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Peer Review

Review of: "Coexistence of Ergodic and Non-Ergodic Behaviour and Level Spacing Statistics in a One-Dimensional Model of a Flat Band Superconductor"

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Dear Editor,

The manuscript deals with superconductivity in flat-band systems. The paradigmatic model is the twisted bilayer graphene at the magic angle, and the problem now is to understand if the excitations behave or not ergodically. Due to conservation laws, there might be local integrals of motion that hinder thermalization. In order to inquire into this point, the authors construct a one-dimensional model and argue that it is related to other models that are related to the two-dimensional superconductivity problems mentioned above. Then the authors perform an exact-diagonalization ETH analysis to study the range where the system breaks quantum chaos and ergodicity. The model has an unperturbed integrable part and some integrability-breaking terms, and the question is for which value of the strength of these terms the system becomes ergodic.

The first thing a reader can notice is that the manuscript is quite verbose. The discussion gets lost in many details, and one has the impression that a point is never reached. Many things could be put in the appendix in order to make the reading sharper and smoother. (For instance, is it really necessary to put the extensive discussion of the symmetries in the main text?) The Introduction is almost three pages long and should be shortened, putting, for instance, many of those details in the initial sections.

I am not an expert in unconventional superconductivity – and I am sure you have also asked the opinion of an expert on that subject --, so I will express opinions only about the ETH analysis. Here, there are some things that can be improved. First of all, the level-spacing ratio versus \lambda_2 analysis (see Fig. 10,11) should be done for many system sizes in order to perform a finite-size scaling and see if the ergodicity breaking is a finite-size effect or is something robust in the thermodynamic limit. (This analysis is performed in Ref. [42], for instance.)

Another important thing that should be considered is the half-system entanglement entropy of the eigenstates. This is a very powerful tool to see if ETH is respected (see, for instance, PhysRevB.102.144302) and if there are isolated nonthermal eigenstates, the so-called quantum scars, that in a system full of symmetries like the one analyzed can possibly occur. Quantum scars are isolated eigenstates with very low half-system entanglement entropy, significantly lower than the ETH

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value (see Fig. 3 of [PhysRevB.98.155134], Fig. 2 of [Annual Review of Condensed Matter Physics 14, 443 (2023)] and Fig. 2 of [PhysRevB.106.035123]). Such low-entanglement states are most likely to occur when the perturbation is small, and the degeneracy in the unperturbed multiplets (see Figs. 3,4) is broken without mixing them. States at the lower and upper boundary of the perturbed multiplets have lower entanglement and might behave as quantum scars [PhsRevB 104, 094309 (2021), arXiv:2309.12504].

Other nice things to look at could be the scaling with the system size of a smoothness indicator (the average of the absolute value of the difference of the density expectations on nearby eigenstates – see PhysRevB.82.174411). This quantity directly probes ETH and can distinguish between full eigenstate thermalization, many-body localization, and the presence of quantum scars [Phys. Rev. B 99, 161101 (2019)].

They might also look at the scaling with the system size of the average inverse participation ratio of the eigenstates (see, for instance, PhysRevB.102.144302). This quantity indirectly probes ETH by indirectly looking at the delocalization of energy eigenstates on a basis of eigenstates of the unperturbed integrable Hamiltonian (in their case \mathcal{H}_1).

In summary, what is missing in the ETH analysis is a scaling with the system size, in order to assess if ergodicity breaking persists in the large-size limit. This is usually a very interesting question in ETH studies, being ergodicity breaking the exception rather than the rule, especially at large system sizes. It is also missing an analysis of quantities (like the entanglement entropy or the smoothness quantifier) that can probe the presence of quantum scars. Due to the many symmetries, I think that these analyses based on exact diagonalization can be pushed up to quite large system sizes.

From the point of view of the unconventional superconductivity aspect, I can express only a non-expert opinion. As a non-expert, I would like to say that the discussion seems somewhat heavy and full of details, and I feel it gets lost in one thousand streams. I suggest again to shorten and sharpen the discussion around the main point, putting many details in an appendix. In the present form, the manuscript seems not interesting even for non-experts.

Declarations

Potential competing interests: No potential competing interests to declare.

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