On Qubits and Quantum Information Technologies

Guang-Liang Li

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Guang-Liang Li
University of Hong Kong
glli@eee.hku.hk

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Abstract

The basic assumption underlying various quantum information technologies, such as quantum computation and quantum communication, is that qubits are physical objects. However, this assumption is false. A qubit is a quantum superposition, which is a purely mathematical entity in a Hilbert space for description of a two-level quantum system. There is no theoretical or experimental evidence for quantum superpositions in general, and for qubits in particular, having physical counterparts in the real world, although quantum systems themselves are all physical objects. Nevertheless, quantum superpositions failing to be physical objects does not necessarily imply quantum mechanics failing to be correct. Hilbert spaces differ essentially from the three-dimensional Euclidean space; the latter is the model of space in which we live and measure physical quantities. Associated with time or space coordinates, measurements of all quantum systems must be performed in the real world. However, precise time and space coordinates are not attainable by measurement. Based on a comparison between the Euclidean space and Hilbert spaces, this paper shows that no quantum systems in the real world can realize qubits, and hence, quantum information technologies are not physically realizable; their so-called advantages over classical information technologies make little sense.

Keywords: Qubit, Quantum computation, Quantum communication, Quantum information, Unattainability of precise time and space coordinates

1 Introduction

For so-called quantum information technologies, such as quantum computation and quantum communication, the notion of “quantum bit”, or “qubit” for short, plays a role purportedly analogous to the well-known notion of “bit” in classical information technologies [1]. Unlike a bit, which represents classical information and is either 0 or 1 but cannot be both simultaneously, a qubit, treated as a physical object for representing so-called quantum information, is supposed to be both 0 and 1 at the same time, purportedly making quantum information technologies much superior to their classical counterparts. The so-called advantages of quantum information technologies sound very attractive. In the past several decades, various attempts to realize such technologies consumed a huge amount of funding and investment.

Expressed by an abstract vector (α, β) in a two-dimensional Hilbert space over the field of complex numbers, where α and β satisfy

$$|\alpha|^2 + |\beta|^2 = 1,$$

a quantum superposition, denoted by $|\psi\rangle$, is a purely mathematical entity, which represents the general state of a two-level quantum system before measurement in current quantum theory. In the literature of quantum computation and quantum information [1], the general state $|\psi\rangle$ is referred to as a qubit, given by

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{1}$$

with mutually exclusive properties of the system corresponding to the orthonormal basis vectors denoted by $|0\rangle$ and $|1\rangle$. The real numbers $|\alpha|^2$ and $|\beta|^2$ are probabilities of finding the system in a specific state when a measurement is performed. The state is $|0\rangle$ with probability $|\alpha|^2$; otherwise it is $|1\rangle$ with probability $|\beta|^2$.

In general, a Hilbert space describes a quantum system in mathematical terms; its elements are vectors representing various possible states of the system. Thus, as a purely mathematical entity, a quantum superposition merely serves as a mathematical description of a quantum system in current quantum theory, while not necessarily having a physical counterpart in the real world. Nevertheless, quantum superpositions failing to be physical objects must not be considered as quantum mechanics failing to be correct. Needless to say, the correctness of quantum mechanics, which is an extremely successful scientific theory with many applications in practice, will not be affected at all by incorrectly interpreting quantum superpositions as physical objects.

However, by treating qubits as physical objects, some two-level quantum systems are widely believed to be capable of realizing qubits for processing quantum information in quantum computation and quantum communication. The door to such quantum information technologies is opened by Bell experiments for testing Bell inequalities against quantum mechanics [2]. The entangled state in Bell experiments is given by a quantum superposition. The legitimacy of treating quantum entanglements or quantum superpositions in general as physical entities should have been tested by Bell experiments; unfortunately, in Bell experiments, it is taken for granted [3]. This is a fatal logical flaw and largely responsible for ineligible applications of quantum mechanics, including so-called quantum information technologies.

This paper aims to show that no quantum systems in the real world are capable of realizing qubits. Needless to say, quantum systems themselves are all physical objects, but quantum superpositions in general, and qubits in particular, have no physical counterparts in the real world. The above conclusion is a consequence of a well-established mathematical fact following from the properties of the three-dimensional Euclidean space, i.e., *precise time and space coordinates are not attainable by measurement.* Unfortunately, this important fact is entirely omitted in current quantum theory. After all, instead of Hilbert spaces, the model of space in which we live and measure physical quantities is the Euclidean space. Nevertheless, for measurements of macroscopic objects, the unattainability of precise time and space coordinates is hardly noticeable and may be safely ignored.

In Section 2, the Euclidean space and Hilbert spaces are compared, then based on the comparison, the omission of the unattainability of precise time and space coordinates in current quantum theory is elucidated. In Section 3, the reason why neither single quantum objects nor composite quantum systems can realize qubits is further elucidated. In Section 4, the paper is concluded with a brief discussion of the presented results and future studies.

2 Euclidean Space and Hilbert Space

The unattainability of precise time and space coordinates is a well-established mathematical fact, which follows from the properties of metric topologies for the three-dimensional Euclidean space and its subspace equipped with the corresponding distance functions. The mathematical model of space in which we live and measure physical quantities is the Euclidean space, endowed with a metric that is the distance function d between two arbitrary points $\mathbf{r} = (r_1, r_2, r_3)$ and $\mathbf{r}' = (r'_1, r'_2, r'_3)$. By adopting the statement given in the Theorem of Pythagoras, the distance function is defined as

$$d(\mathbf{r}, \mathbf{r}') = \sqrt{(r_1 - r'_1)^2 + (r_2 - r'_2)^2 + (r_3 - r'_3)^2}.$$

By definition, $d(\mathbf{r}, \mathbf{r}') = 0$ if and only if $\mathbf{r} = \mathbf{r}'$. Similarly, the mathematical model of time elapsed in the real world is the set of nonnegative real numbers. This set is typically represented by the interval $[0, \infty)$. For ease of exposition, the interval might be considered as a subspace of the Euclidean space endowed with the corresponding metric, namely, the distance function given by $|s–t|$, where s, t are nonnegative real numbers.

Of course, the Euclidean space and its subspace endowed with the corresponding distance functions are not Hilbert spaces equipped with metrics induced by inner products. Hilbert spaces are models for description of quantum systems. Evidently, metrics associated with various Hilbert spaces in quantum mechanics differ essentially from the distance functions defined for the Euclidean space and its subspace. Needless to say, all physical quantities are necessarily measured in space and time in the real world rather than in Hilbert spaces devised for describing quantum systems.

Although probabilistic predictions of quantum mechanics formulated based on Hilbert spaces are always in agreement with measurement results obtained by experiment, Hilbert spaces cannot capture the unattainability of precise time and space coordinates. By definition, elements of a Hilbert space in quantum mechanics are possible states of a quantum system rather than points with precise coordinates in the Euclidean space or its subspace.

On the other hand, if a physical quantity concerning a quantum system can be measured, it is necessary for the measurement of the physical quantity to be associated with precise time or space coordinates; the association can be implicit, in the sense that the time or space coordinates may not necessarily appear explicitly in the state function of the system. For instance, when the space coordinates represent a direction along which a quantum object (such as a photon) moves in space, or an orientation of an apparatus (such as a polarizer for measuring the polarizations of photons), the time coordinate usually will not appear in the state function explicitly.

Nevertheless, while the state function may or may not depend on a time or space variable explicitly, the physical quantity is necessarily measured in time and space. In general, measurements of quantum systems must be explicitly associated either with time or with space. This fact, together with the unattainability of precise time and space coordinates, is crucially important for us to understand why qubits are not physical objects, as will be further elucidated in the next section.

Clearly, no metric in any Hilbert space is devised to measure time or space. In contrast, the distance function adopted from the Theorem of Pythagoras is the only available tool for us to measure the distance between two arbitrarily given points in space. Similarly, the length of an arbitrarily given time interval must be measured by using the corresponding metric, namely, the distance function defined for a subspace of the Euclidean space consisting of all nonnegative real numbers. Now, it is not difficult to see why the unattainability of precise time and space coordinates is entirely omitted in the current quantum theory: Precise time or space coordinates associated with measurements of quantum systems are taken for granted!

As can be readily seen, because precise time and space coordinates are not attainable by measurement, which is exactly the cause of quantum randomness exhibited in outcomes obtained in experiments with quantum objects, quantum mechanics is not inherently probabilistic; the omission of the unattainability of precise time and space coordinates can explain why predictions of quantum mechanics are random rather than deterministic, although quantum-mechanical predictions are always correct.

Unfortunately, omitting the unattainability of precise time and space coordinates already resulted in serious consequences nowadays: In current quantum theory, quantum superpositions describing some two-level quantum systems are mistaken for physical objects with a glamorous new name called “qubits”, as if they were physical resources “out there” in the real world to be exploited for developing so-called quantum information technologies.

3 Quantum Systems and Qubits

In the literature of quantum computation and quantum information [1], two mutually exclusive properties of a quantum system are considered as corresponding to the superposed states of a qubit needed to process so-called quantum information. The system may be either a single quantum object or composed of multiple quantum objects. In this section, it is shown that neither single quantum objects (Subsection 3.1) nor composite quantum systems (Subsection 3.2) can realize qubits.

3.1 Single Quantum Objects and Qubits

The three-dimensional unit sphere, called the Bloch sphere, is considered in the literature of quantum computation and quantum information as a geometric representation of a single qubit in general [1]. With this geometric representation, (1) can also be expressed as

$$|\psi\rangle = \frac{\cos\theta}{2}|0\rangle + e^{i\varphi}\frac{\sin\theta}{2}|1\rangle$$

after ignoring a global phase factor. Thus, $|\psi\rangle$ is represented by a point (θ, φ) on the sphere. However, this geometric representation and various operations performed on $|\psi\rangle$ illustrated within the picture of the Bloch sphere are all irrelevant to actual measurements of quantum systems in the real world.

To see this, let $|0\rangle$ and $|1\rangle$ be $|\uparrow\rangle$ and $|\downarrow\rangle$, respectively, which are the eigenvectors belonging to the Pauli spin matrix

$$\hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The eigenvectors span a two-dimensional Hilbert space in which a spin-1/2 particle lives. Consider the direction represented by an arbitrarily given z-axis in the notation $\hat{\sigma}_z$. Clearly, this direction must be specified by the corresponding coordinates in the Euclidean space rather than in the Hilbert space. Actually, every direction in the space modeled by the Euclidean space can be determined by the coordinates of a unique point $\mathbf{r}$ on a unit sphere D. Having nothing to do with the Bloch sphere, the unit sphere D is a subset of the Euclidean space given below.

$$D = \{\mathbf{r} : d(\mathbf{r}, 0) = 1\},$$

where $d(\mathbf{r}, 0)$ is the distance between $\mathbf{r}$ and the origin. Evidently, the distance function d is not the metric induced by the inner product $\langle\cdot|\cdot\rangle$ defined for the corresponding Hilbert space; where $\langle\cdot|\cdot\rangle$ is an ordinary scalar product of vectors, except the entries in $\langle\cdot|$ are complex conjugated.

Equipped with the distance function d, the Euclidean space is a metric space with its open subsets forming a metric topology. The neighborhood of a point $\mathbf{r}$ is a subset $V(\mathbf{r})$ of the Euclidean space, such that $\mathbf{r} \in U \subset V(\mathbf{r})$, where U is a member of the metric topology. For every sufficiently small real number $\gamma > 0$,

$$V(\mathbf{r}) \cap B(\mathbf{r}, \gamma) = B(\mathbf{r}, \gamma) \neq \{\mathbf{r}\},$$

where $B(\mathbf{r}, \gamma)$ is an open ball with center $\mathbf{r}$ and radius γ, such that $d(\mathbf{r}, \mathbf{r}') < \gamma$. Consequently, so long as $\gamma > 0$, there always exist uncountably many points different from and arbitrarily close to $\mathbf{r}$ in $B(\mathbf{r}, \gamma)$, and hence the distance between each of such points and $\mathbf{r}$ given by d is strictly greater than zero. In other words, $\mathbf{r}$ is not an isolated point of every $V(\mathbf{r})$. Note that $\mathbf{r}$ considered above can be arbitrary.

As a result, by using the distance function d, we cannot obtain any desired point $\mathbf{r}$. Instead of the desired point $\mathbf{r}$, we can only obtain a neighborhood $V(\mathbf{r})$ as an approximation. The approximation is, at best, an infinitesimal spatial volume. In this sense, precise coordinates are unattainable. Nevertheless, the approximation is much better than the omission of the unattainability of precise coordinates while taking such coordinates for granted in current quantum theory.

To be specific and without loss of generality, consider a sequence $(q_k)_{k\geq 1}$ of identically prepared spin-1/2 particles as an example. According to quantum mechanics in its current form, when described by the state given by the quantum superposition $|\psi\rangle$, any particle in $(q_k)_{k\geq 1}$ has no definite spin in any direction but has two states $|\uparrow\rangle$ and $|\downarrow\rangle$ in every direction before measurement. For arbitrarily given particles q_i and q_j in $(q_k)_{k\geq 1}$ where $i \neq j$, measuring the spins of q_i and q_j along purportedly the same direction specified by the z-axis yields two deterministic outcomes, which may not necessarily be identical, but correspond to either $|\uparrow\rangle$ with probability $|\alpha|^2$ or $|\downarrow\rangle$ with probability $|\beta|^2$.

As can be readily seen, the randomness exhibited in the outcomes is due to the unattainability of precise space coordinates. Because precise coordinates for representing directions in space cannot be obtained by measurement, the exact directions are approximated by small volumes containing the corresponding coordinates. The particles q_i and q_j are actually measured in unknown directions z_i and z_j with their precise but practically unattainable coordinates contained in the corresponding small volume, which also contains the coordinates of the z-axis; the directions z_i, z_j and z are almost surely different. Clearly, there is no experimental evidence for the claim that every single particle in $(q_k)_{k\geq 1}$ has two states in every direction. Therefore, such particles cannot realize qubits.

The above analysis also applies to quantum systems when their measurements are explicitly associated with time. As already mentioned, time elapsed in the real world is modeled by the set of nonnegative real numbers, which is a subspace of the Euclidean space equipped with the metric given by $|s - t|$, and an "open ball" with center $t > 0$ and radius γ is simply an open interval $(t - \gamma, t + \gamma)$, where $\gamma < t$. Therefore, the unattainability of precise time coordinates must be taken into account.

As elucidated above, mistaking qubits as physical objects is mainly due to omitting the unattainability of precise time and space coordinates. In addition to the unattainability of precise time and space coordinates, there is also an important physical constraint imposed on measuring single quantum objects: *The same single quantum object can at most be detected or measured only once.* Unfortunately, this constraint is violated by Bell inequalities, which attempt to provide a more complete description of the physical world than the quantum-mechanical description, although actual measurements performed in real experiments will not violate the constraint. Note the difference between Bell inequalities and Bell experiments; as an alternative description of the physical world, Bell inequalities are not Bell experiments. Note also the difference between "preparing quantum systems identically" and "measuring time or space coordinates precisely"; it is impossible to obtain precise time or space coordinates by measurement, no matter how identically quantum systems are prepared. Because the unattainability of precise time and space coordinates is omitted in the current quantum theory, *identically prepared quantum systems of the same kind are confused with the same quantum system.* Such confusion is also responsible for mistaking qubits for physical objects.

3.2 Composite Quantum Systems and Qubits

Unlike single quantum objects, composite quantum systems may be measured repeatedly without being destroyed. In general, measurements of such systems are explicitly associated with time. Because precise time coordinates are unattainable by measurement, which is sufficient to rule out the possibility of finding quantum systems that possess mutually exclusive properties *at the same time*, when we explain the outcomes of measuring such systems, the unattainability of precise time coordinates must not be omitted. Unfortunately, this is not the case.

Consider composite systems composed of non-Abelian anyons [4]. Systems of this kind can only exist, or more precisely, be constructed in two-dimensional spaces, which are subspaces of the three-dimensional Euclidean space. The construction is guided by numerical studies [4]. Using non-Abelian quantum phases of matter, such systems are purportedly capable of encoding qubits in a non-local manner, so it is widely believed that the systems might be used for so-called topological quantum computing, which could topologically protect quantum information not only from imperfections in the implemented protocols but also from interactions with the environment [5]. By encoding qubits in the so-called topologically-protected way, systems of this kind are considered as promising candidates for building topological quantum computers [6]. However, the possibility of using some specific systems of this kind (i.e., Majorana fermions in two-dimensional p + ip Fermi superfluids) for topological quantum computation has been questioned recently [7]. In fact, because qubits are not physical objects, no quantum systems in the real world have the capability of realizing or encoding qubits; composite systems are not exceptions, even though they can be repeatedly measured without being destroyed. Any effort to realize qubits is doomed to failure.

As purely mathematical entities in Hilbert spaces for description of quantum systems, quantum superpositions in general and qubits in particular do not necessarily have physical counterparts in the real world; there is no theoretical or experimental evidence for the opposite. Any claimed experimental evidence for qubits having physical counterparts in the real world stems from confusing mathematical entities with physical objects. For measurements of composite systems, the confusion is due to the omission of the unattainability of precise time coordinates. Therefore, topological quantum computation, just like all other quantum information technologies, is not an eligible application of quantum mechanics, and hence cannot be realized physically. Eligible applications of quantum mechanics will not involve anything in any way that does not exist physically.

4 Discussion and Conclusion

With a metric induced by an inner product defined for states of a quantum system, a Hilbert space in quantum theory is the model for the description of the system in terms of mathematics. Mathematically, each quantum system is described by a quantum superposition, which is a purely mathematical entity in a Hilbert space without any physical counterpart in the real world. Unfortunately, quantum superpositions for the description of some two-level systems are mistaken for physical objects called qubits. Nevertheless, quantum superpositions failing to be physical objects does not necessarily imply quantum mechanics, as a supremely successful scientific theory with a large number of practical applications, failing to be correct.

By assuming that qubits have physical counterparts in the real world, "quantum circuits" composed of various "quantum gates" are devised for processing so-called quantum information. Various performance indexes of quantum information technologies are all estimated based on the assumption that qubits are physical objects. Relying on Hilbert spaces for description of the corresponding quantum systems, the estimated indexes, such as the speed of quantum computing, the channel capacity of quantum communication, and the fidelity of quantum data compression, purportedly demonstrate the so-called advantages of quantum information technologies over their classical counterparts. However, because quantum information technologies are all based on the notion of qubit, which does not exist physically, such technologies are not eligible applications of quantum mechanics; by no means will eligible applications of quantum mechanics involve anything in any way that does not exist physically. Consequently, "quantum gates" and various estimated performance indexes of quantum information technologies are not physically meaningful.

The model of space in which we live and measure physical quantities is the Euclidean space equipped with the metric given by the usual distance function. The model of time elapsed in the real world is a subspace of the Euclidean space endowed with the corresponding metric, which is the distance function defined on the set of nonnegative real numbers. The metrics above are the only available tools for us to measure the distance between two points in space or the length of a time interval between two instants in the real world.

In sharp contrast to the distance function defined for the Euclidean space or its subspace, metrics induced by inner products defined for Hilbert spaces have little use for measuring spatial distances or lengths of time intervals elapsed in the real world. Because precise time and space coordinates are not attainable by measurement, we have to be content with some approximation, which is, at best, an infinitesimal spatial volume or an infinitesimal time interval. Compared with omitting the unattainability of precise time and space coordinates, having such an approximation is much better than taking precise but practically unattainable time and space coordinates for granted.

The state of a quantum system may also depend on some parameter, which is not a variable representing time or space, but its values form a continuum. For instance, the relative phases of different quantum states are parameters of this kind; their values are real numbers constituting a continuum. Just like precise time or space coordinates, precise values of continuous parameters cannot be obtained by measurement, but they are simply taken for granted. The unattainability of such values may be safely ignored in experiments with macroscopic objects. However, if the state of a quantum system relies on a continuous parameter, the following questions then arise themselves naturally: When such systems are measured by experiment, if the unattainability of precise values of the parameter is omitted, will the omission cause serious consequences? If the answer is yes, then what are the consequences? These questions are certainly important and should be answered by future studies.

References

[1] M. A. Nielsen and I. L. Chuang, **Quantum Computation and Quantum Information**, 2000, Cambridge University Press, Cambridge.

[2] A. Aspect, Closing the door on Einstein and Bohr's quantum debate, *Physics*, 8(2015), DOI: 10.1103/Physics.8.123.

[3] Guang-Liang Li, On Bell experiments and quantum entanglement, DOI: https://doi.org/10.32388/QTHWYK, Sep 4, 2023.

[4] Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, and Sankar Das Sarma, Non-Abelian anyons and topological quantum computation, *Review of Modern Physics*, 2008, DOI: 10.1103/RevModPhys.80.1083.

[5] Ady Stern and Netanel H. Lindner, Topological quantum computation – from basic concepts to first experiments, *Science*, 339 (6124), 1179-1184, 8 March 2013, DOI: 10.1126/science.1231473.

[6] Sankar Das Sarma, Michael Freedman, and Chetan Nayak, Topologically-protected qubits from a possible Non-Abelian fractional quantum hall state, *Physical Review Letters*, 94(16), 166802, 2005, DOI:https://doi.org/10.1103/PhysRevLett.94.166802.

[7] Yiruo Lin and A.J. Leggett, Some questions concerning Majorana fermions in 2D (p + ip) Fermi superfluids, *Quantum Frontiers*, 2022, DOI: https://doi.org/10.1007/s44214-022-00006-w.