery, “A New View of the High Energy Gamma-ray Sky with the Fermi Gamma-Ray Space Telescope,” *217th meeting of the American Astronomical Society*, Seattle, WA, 9 – 13 January (2011).

22. J. Adam *et al.* (STAR Collaboration), “Measurement of e^+e^- Momentum and Angular Distributions from Linearly Polarized Photon Collisions”, *Phys. Rev. Lett.* 127, 052302, July (2021), [DOI: 10.1103/PhysRevLett.127.052302].

23. G. Breit and John A. Wheeler, "Collision of Two Light Quanta". *Physical Review*, 46, (12), 1087–1091, December, (1934), [DOI: 10.1103/PhysRev.46.1087].

24. L. Allen, et al, “Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes,” *Phys. Rev. A,* 45, 8185, June (1992), [DOI:10.1103/PhysRevA.45.8185].

25. M. Padgett *et al.* “Light’s Orbital Angular Momentum,” *Physics Today*, 57, 5, 35, (2004); [DOI: 10.1063/1.1768672].

26. S. Kruk *et al.* “Invited Article: Broadband highly efficient dielectric meta devices for polarization control,” *APL Photonics* 1, 030801 (2016), [DOI: 10.1063/1.4949007].

27. Deng Pan, *et al.*, “Strong Spin-Orbit Interaction of Light in Plasmonic Nanostructures and Nanocircuits,” *Phys. Rev. Lett.* Vol 117, 166803, 14 Oct. (2016). [DOI: 10.1103/PhysRevLett.117.166803].

28. K. Y. Bliokh, et al., “Spin-orbit interactions of light”, *Nature Photonics*, 9, 796-808, (2015), [DOI: 10.1038/nphoton.2015.201].

29. Yijie Shen, Xuejiao Wang, Zhenwei Xie, Changjun Min, Xing Fu, Qiang Liu, Mali Gong & Xiaocong Yuan, “Optical vortices 30 years on: OAM manipulation from topological charge to multiple singularities”, *Nature,* 8:90 (2019), [DOI: 10.1038/s41377-019-0194-2].

30. Jian Wang, Jeng-Yuan Yang, Irfan M. Fazal, Nisar Ahmed, Yan Yan, Hao Huang, Yongxiong Ren, Yang Yue, Samuel Dolinar, Moshe Tur and Alan E. Willner, “Terabit free-space data transmission employing orbital angular momentum multiplexing”, *NATURE PHOTONICS* , 6, 488-496, (2012), [DOI: 10.1038/nphoton.2012.138].

31. https://physics.nist.gov/cuu/Constants/index.html, Retrieved on 18, March 2023.

32. C. Greenshields et al., “Vacuum Faraday effect for electrons*,” New Journal of Physics,* 14 (2012), 103040, [DOI:10.1088/1367-2630/14/10/103040].

33. By L. Allen, Stephen M. Barnett, Miles J. Padgett**,** *Optical Angular Momentum*, IoP, institute of physics Publishing, Bristol and Philadelphia, (2003), [ISBN 0 7503 0901 6].

34. A. Bekshaev, M. Soskin, M. Vasnetsov, *Paraxial Light Beams with Angular Momentum*, Nova Science Publishers, Inc, 30. December, (2008), ISBN 978-1604561142

35. Juan P. Torres, Lluis Torner, *Twisted Photons: Applications of Light with Orbital Angular Momentum*, Wiley, VCH, ISBN: 978-3-527-63537-5, March (2011).

36. David L. Andrews, Mohamed Babiker, *The Angular Momentum of Light*, Cambridge university press, December, (2012), ISBN: 9780511795213, [DOI: 10.1017/CBO9780511795213].

37. Hartmann Römer, Michael Forger, *Elementare Feldtheorie: Elektrodynamik, Hydrodynamik, spezielle Relativitätstheorie*, VCH Verlagsgesellschaft, p.109-110, (1993), ISBN: 3527290656.

38. H.J. Brochers and G.C. Hegerfeldt, “The Structure of Space-Time Transformations”, *Commun. Math. Phys.* 28, 259-266, (1972).