Commentary

The Sagnac Effect and General Relativity Foundations

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The present note is motivated by an article published in the Annales de la Fondation Louis de Broglie. In that piece, Wolfgang Engelhardt presented an interesting and straightforward argument claiming the application of the Lorentz transformations predicts a null effect for a Sagnac-type experiment, thereby purportedly disproving relativity theory through empirical evidence. Given that outside specialists' circles, there is widespread confusion over similar arguments involving rotational motion, discussing a correct relativist explanation has didactic relevance. It also allows for revisiting some aspects of the foundations of general relativity.

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1. Introduction

This article complements Ref.[1], which stresses the consistency of relativity theory against claims to the contrary based on the erroneous analysis of noninertial motion along the lines of the so-called Selleri's paradox[2].

Although Engelhardt's argument in Ref.[3] is also based on an incorrect relativistic analysis of rotational motion, his specific argument is distinct from Selleri's[4].

This situation offers an excellent opportunity to explain further the correct application of special relativity to noninertial motion, hopefully dispelling the widespread belief that special relativity can not deal with accelerated frames of reference.

The former point was disclosed in a fairly recent research by Pepino and Mabile who found from a survey of physics faculty and graduate students from more than 22 physics departments of prestigious institutions in the United States and United Kingdom, that only 37% of faculty members and 10% of the graduate students correctly answered *yes* to the question “Is SR capable of describing physics in accelerated reference frames? (*yes or no*)”[5].

To set the appropriate context, we begin with a brief history of the Sagnac effect and its interpretation before addressing Engelhardt’s concrete argument against relativity in Ref.[3].

Around 1910, the French physicist Georges Sagnac[1] proposed an experiment to prove the existence of the luminiferous ether based on what is now known as the Sagnac effect. By 1913, he performed and published the experiment results confirming his ideas about the aether[6].

Later, in 1925, Michelson and Gale performed another Sagnac-type experiment, this time, to detect the Earth’s rotation. Michelson-Gale’s experimental result can be explained only if the ether is not dragged along with the Earth’s rotation[7], thus contradicting the 1887 Michelson-Morley experiment, which requires the hypothetical ether to be dragged along with Earth’s motion[8]. The unavoidable conclusion is that, when considered together, both experiments, Michelson-Gale’s and Michelson-Morley’s, prove the aether hypothesis untenable.

Despite the many scientific and technological applications of the Sagnac effect and the fact they all are consistently explained within the scope of the theory of relativity, there still exists outside mainstream physics, an unorthodox current claiming that the Sagnac effect disproves the theory of relativity[4][9][10][11][12][13]. Perhaps, the best-known proponent of the idea was Franco Selleri[4].

Although Selleri’s and Engelhardt’s approaches are different, both are based on the claim that light-speed invariance leads to contradictions, so the theory of relativity must be incorrect.

Section 2 recalls that when the experiment is described from an inertial frame, the classical Newtonian and the relativistic predictions are equivalent in first-order approximation. In Section 3 we formulate Engelhardt’s argument and observe that a previous comment by Sfarti[14] did not address Engelhardt’s claim in a direct form. Section 4 expounds on the relativist correct explanation of the apparent paradox raised by Engelhardt thus refuting his argument.

2. The classical and relativistic explanations

The simplest relativist explanation of the Sagnac effect is achieved when we describe it from an inertial frame where the center of the rotating platform is stationary. In this case, the relativist and the classical

Newtonian explanations are very similar, and we can easily see that both theories predict the same first-order result, so the Sagnac experiment alone cannot disprove either theory.

The difference in the arrival times of the clock and anti-clock-wise beams, from the perspective of an inertial observer is given by equation five of ref.[3]

$$t^{+} - t^{-} = \frac{2Lv_0}{c^2(1-(\frac{v_0}{c})^2)} \tag{1}$$

where $L = 2\pi R$ and $v_0 = \Omega R$.

The relativistic result is obtained by introducing in (1) the factor $1/\gamma = \sqrt{1-(v_0/c)^2}$ corresponding to the time dilation effect owing to the motion of the detecting device rotating with the platform,

$$\begin{aligned} t^{+} - t^{-} &= \frac{1}{\gamma}\frac{2Lv_0}{c^2(1-(\frac{v_0}{c})^2)} \\ &= \frac{2Lv_0}{c^2\sqrt{1-(\frac{v_0}{c})^2}} \end{aligned} \tag{2}$$

Since to first-order in v_0/c,

$$\frac{1}{1-(\frac{v_0}{c})^2} \approx \frac{1}{\sqrt{1-(\frac{v_0}{c})^2}} \approx 1 \tag{3}$$

the Newtonian (1) and relativist (2) results coincide.

3. Engelhardt's main argument

Engelhardt's paper[3] contains some puzzling statements, such as that the Michelson-Morley experiment does not contradict the Michelson-Gale result regarding the existence of an aether wind, or that, in Ref. [15], the renowned GPS expert Neil Ashby indeed used the Galilean transformations despite Ashby's explicit statement that "*These clocks have gravitational and motional frequency shifts that are so large that, without carefully accounting for numerous relativistic effects, the system would not work.*"

For conciseness and clarity, we shall only concentrate on what we call here *Engelhardt's main argument* (EMA). To facilitate the analysis we separate EMA into two parts:

a. Applying the formula for adding velocities based on the Lorentz transformations (LT), observers fixed to the platform and rotating with it, find that the speed of the light beams is "*c*', either in the direction of rotation or the opposite direction.

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b. Since the speed of both light beams with respect to the rotating platform is always “*c*”, after following symmetrical paths, they must arrive simultaneously at the point from where they were initially emitted, so according to special relativity, an observer fixed within the platform does not predict any displacement of interference fringes, thus contradicting the experiment outcome.

We see that Engelhardt’s argument is straightforward and may seem unassailable for the unwary. Further, in his comment[14] to Engelhardt’s paper, Sfarti presented a derivation from an inertial frame as we explained in section 2, purportedly refuting Engelhardt’s argument. Although Sfarti correctly observed that the theory advocated by Engelhardt (Galilean relativity) had been widely falsified by now, Engelhardt rightly pointed out in his response[16] that Sfarti did not refute his specific argument, which we call EMA here.

The direct refutation of EMA is relevant and timely from a didactic viewpoint because it clarifies common misunderstandings sustained not only by those rejecting but also advocating the theory of relativity.

The next section explains why EMA fails to disprove special relativity when the LT is correctly applied to Engelhardt’s proposed scenario.

4. Rebuttal of EMA

What follows is well-known by relativists, albeit we must warn the non-specialist that a direct explanation of why the apparent contradiction raised by EMA fails is not as evident and intuitive as one might naively expect.

Owing that the rotating platform is a noninertial frame, it is often believed that observers within the rotating platform require the equivalence principle for a proper description of the phenomenon[17][18].

Although part of the formalism used in general relativity is indeed necessary to describe the phenomenon properly, a foundational analysis shows that, from a purely conceptual stance, we do not need to go beyond special relativity for establishing that formalism.

That special relativity can deal with noninertial reference frames was, for instance, explicitly proved by Logunov and Chugreev[19]. They based their proof on the sole postulate that spacetime is a Minkowskian manifold. That could be criticized as being too abstract and mathematical.

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Moreover, to deal with EMA directly, it is necessary to return to the most basic and immediate physical postulates. That is why we will base our arguments only on Einstein's 1905 approach to special relativity.

Before we do that, it is necessary to revisit the relationship between acceleration, gravitation, and inertial reference frames.

4.1. Relationship between noninertial frames, gravitation and special relativity

As revealed by the Pepino and Mabile study[5], there exists widespread confusion regarding the need for general relativity in the description of noninertial reference frames.

Of course, general relativity contains Minkowskian spacetime as a special case and can be used to deal with noninertial reference frames in flat spacetime. However, the discussion here is whether special relativity alone suffices to accomplish that.

A possible source of confusion may be that general relativity is commonly taught axiomatically. A constructive approach based on heuristic arguments and foundational concepts has only a historico-philosophical interest and is, in a sense, irrelevant, since it leads to the same mathematical formalism.

Thus, it is common to approach noninertial motion invoking the equivalence principle to equate inertial forces to a certain "fictitious" gravitational field, hence allowing the use of the formalism of general relativity once a gravitational field is present[18].

However, we shall see that it is possible to describe noninertial frames without invoking general relativity and Einstein Equivalence Principle (EEP). Noninertial frames can be wholly described with special relativity by "implicitly" assuming that an accelerated frame is locally equivalent to a comoving inertial frame.

Relativistic gravitation is "solved" by the EEP because a gravitational field is "locally" equivalent to an accelerated frame. At the same time, the last frame is equivalent to a freely falling frame, which is a comoving inertial frame. That is why Einstein dubbed the equivalence principle "The happiest thought of my life"[20] allowing him to construct general relativity through the local application of special relativity to noninertial frames.

It is remarkable that already in 1905 in his first relativity paper, Einstein extended special relativity to accelerated frames when explaining that *"It is at once apparent that this result still holds good if the clock moves from A to B in a polygonal line, and also when the points A and B coincide"*[21]. His obvious reasoning is

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that he was partitioning a closed continuous path into a finite number of polygonal sections, considering each section in inertial motion.

In other words, Einstein was approaching accelerated motion as mathematicians rectify a curve that is not a straight line, only that mathematicians introduce an independent definition while Einstein's heuristic physical mind considered it as evident and *"at once apparent"* instead of introducing the procedure as a new principle, a principle that Mashhoon appropriately called "The Hypothesis of Locality (HL)"[22].[2]

On the other hand, if we use the EEP to produce a fictitious gravitational field to describe acceleration through general relativity, we are reversing the conceptual steps that allowed Einstein to arrive at general relativity.

To restrengthen our arguments, besides the authoritative references given by Pepino and Mabile[5], we quote two more. One by Einstein himself[23]:

> *The general theory of relativity rests entirely on the premise that each infinitesimal line element of the spacetime manifold physically behaves like the four-dimensional manifold of the special theory of relativity. Thus, there are infinitesimal coordinate systems (inertial systems) with the help of which the ds are to be defined exactly like in the special theory of relativity. The general theory of relativity stands or falls with this interpretation of ds. It depends on the latter just as much as Gauss' infinitesimal geometry of surfaces depends on the premise that an infinitesimal surface element behaves metrically like a flat surface element...*

The other one by Misner, Thorne, and Wheeler[24]:

> *A tourist in a powered interplanetary rocket feels "gravity." Can a physicist by local effects convince him that his "gravity" is bogus? Never, says Einstein's principle of the local equivalence of gravity and accelerations. But then the physicist will make no errors if he deludes himself into treating true gravity as a local illusion caused by acceleration. Under this delusion, he barges ahead and solves gravitational problems by using special relativity.*

The key point above is "... and solves gravitational problems by using special relativity" because he is deluded into believing that true gravitation is acceleration; i.e., he treats acceleration with special relativity, and that allows him to describe gravitation through the EEP. Therefore, it is acceleration that

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explains gravitation, not the other way around. Thus, from this heuristic viewpoint, using gravitation to describe acceleration leads to circular reasoning.

Unfortunately, the heuristic insight given by Misner, Thorn, and Wheeler above is lost when we first learn the theory axiomatically. However, that insight is the reason why the experts quoted by Pepino and Mabile assert that general relativity is not necessary to explain the Twin Paradox and that the widespread belief in the need for general relativity because acceleration is involved is misleading.

Next we compare the explicit use of the HL when applying the EEP with its implicit application. Let at a given spacetime point, G stand for the gravitational field, LAF for local accelerated reference frame, CIF for comoving inertial reference frame, and FFF for freely falling reference frame. Then we have the following locally valid equivalences,

$$G \xrightarrow{EEP} LAF \xrightarrow{HL} CIF \equiv FFF \qquad (4)$$

On the other hand, when implicitly assuming the HL, the above chain of implications reduces to,

$$G \xrightarrow{EEP} FFF \qquad (5)$$

The short implication (5) overlooks the fact that EEP only allows the replacement of gravitation by an accelerated reference frame and that is the application of the HL that finally allows the substitution of the accelerated reference frame by a comoving inertial reference frame represented by a freely falling reference frame.

4.2. Formal description

The fact that the equivalence between acceleration and inertial motion is only local and infinitesimal, introduces serious consequences for the global interpretation of spacetime properties in a noninertial reference frame. Consequently, the equivalence between gravitation and acceleration, postulated as the EEP, is also a local principle, and its loose application can easily lead to incorrect inferences[25][26].

The local properties of spacetime referred to an inertial reference frame is expressed in special relativity with the invariant interval ds,

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \qquad \text{(repeated indices are summed)} \qquad (6)$$

where $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$. The fact that $\eta_{\mu\nu}$ are constants is a consequence of the homogeneity and isotropy of space time.

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In a noninertial reference frame (or in a gravitational field through the EEP), special relativity has only a local validity and the usual global spacetime properties are lost. This fact is formally expressed by a change in the expression of the metric,

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{7}$$

where $g_{\mu\nu}(x^0, x^1, x^2, x^3) \neq const.$

As a consequence, in noninertial reference frames, the intuitive meaning of the inertial coordinates are no longer valid, our everyday Newtonian intuition of global spacetime properties are lost and puzzling effects arise, such as, 3-space is no longer Euclidean, distant simultaneity and distant synchronization of clocks lose their meanings even in a fixed reference frame, see for instance the discussion in Refs.[17][27].

The interpretative problems posed to our intuition by the above observation should not be taken lightly. Here again, we quote Einstein on this point[28]:

> *All this happened in 1908. Why were another seven years required for the construction of the general theory of relativity? The main reason lies in the fact that it is not so easy to free oneself from the idea that coordinates must have a direct metric significance.*

Note that although Einstein is referring to gravitation theory based on EEP, the same observation is valid for the description of noninertial frames without gravitation based on HL, the interpretative difficulties arise from the “only local” applicability of special relativity and the LT irrespective of the existence of a gravitational field. It is not a minor point that Einstein considered the difficulties with interpreting spacetime coordinates to be the “main reason” for the complications encountered when constructing the general theory of relativity.

All that means we can no longer naively interpret differences of space coordinates as directly representing physical distances, and what is even perhaps most puzzling, differences of the time coordinate cannot be straightforwardly interpreted as the time indicated by physical clocks synchronized and distributed throughout space as if spacetime were a homogeneous and isotropic abstract entity.

Notwithstanding the difficulties mentioned above, there is a concrete method for interpreting local coordinates. This method is based on the physical interpretation of the invariant differential interval ds.

For the case that concerns us, i.e., the prediction of the Sagnac effect from within the rotating platform, we transform from the coordinates in the inertial frame with origin in the disc center (ct', r', ϕ', z') to coordinates in the rotating platform (ct, r, ϕ, z),

$$t=t', \qquad r=r', \qquad \phi=\phi'-\Omega t, \qquad z=z' \tag{8}$$

Here we remark a usual misinterpretation. As we noted in section 3, Engelhardt claimed that when Ashby used transformation (8) in[15], he (Ashby) was using a Galilean transformation instead of a LT because $t=t'$.

However, that is an incorrect interpretation of the time coordinate, which is shared by others who reject relativity theory[29]. When referred to an arbitrary (noninertial) reference frame, the time coordinate is a mere parameter used to calculate the actual proper time measured by physical clocks in that frame. Although according to transformation (8), the time coordinate t can be interpreted as the time of the background inertial frame, inside the platform it is only an auxiliary parameter that allows the calculation of the true time indicated by physical clocks attached to the rotating platform.

Next, we present a concise relativist calculation of the Sagnac effect from within the rotating frame. For a more detailed exposition, we refer the reader to Ref.[1].

As we already observed before, the physical meaning of the local coordinates is obtained through the expression of the general metric,

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu \tag{9}$$

In the case of the rotating platform, according to (8) we have,

$$ds^2=\underbrace{\left(1-\beta^2\right)}_{g_{00}}c^2dt^2-dr^2-r^2d\phi^2-dz^2-2\beta rc\,dt\,d\phi \tag{10}$$

where $\beta=v_0/c$, $v_0=r\Omega$. The time elapsed on a physical clock fixed in the same position inside the rotating platform is obtained from (10) by putting, $ds=cd\tau$, and $dr=d\phi=dz=0$,

$$\tau_2-\tau_1= \int_{x_1^0}^{x_2^0}\frac{\sqrt{g_{00}}}{c}\,dx^0 \tag{11}$$

$$\tau_2-\tau_1= \int_{t_1}^{t_2}\frac{dt}{\gamma} \tag{12}$$

To evaluate the Sagnac effect, we need to calculate the elapsed times τ^+ and τ^- in the location of the interferometer.

According to HL the light paths must satisfy $ds=0$, which for $r=R$ leads to,

$$dt^+ = +\frac{Rd\phi^+}{c(1-\beta)}, \qquad d\phi^+>0, \text{ (forward beam)} \tag{13}$$

$$dt^- = -\frac{Rd\phi^-}{c(1+\beta)}, \qquad d\phi^-<0, \text{ (backward beam)} \tag{14}$$

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Replacing (13) in (12),

$$\tau^{+} = \int_0^{2\pi} \frac{1}{\gamma} \frac{R d\phi^{+}}{c(1-\beta)} \tag{15}$$

$$= \frac{1}{\gamma} \frac{L}{c(1-\beta)} \tag{16}$$

Analogously, from (14) and (12),

$$\tau^{-} = \int_0^{-2\pi} -\frac{1}{\gamma} \frac{R d\phi^{-}}{c(1+\beta)} \tag{17}$$

$$= \frac{1}{\gamma} \frac{L}{c(1+\beta)} \tag{18}$$

The interference fringes are determined by the difference,

$$\tau^{+} - \tau^{-} = \frac{2Lv_0}{c^2\sqrt{1-(\frac{v_0}{c})^2}} \tag{19}$$

which is the same as (2) and equivalent to the classical Newtonian result to first-order.

Finally, we remark another common mistake. It is often believed that from (13) and (14) we obtain speeds of the light beams measured inside the platform equal to $R d\phi^{\pm}/dt^{\pm} = c \mp v_0$. However, the correct derivation according to the general metric formalism gives the invariant value c, coincident with the relativistic addition law for velocities (cf. Ref.[1] for a detailed explanation).

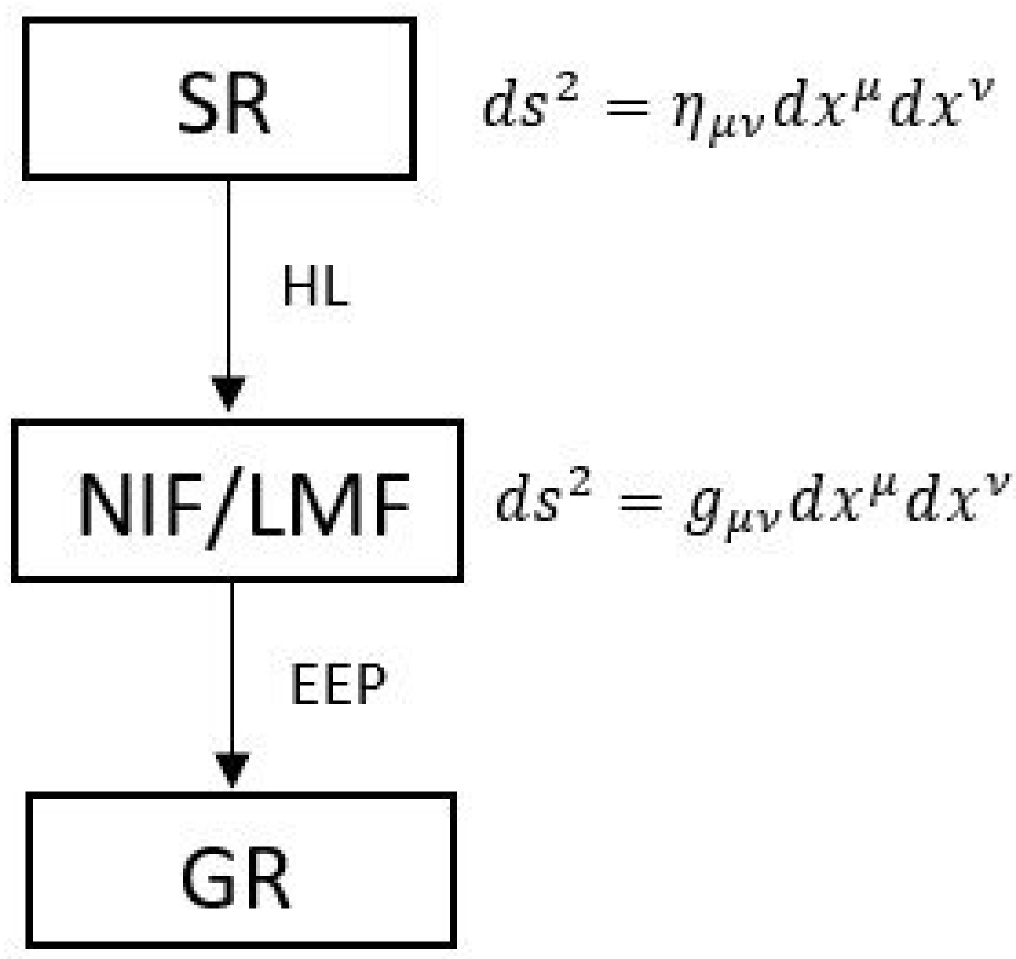

Figure 1. SR ≡ special relativity; HL ≡ hypothesis of locality; NIF ≡ noninertial reference frame; LMF ≡ local metric formalism; EEP ≡ Einstein equivalence principle ; GR ≡ general relativity

5. Conclusions

The correct local application of the LT proves that observers distributed along the rim of the platform and rotating with it measure a speed of light equal to c. That makes part a) of EMA correct.

But, part b) of EMA is incorrect because, within a noninertial frame, a consistent local application of special relativity leads to a spacetime not globally isotropic and homogenous.

For observers fixed to the platform, spacetime relations are determined by the differential interval,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{20}$$

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We call this method “Local Metrical Formalism (LMF)”. Figure 1 shows the relationship among special relativity, its local application to noninertial reference frames through the HL realizing the LMF, and the EEP to describe gravitation.

Note that the implementation of the metric formalism does not need the EEP, but that the latter is necessary for applying the metric formalism to gravitation. The difference between a true gravitational field and the one mimicked by noninertial motion is that the former cannot be made to disappear with a single LT, which is mathematically expressed through the nonvanishing of the Riemann tensor.

When we apply the LMF to the propagation of light in the rotating platform, the mean speed of light measured with one local clock in a forth and back finite journey is not equal to the instantaneous speeds measured with fixed clocks placed along their path.

A similar effect also takes place in the gravitational field of the sun. It is known as the Shapiro effect and is considered the fourth classical test of general relativity[30]. This analogy is also mentioned in Ref.[31].

Furthermore, for the concrete case of a “stationary spacetime” like the rotating platform, the clockwise and counterclockwise directions are not equivalent[32], which explains the difference between τ^+ and τ^-.

A consistent “local” application of special relativity is incompatible with the global properties naively suggested by the scale of our everyday human experience of slow velocities, near “inertiality”, and the weak Earth’s gravitational field.

Although human intuition is also valuable, physicists have learned that sometimes it must give way to strict rational thinking and empirical evidence, accepting nature as it is.

Footnotes

[1] Sagnac was an opponent of relativity. Although that was comprehensible in the early years after 1905, surprisingly, some paradigmatic examples of relativity denial persisted much later[33].

[2] A particular instance of the HL is “the clock hypothesis.”

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