

Peer Review

Review of: "A Quasilinear Algorithm for Computing Higher-Order Derivatives of Deep Feed-Forward Neural Networks"

Pushkar Khairnar¹

1. Michigan Technological University, United States

The paper presents a well-structured and thorough investigation into the limitations of autodifferentiation for high-order derivatives in PINNs and proposes *n-TANGENTPROP* as an efficient alternative. The methodology is clearly explained, and the author support their claims with theoretical insights and empirical validation. The inclusion of an appendix and an open-source GitHub repository enhances the transparency and reproducibility of the work.

The author successfully addresses the computational inefficiencies of autodifferentiation in high-order derivative calculations, providing a concrete solution that enhances the practicality of neural networks for solving differential equations. Their approach directly tackles the exponential runtime problem, making higher-order derivative computations more feasible.

The theoretical foundation of *n-TANGENTPROP* is thoroughly explained, with a structured presentation of the algorithm's implementation details. The paper offers a clear and rigorous breakdown of the method, ensuring that readers can understand both the mathematical underpinnings and the practical execution of the approach.

The author provides strong empirical validation, demonstrating that *n-TANGENTPROP* significantly outperforms traditional autodifferentiation in computing higher-order derivatives. Their experiments highlight the efficiency of the proposed method, particularly in resource-limited environments where traditional approaches struggle.

This work has important implications for PINNs, as improving training efficiency enables PINNs to scale to more complex and computationally demanding problems. By reducing computational

bottlenecks, *n*-TANGENTPROP enhances the feasibility of using PINNs for high-order solutions, contributing to their broader applicability.

Finally, the open-source contribution of providing code on GitHub is a valuable addition, allowing for further exploration and adoption of the proposed method. By making their implementation publicly available, the author encourage transparency, reproducibility, and further advancements in the field.

The paper is well-structured and technically sound, with a few minor refinements that could improve clarity and presentation.

Suggested Improvements:

1. **Figure Labeling Issue** – In *Figure 1, 2 and 3*, the caption states that the bottom frame is plotted with a logarithmic axis, but the labeling seems unclear. The distinction between the two plots should be explicitly marked in the figure or clarified in the caption.
2. **Explicit Definition of *ad/ntp* Ratio** – While the paper defines the *autodifferentiation / n-TANGENTPROP ratio*, it would be clearer if this full form were explicitly stated upon first mention.
3. **Typographical Error** – In *Figure 6*, line 5 of the caption contains a typographical error ("pannel" should be "panel").

Overall, this paper presents a significant contribution to the field of PINNs by providing an efficient alternative to traditional autodifferentiation for high-order derivatives. The work is well-motivated, rigorously analyzed, and supported by strong empirical results. Addressing the minor issues mentioned above would further enhance clarity and presentation.

Declarations

Potential competing interests: No potential competing interests to declare.