Research Article

Conditions for Emergent Gravitational Light Bending from a Logarithmic Superfluid Vacuum

Boris Kulangiev[1]

1. Independent researcher

Purely scalar theories of gravitation predict a Parameterized Post-Newtonian (PPN) parameter $\gamma = 0$, yielding only half the observed light deflection. We trace this failure to a structural feature of real scalar wave equations: the characteristic propagation speed is fixed by the D'Alembertian operator and cannot depend on the background field, regardless of the self-interaction potential. We show that modeling the vacuum as a complex superfluid with a relativistic logarithmic nonlinearity, analyzed via the Madelung transformation, introduces two new degrees of freedom absent in the wave equation form: a density-dependent sound speed and a macroscopic flow velocity. Using the standard relativistic acoustic-metric formalism of Barceló-Liberati-Visser, we derive the sound speed $c_s^2 = [2\ln(\overline{\rho}/\rho_c) + 3]^{-1}$ from the logarithmic equation of state, demonstrating its dependence on the local vacuum density—an effect absent in the non-relativistic limit. We compute the PPN parameter for the static acoustic metric, finding $\gamma = (1-\alpha)/(1+\alpha)$ where $\alpha = d\ln c_s / d\ln\rho$. The logarithmic equation of state at the background density gives $\alpha = -1$, placing γ_{static} exactly at the pole of this expression: the static acoustic metric is not merely numerically wrong but mathematically ill-defined. We then demonstrate that a non-static acoustic metric with macroscopic vacuum flow regularizes this pathology and yields $\gamma = 1$ exactly, provided the flow velocity satisfies $v(r) = \sqrt{2G_{eff}M/r}$—the Painlevé-Gullstrand profile. Using a self-consistent Bondi accretion calculation, we show that the logarithmic equation of state does not naturally produce this profile in the far field, where the flow decays as r^{-2} rather than the required $r^{-1/2}$. We further show that the Painlevé-Gullstrand flow is the *only* self-consistent weak-field solution of the acoustic metric equations: the static metric is divergent, whereas the flowing metric closes the self-consistency loop for any barotropic fluid. The remaining open problem is to derive this macroscopic flow as a collective effect of the microscopic soliton dynamics.

 doi.org/10.32388/OWEUNJ.4

Correspondence: papers@team.qeios.com — Qeios will forward to the authors

1. Introduction

The bending of light by gravity is one of the most precise tests of gravitational theory. In the PPN formalism [1], the deflection of a light ray with impact parameter b by a mass M in the grazing-ray limit is

$$\delta\theta = \frac{(1+\gamma)\,2GM}{bc^2}, \tag{1}$$

where γ encodes the ratio of spatial to temporal curvature in the weak-field metric. General relativity predicts $\gamma = 1$; the Cassini spacecraft confirmed $\gamma = 1.000021 \pm 0.000023$ [2].

Purely scalar theories generically fail this test. Nordström's conformally flat theory gives $\gamma = -1$ [3]. A real scalar field with standard kinetic term gives $\gamma = 0$ [1]—Einstein's 1911 result [4], using only time dilation without spatial curvature. This half-deflection led to the conclusion that gravity requires a rank-2 tensor field.

In this paper, we examine whether a superfluid vacuum model—specifically, a complex scalar field with logarithmic self-interaction analyzed in the Madelung hydrodynamic representation [5][6]—can overcome this constraint. We find that the Madelung formulation introduces two structural ingredients absent in the wave equation form: (i) a density-dependent sound speed, and (ii) a macroscopic flow velocity. Both are needed for $\gamma = 1$. We derive the precise conditions required, and show via self-consistent computation that these conditions are not automatically satisfied, identifying the central open problem for superfluid vacuum gravity.

2. The Kinematic Trap: Why $\gamma = 0$ for Real Scalar Fields

Consider a real scalar field $D(x,t)$ satisfying

$$\Box D + V'(D) = J(x,t), \tag{2}$$

where $\Box = \nabla^2 - c^{-2}\partial_t^2$. We show that the geometric-optics propagation speed of perturbations on any background $\overline{D}(x)$ is identically c.

Linearizing $D = \overline{D} + \delta D$:

$$\Box(\delta D) + V''(\overline{D})\delta D = 0. \tag{3}$$

In the WKB limit $\delta D \sim Ae^{i\phi}$ with $|\nabla\phi| \gg |\nabla A/A|$, the eikonal equation is

 doi.org/10.32388/OWEUNJ.4

$$|\nabla\phi|^2 - \frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 = 0, \tag{4}$$

giving $v_{ph} = c$ everywhere. The term $V''(\overline{D})$ contributes only a subleading effective mass. Figure 1 confirms this numerically: a wave packet in a Klein-Gordon field propagates at identical velocity for background densities spanning an order of magnitude.

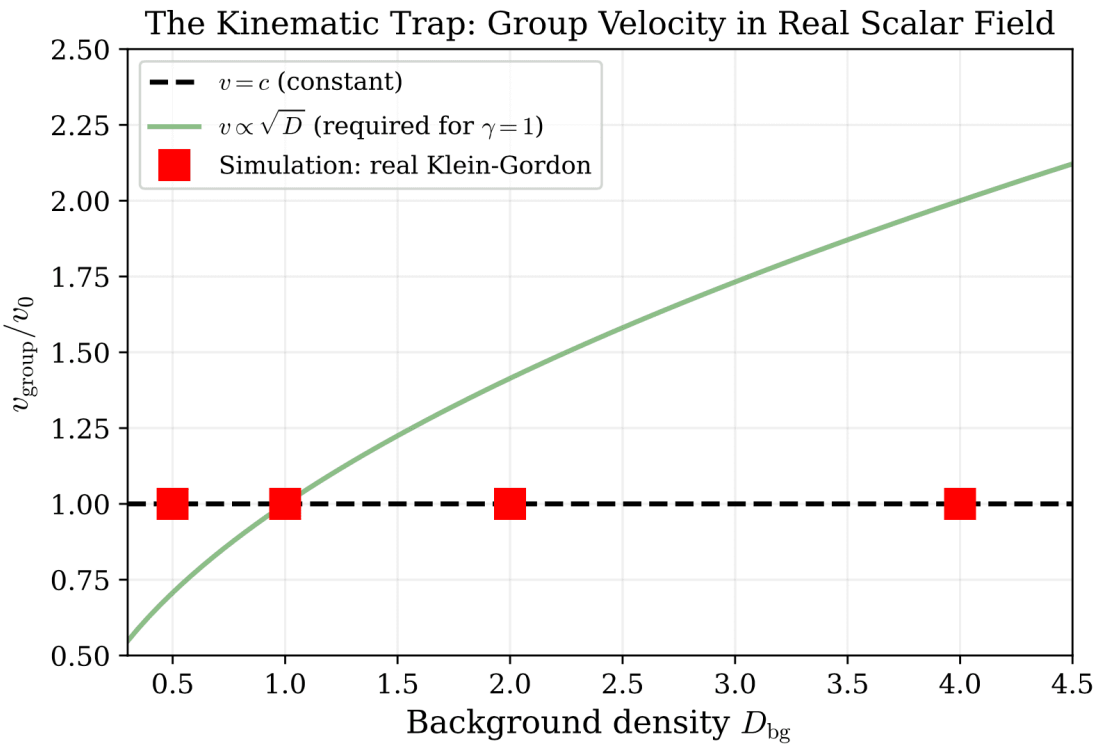


Figure 1. The kinematic trap. Group velocity of a wave packet in a Klein-Gordon field vs. background density D_{bg}. The velocity is constant (red squares), confirming Eq. (4). The green curve shows the density dependence needed for $\gamma = 1$.

Since the coordinate speed of light cannot vary with the background, the only deflection contribution comes from time dilation:

$$\delta\theta_{scalar} = \frac{2GM}{bc^2}, \tag{5}$$

giving $\gamma = 0$. This holds for *any* potential $V(D)$: the characteristic surfaces of (2) are determined by the principal symbol (the D'Alembertian), not by V. Escaping this constraint requires either modifying the

doi.org/10.32388/OWEUNJ.4

kinetic term, as in scalar-tensor theories [7], or reinterpreting the field in a formulation where the propagation speed emerges dynamically.

3. The Madelung Superfluid Formulation

Interpretation of the order parameter

We take a deliberately classical-field stance toward Eq. (6) below: $\psi(x,t)$ is a complex *order parameter* for a deterministic vacuum substrate, not a quantum-mechanical wavefunction in the standard sense. Its squared magnitude $|\psi|^2 = \rho$ is a number density of the substrate's microscopic constituents, not a probability density, and is not subject to the unitary normalization $\int |\psi|^2 d^3x = 1$. The Madelung decomposition $\psi = \sqrt{\rho} e^{iS/\hbar_{eff}}$ is therefore a hydrodynamic ansatz for a real classical fluid, in the spirit of de Broglie's pilot-wave program [8] and the Madelung-Bohm hydrodynamic interpretation of quantum mechanics [9][10][11]. The relativistic Lorentz-covariant form of Eq. (6) reflects an emergent symmetry of small fluctuations around the condensate's rest frame—identified physically with the cosmic vacuum frame in which the cosmic microwave background dipole vanishes—rather than an exact symmetry of the underlying theory. We do not specify a UV completion or a quantization procedure for the underlying field; the framework is studied here as a classical effective theory whose hydrodynamic predictions are compared with weak-field gravitational observations.

3.1. The logarithmic superfluid

We model the vacuum as a complex order parameter $\psi(x,t)$ evolving under [5][6]:

$$\Box\psi + \mu^2\psi - b\psi \ln\left(\frac{|\psi|^2}{\rho_0}\right) = 0. \tag{6}$$

The logarithmic nonlinearity produces Gaussian-localized solitons ("Gaussons") [12][13] and preserves the Landau roton-maxon spectrum [14].

Units of the order parameter

A clarification on dimensions. In the relativistic k-essence Lagrangian $\mathscr{L} = \rho X - V(\rho)$, with $X = \partial_\mu S \partial^\mu S$ and S having dimensions of action, the variable $\rho = |\psi|^2$ does not have units of mass density [kg/m^3]. Demanding $[\mathscr{L}] = [\text{energy}/\text{volume}]$ together with $[X] = [\text{momentum}^2]$ forces $[\rho] = [(\text{energy}\cdot\text{volume})^{-1}]$. The Lagrangian variable ρ is therefore an inverse-energy-volume quantity;

the physical mass density of the condensate, which is what enters the gravitational source term and what one would measure as kg/m^3, is a *distinct* quantity computed from the time-time component of the stress-energy tensor:

$$\rho_{mass} = \frac{T_{00}}{c^2} = \frac{\rho V'(\rho) + V(\rho)}{c^2}, \tag{7}$$

where the right-hand side is evaluated using the Lagrangian variable ρ and reduces, at the background, to a constant ρ_0^{phys}. We retain the conventional notation ρ for the order parameter throughout, but the reader should keep in mind that physical observables involving mass density use $\rho_{mass} = T_{00}/c^2$, not ρ itself. The Madelung decomposition below treats ρ as the field variable, with all subsequent equations dimensionally consistent under this identification.

3.2. The Madelung decomposition

Applying $\psi = \sqrt{\rho} e^{iS/\hbar_{eff}}$ and separating real and imaginary parts yields two coupled equations:

Continuity (conservation of vacuum flux):

$$\partial_\mu(\rho \partial^\mu S) = 0. \tag{8}$$

Hamilton-Jacobi (relativistic phase dynamics):

$$\frac{1}{\hbar_{eff}^2}\partial_\mu S \partial^\mu S = \mu^2 - b\ln\left(\frac{\rho}{\rho_0}\right) + \frac{\Box\sqrt{\rho}}{\sqrt{\rho}}. \tag{9}$$

The last term is the quantum potential (Bohm potential), negligible in the macroscopic limit. The fluid four-velocity is $u_\mu = \partial_\mu S/m_{eff}$, irrotational by construction except at topological defects (vortices).

The crucial point is that Eq. (9), unlike a wave equation, couples the phase gradient $\partial_\mu S$ to the density ρ through a constraint that depends on the local thermodynamic state. This coupling produces a density-dependent sound speed, as we now show.

Origin of the continuity equation

The d'Alembertian in Eq. (6) contains second-order time derivatives, so Eq. (6) is a wave equation, not directly a continuity equation. However, the Madelung decomposition $\psi = \sqrt{\rho} e^{iS/\hbar_{eff}}$ separates Eq. (6) into real and imaginary parts that have distinct physical content. The real part yields the relativistic Hamilton-Jacobi equation (9), which is second-order in time and governs phase dynamics. The imaginary part yields the first-order conservation law in Eq. (8) which is the relativistic continuity equation for the four-current $J^\mu = \rho\partial^\mu S/\hbar_{eff}$ associated with the global U(1) symmetry $\psi \to e^{i\theta}\psi$ of Eq. (6). In the rest

frame of the condensate, where $\partial_t S \approx -\hbar_{eff}\omega_0$ is approximately constant and spatial gradients are small, Eq. (8) reduces to the familiar non-relativistic continuity form $\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$ with $\mathbf{v} = \nabla S / m_{eff}$. The wave-equation character of Eq. (6) is therefore consistent with the existence of a first-order conservation law for the U(1) charge density.

4. Relativistic Sound Speed from the Logarithmic Equation of State

For a relativistic scalar field with Lagrangian density $\mathscr{L} = \rho X - V(\rho)$ where $X \equiv \partial_\mu S \partial^\mu S$ and $\rho = |\psi|^2$, the Hamilton-Jacobi equation in the macroscopic limit reads $X = V'(\rho)$, which allows the density to be integrated out in favor of X. The resulting effective Lagrangian $\mathscr{L}_{eff}(X) = \rho(X) X - V(\rho(X))$ is of k-essence type, with pressure $p = \mathscr{L}_{eff}$ and energy density $\epsilon = 2X\mathscr{L}'_{eff} - \mathscr{L}_{eff}$ [15]. Using the Barceló-Liberati-Visser definition $c_s^2 = dp/d\epsilon$ for relativistic barotropic fluids [16], one finds

$$c_s^2 = \frac{\rho_0 V''(\rho_0)}{\rho_0 V''(\rho_0) + 2V'(\rho_0)}. \tag{10}$$

As a check, the radiation-gas potential $V(\rho) = \frac{1}{2} g \rho^2$ gives $V' = g\rho$, $V'' = g$, $\rho V'' = g\rho = V'$, and Eq. (10) recovers the standard result $c_s^2 = 1/3$.

For the logarithmic potential $V(\rho) = -b\rho \ln(\rho/\rho_c)$, where the minus sign reflects the attractive (self-binding) interaction in Eq. (6):

$$V' = -b\left[\ln\left(\frac{\rho}{\rho_c}\right) + 1\right], V'' = -\frac{b}{\rho}, \rho_0 V'' = -b. \tag{11}$$

Substituting (the minus signs cancel in the ratio):

$$\boxed{c_s^2 = \frac{1}{2\ln(\overline{\rho}/\rho_c) + 3}} \tag{12}$$

The coupling b cancels, and c_s depends on the local vacuum density $\overline{\rho}$ through the logarithm. Normalizing so that $c_s = c$ at the background density ρ_∞ fixes $\rho_c = e\rho_\infty$. This is a normalization convention rather than a prediction of the theory: ρ_c sets the dimensional scale of the logarithmic potential, and the choice $\rho_c = e\rho_\infty$ makes the speed of light at the cosmological background coincide with the parameter c. The precise physical value of ρ_∞—which determines the absolute scale of the framework—is fixed by the gravitational coupling derivation in companion work, where dimensional self-consistency forces ρ_∞ to the Planck mass density; we use the present convention throughout this paper and defer the cosmological identification to dedicated work.

This density dependence is a relativistic effect. In the non-relativistic logarithmic Schrödinger equation, the denominator of (10) is dominated by $|V'(\rho_0)| \approx m_{eff}c^2 \gg |\rho_0 V''|$ (where ρ_0 here denotes the background value of the Lagrangian variable, not the physical mass density), yielding $c_s^2 \approx |\rho_0 V''|/(2m_{eff}c^2) \propto b/(m_{eff}c^2)$, independent of density [13]. In the relativistic regime, V' and $\rho_0 V''$ are comparable, and the density dependence survives (Fig. 2). Equation (12) also reveals a hard minimum density: at $\bar{\rho} = \rho_c e^{-3/2}$ the sound speed diverges, and below this threshold $c_s^2 < 0$, signaling breakdown of the hydrodynamic description.

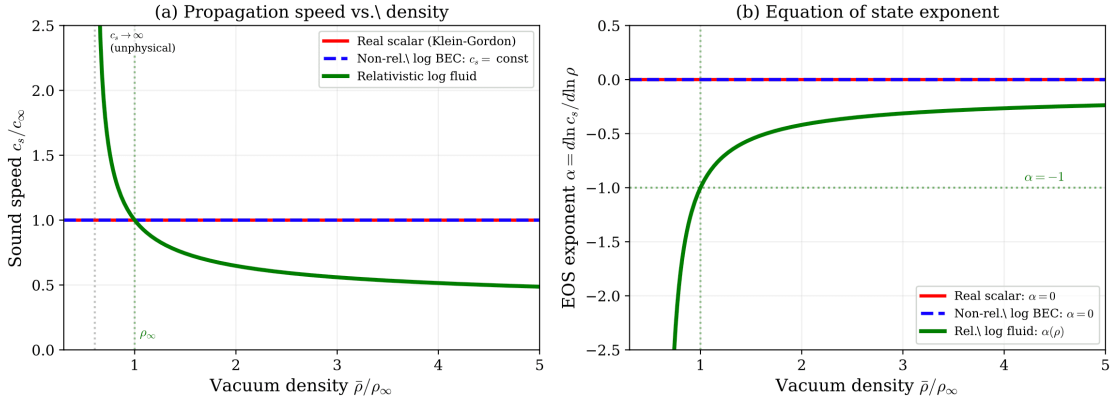


Figure 2. (a) Sound speed vs. density. The relativistic logarithmic fluid (green) has density-dependent c_s, while the real scalar KG and non-relativistic BEC give constant propagation speed. A singularity at $\bar{\rho}/\rho_c = e^{-3/2} \approx 0.22$ marks the minimum physical density. (b) The equation of state exponent $\alpha = d\ln c_s/d\ln\rho$, crossing $\alpha = -1$ at $\bar{\rho} = \rho_\infty$.

The equation of state exponent, which controls the PPN structure, is:

$$\alpha \equiv \frac{d\ln c_s}{d\ln\rho} = -\frac{1}{2\ln(\bar{\rho}/\rho_c) + 3}. \tag{13}$$

At the cosmological background $\bar{\rho} = \rho_\infty$ (where $\ln(\rho_\infty/\rho_c) = -1$): $\alpha = -1$.

Comparison with non-relativistic SVT

We note that this density-dependent sound speed differs from the result obtained in the non-relativistic superfluid vacuum theory (NR-SVT) program [5], where the first-sound speed of the logarithmic superfluid is found to be density-independent: $c_s^2 = -Db/\hbar$ with b approximately constant. The two results are not in conflict; they describe different regimes. NR-SVT analyzes the non-relativistic

logarithmic Schrödinger equation, in which the rest-mass term dominates the dispersion relation and the sound speed reduces to a fixed scalar function of the coupling. The relativistic logarithmic equation (6) used here couples the phase dynamics to the density through the d'Alembertian operator, producing the density dependence of Eq. (12). The density dependence is therefore a relativistic effect, absent in the non-relativistic limit. As established in Sec. 2, this density dependence is essential for the framework to recover $\gamma = 1$: a density-independent sound speed cannot, by itself, produce the spatial-curvature contribution to light bending observed by Cassini [2]. The relativistic starting point is therefore a deliberate methodological choice with observational motivation, not an oversight of the non-relativistic case.

5. The Static Acoustic Metric and Its Limitations

5.1. PPN parameter from the static metric

For a static irrotational fluid with density $\overline{\rho}(x)$ and sound speed $c_s(x)$, the acoustic metric is [16], [17]:

$$g_{tt} = -\overline{\rho}c_s, g_{ij} = \frac{\overline{\rho}}{c_s}\delta_{ij}. \tag{14}$$

With $c_s \propto \overline{\rho}^{\alpha}$, a small density perturbation $\overline{\rho} = \rho_\infty(1+\epsilon)$ gives:

$$\frac{\delta g_{tt}}{g_{tt}} = (1+\alpha)\epsilon, \tag{15}$$

$$\frac{\delta g_{rr}}{g_{rr}} = (1-\alpha)\epsilon. \tag{16}$$

To extract γ we first rescale the acoustic metric so that the background is Minkowski-normalized, $\eta_{\mu\nu} = diag(-1,+1,+1,+1)$, as required by the standard PPN form [1]. With $c_s(\rho_\infty) = c$ this fixes the conformal factor to unity at the background and the perturbations of Eqs. (15)-(16) become directly comparable to the PPN expansion $g_{00} = -1 + 2U + \mathcal{O}(U^2)$, $g_{ij} = (1+2\gamma U)\delta_{ij}$ [1]. Reading $2U$ from the temporal perturbation and $2\gamma U$ from the spatial perturbation, the ratio gives

$$\gamma_{static} = \frac{\delta g_{rr}/g_{rr}}{\delta g_{tt}/g_{tt}} = \frac{(1-\alpha)\epsilon}{(1+\alpha)\epsilon}, \tag{17}$$

which yields

$$\boxed{\gamma_{static} = \frac{1-\alpha}{1+\alpha}} \tag{18}$$

This is the PPN parameter for any static barotropic acoustic metric with equation of state exponent α (Fig. 3).

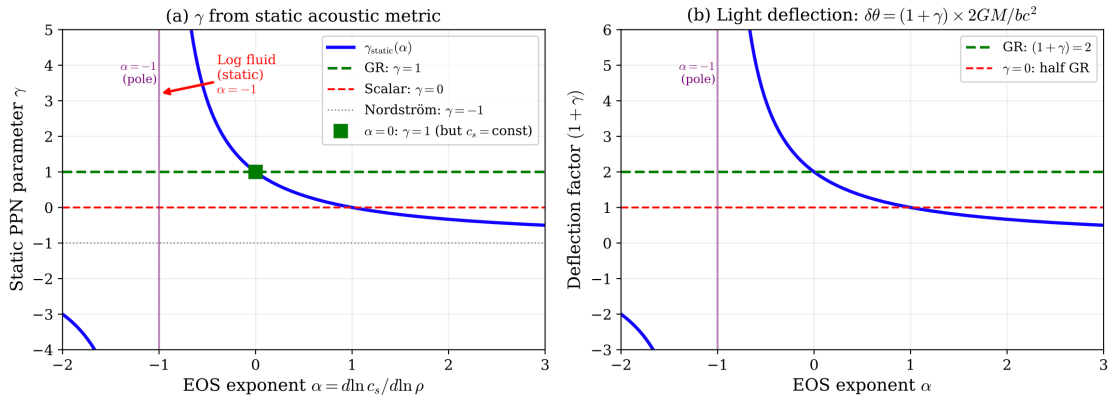


Figure 3. (a) Static PPN parameter $\gamma(\alpha) = (1-\alpha)/(1+\alpha)$. The logarithmic fluid at $\bar{\rho} = \rho_\infty$ has $\alpha = -1$, a pole of γ_{static}: the static acoustic metric is ill-defined. GR's $\gamma = 1$ is attained at $\alpha = 0$, but this corresponds to density-independent c_s, which produces zero refractive deflection. (b) The same pathology in the deflection factor $(1+\gamma)$.

5.2. The static acoustic metric is pathological

Equation (18) reveals a fundamental obstruction. For the logarithmic equation of state at the background density, $\alpha = -1$, and the denominator of γ_{static} vanishes:

$$\gamma_{static} \to \infty \quad \text{at} \quad \alpha = -1. \tag{19}$$

The static acoustic metric does not merely give the wrong numerical value of γ; the spatial metric perturbation diverges relative to the temporal one, and no finite weak-field expansion around the static background is possible. Physically, the pole signals that the static configuration is not a stable background for perturbations: an arbitrarily small density gradient produces an arbitrarily large metric response.

This is, at a deeper level, a consequence of conformal invariance. For a static fluid with no flow, the acoustic metric (14) is conformally flat: $g_{\mu\nu} \propto (\bar{\rho}/c_s)\eta_{\mu\nu}^{(c_s)}$. Null geodesics are insensitive to the conformal factor, so light follows paths determined only by $c_s(x)$. Moreover, the PPN value $\gamma = 1$ is attained in Eq. (18) only at $\alpha = 0$ (constant c_s), which produces no refractive deflection. The static acoustic metric therefore presents a twofold obstruction: the logarithmic model sits at the pole $\alpha = -1$, and even the regular point $\alpha = 0$ would give vanishing deflection. Any consistent treatment must therefore regularize

the divergence by introducing additional dynamical content beyond the static density channel; this is the role of the Painlevé-Gullstrand flow developed in Sec. 6.

6. The Flowing Acoustic Metric and the Path to $\gamma = 1$

6.1. The Painlevé-Gullstrand structure

The static metric exhausts only one of the two degrees of freedom provided by the Madelung decomposition. The second—the macroscopic flow velocity $\mathbf{v} = \nabla S/m_{eff}$—contributes off-diagonal terms. For a spherically symmetric radial inflow $v(r)$ in a fluid with $\overline{\rho}/c_s \approx \text{const}$ and $c_s \approx c$, the acoustic metric takes the Painlevé-Gullstrand form [17][18][19]:

$$ds^2 = -c^2 dt^2 + (dr + v(r)dt)^2 + r^2 d\Omega^2. \tag{20}$$

If the flow velocity satisfies the free-fall profile:

$$v(r) = \sqrt{\frac{2G_{eff}M}{r}}, \tag{21}$$

then the standard coordinate transformation $dt_s = dt - v dr/(c^2 - v^2)$ converts (20) to the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2G_{eff}M}{rc^2}\right) c^2 dt_s^2 + \frac{dr^2}{1 - 2G_{eff}M/(rc^2)} + r^2 d\Omega^2, \tag{22}$$

which gives $\gamma = 1$ exactly. The off-diagonal flow term breaks the conformal flatness of the static metric, introducing genuine spatial curvature through the $g_{rr} = (1 - v^2/c^2)^{-1}$ factor. That this specific velocity profile is not imposed by fiat but follows from a Bernoulli self-consistency condition is established in Sec. 7.2. The physical mechanism is frame-dragging by the inflowing vacuum: light propagating against the current is slowed (increased effective refractive index), while the spatial geometry is distorted by the kinetic energy of the flow. Both effects contribute to the deflection, and their combined weight gives the full GR value.

6.2. Self-consistency test: Bondi accretion

The Painlevé-Gullstrand argument assumes $v(r) = \sqrt{2G_{eff}M/r} \propto r^{-1/2}$. We now ask: does this profile emerge self-consistently from the fluid equations with the logarithmic equation of state?

For steady-state spherical accretion, the continuity equation (8) gives $4\pi r^2 \overline{\rho} v = \dot{M} = \text{const}$. If $\overline{\rho}$ is approximately constant (required for the Painlevé-Gullstrand form), then:

$$v(r) \propto \frac{1}{r^2} \qquad (\text{continuity with } \bar{\rho} \approx \text{const}). \tag{23}$$

This is incompatible with the required $v \propto r^{-1/2}$. The discrepancy grows as $(r_s/r)^{3/2}$, where $r_s = GM/(2c^2)$ is the sonic radius.

We verify this by solving the full Bondi accretion equations [20]:

$$\frac{dv}{dr} = \frac{v(2c_s^2/r - GM/r^2)}{v^2 - c_s^2} \tag{24}$$

with $c_s^2(\bar{\rho})$ given by Eq. (12) and $\bar{\rho}$ from continuity. The solution, passing through the sonic point ($v = c_s$ at $r = r_s$), is shown in Fig. 4.

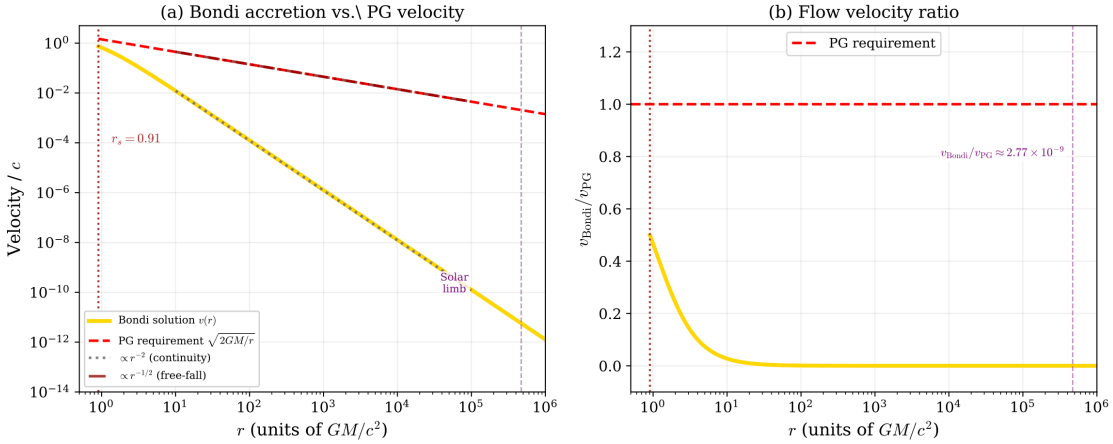


Figure 4. (a) Bondi accretion velocity (solid gold) vs. the Painlevé-Gullstrand requirement $v = \sqrt{2GM/r}$ (dashed red). In the subsonic far field, $v_{Bondi} \propto r^{-2}$, falling far below the $r^{-1/2}$ profile. (b) Ratio v_{Bondi}/v_{PG}. At the solar limb ($r \sim 5 \times 10^5 GM/c^2$), the ratio is $\approx 2.8 \times 10^{-9}$.

This establishes that single-soliton Bondi accretion cannot produce the PG profile required by the acoustic-metric consistency argument. The resolution is that the PG profile must arise from collective, many-soliton dynamics, not from single-source accretion—a point we return to in Sec. 7.2.

7. Discussion

7.1. The circularity objection

A natural objection: by placing c in the D'Alembertian of Eq. (6), we assumed Lorentz invariance. Is $\gamma = 1$ then circular?

No. The D'Alembertian alone guarantees only that the asymptotic propagation speed is c. As shown in Sec. 2, the real scalar equation $\Box D + V'(D) = 0$ also contains c in its D'Alembertian, yet gives $v = c = const$ regardless of background, yielding $\gamma = 0$. Lorentz invariance is necessary but not sufficient for $\gamma = 1$.

What promotes γ from 0 to 1 is the combination of three distinct structural features, none of which is present in the real-scalar theory:

1. *The Madelung decomposition* promotes the field content from one real degree of freedom (D) to two real degrees of freedom (density ρ and phase S), breaking the rigidity of the principal symbol.
2. *The logarithmic nonlinearity*, in the *relativistic* regime, produces a density-dependent sound speed with $\alpha = -1$ at the background (Eq. 13).
3. *The Painlevé-Gullstrand flow* provides a second dynamical channel (the macroscopic flow velocity) that regularizes the static-metric divergence at $\alpha = -1$ and delivers $\gamma = 1$ for any barotropic equation of state.

Each ingredient is individually insufficient: the real-scalar KG with the same D'Alembertian gives $\gamma = 0$; the Madelung decomposition alone in the non-relativistic limit gives $\alpha = 0$ and thus no refractive bending; the static acoustic metric alone at $\alpha = -1$ is pathological. Lorentz invariance is the foundation; the complex field structure, the relativistic log nonlinearity, and the macroscopic flow together build the house.

7.2. The logical chain

Our results establish a clear logical chain:

The kinematic trap is absolute

Any real scalar field equation with a D'Alembertian kinetic term gives $\gamma = 0$, regardless of the self-interaction potential (Sec. 2). This is a property of the equation's characteristic surfaces, not of any particular solution.

The Madelung formulation breaks the trap

The complex field's Madelung decomposition introduces two degrees of freedom—density ρ and flow velocity $\mathbf{v}$—that are hidden in the wave equation form. The relativistic logarithmic equation of state yields

a density-dependent sound speed (Eq. 12), which is the prerequisite for any refractive bending mechanism.

The static acoustic metric is pathological

The PPN parameter of a static barotropic acoustic metric is $\gamma = (1-\alpha)/(1+\alpha)$ (Eq. 18). For the logarithmic equation of state at the background density, $\alpha = -1$, placing γ_{static} exactly at the pole. The static acoustic metric is mathematically ill-defined, not merely numerically wrong.

Background flow is necessary and regularizes the pathology

The Painlevé-Gullstrand form of the flowing acoustic metric reproduces the Schwarzschild geometry and $\gamma = 1$ exactly, provided $v(r) = \sqrt{2G_{eff}M/r}$ (Sec. 6). The off-diagonal flow term $-2vdrdt$ cannot be removed by a conformal rescaling and carries the full $\gamma = 1$ result for any barotropic equation of state, independent of α. In this way the flow regularizes the divergence of the static case: the dynamical content that the static metric could not carry in the (pathological) density channel is transferred to the (well-defined) flow channel.

The PG flow is the only self-consistent solution

In the superfluid vacuum framework, the gravitational potential is not externally specified—it *is* the acoustic metric. This imposes a self-consistency requirement: the density perturbation $\epsilon = \delta\rho/\rho_0$ and flow velocity $v(r)$ must produce an acoustic metric whose effective Newtonian potential $\Phi_{eff} = -\frac{1}{2}\delta g_{tt}/g_{tt}$ matches the potential that generated ϵ and v in the first place.

For the static solution (Solution A: $v = 0$, $\epsilon \neq 0$), the divergence of γ_{static} at $\alpha = -1$ means that no finite density perturbation ϵ produces a finite weak-field metric: the self-consistency equation admits no regular solution in this channel.

For the flowing solution (Solution B: $v = \sqrt{2G_{eff}M/r}$, $\rho \approx \rho_0$), the Bernoulli equation gives $\frac{1}{2}v^2 + \Phi_{eff} = 0$ at leading order, and the acoustic metric in Schwarzschild coordinates gives $g_{tt} \approx -(1-2U)$, $g_{rr} \approx (1+2U)$, yielding $\Phi_{eff} = -U$ and $\gamma = 1$. The Bernoulli relation $v^2 = 2U$ then closes the loop: Φ_{eff} produces the flow that produces Φ_{eff}. This solution *is* self-consistent for any barotropic equation of state, because the Painlevé-Gullstrand acoustic metric is identically the Schwarzschild metric regardless of the specific form of $c_s(\rho)$.

Independent confirmation from Bernoulli dynamics

The Painlevé-Gullstrand requirement can be derived independently from the Bernoulli relation of the logarithmic fluid. Linearizing the Hamilton-Jacobi equation (9) about the background with $|\mathbf{v}| \ll c$ and using $V'(\rho_0) = -b[\ln(\rho_0/\rho_c) + 1]$, one obtains $\ln(\rho/\rho_\infty) = +(m_{eff}^2/b\hbar_{eff}^2)|\mathbf{v}|^2$ to leading order. With the convention $\delta\rho \equiv \rho_\infty - \rho > 0$ for the depletion in a gravitational well, this gives $\delta\rho/\rho_\infty \propto v^2$ in the weak-field limit. Since Newtonian gravity requires a density depletion $\delta\rho \propto 1/r$ (from the Poisson equation combined with the framework's identification $\Phi \propto -\delta\rho$), the flow velocity must satisfy $v^2 \propto 1/r$, recovering the Painlevé-Gullstrand profile $v = \sqrt{2G_{eff}M/r}$ from a purely hydrodynamic argument. This provides a second, independent derivation of the PG condition, complementing the acoustic metric self-consistency argument above.

The continuity constraint

The radially inward PG flow $\mathbf{v} = -\sqrt{2GM/r}\hat{\mathbf{r}}$ has $\nabla \cdot \mathbf{v} = -\frac{3}{2}\sqrt{2GM}r^{-3/2} \neq 0$, corresponding to condensate accumulation $\partial\rho/\partial t = -\rho_0 \nabla \cdot \mathbf{v} > 0$. The accumulation rate is small: at the solar limb, $|\nabla \cdot \mathbf{v}|^{-1} \sim 10^3$ s, but this timescale applies to fractional changes $\delta\rho/\rho_0 \sim U \sim 10^{-6}$, so the absolute density change per observation epoch is negligible. The PG flow is a valid quasi-steady approximation on all timescales short compared to the gravitational collapse time. A fully steady-state treatment requires a mechanism that redirects the inflowing condensate without local accumulation; candidate mechanisms include two-fluid counterflow, vortex-core absorption, and geometric redirection into non-radial outflow. The choice of mechanism is deferred to future work.

Single-particle vs. collective flow

The Bondi accretion solution for the logarithmic equation of state does not produce the required velocity profile in the far field. Continuity enforces $v \propto r^{-2}$ for nearly-incompressible steady flow, while $\gamma = 1$ requires $v \propto r^{-1/2}$. This discrepancy applies to single-soliton configurations. However, the self-consistency argument above does not depend on the single-particle velocity field. The PG flow must arise as a *collective* effect of $N \sim M/m_{particle}$ solitons: individual particles contribute microscopic density perturbations that sum to the macroscopic potential $\Phi = -GM/r$, and the resulting vacuum inflow adjusts to maintain self-consistency of the acoustic metric. In this picture, the Bondi calculation describes the flow around a single vortex core (irrelevant at astrophysical distances), while the PG flow describes the macroscopic vacuum response to the collective mass distribution. The relationship between the

microscopic and macroscopic descriptions—analogous to the Feynman-Onsager relation in rotating superfluids—remains an open problem.

Two open problems remain. First, deriving the macroscopic PG flow from the microscopic soliton dynamics: showing that a collection of N vortex-Gaussons in the logarithmic condensate produces a collective vacuum inflow $v = \sqrt{2GM/r}$ as a mean-field effect, analogous to how a lattice of quantized vortices in superfluid helium produces solid-body rotation [19]. Second, establishing the steady-state mechanism: identifying whether the two-fluid counterflow (superfluid infall balanced by normal-component outflow) or vortex-core absorption provides the sink term required for continuity. The first problem is amenable to a Thomas-Fermi mean-field calculation; the second requires understanding the quantum depletion of the condensate in the presence of topological defects.

7.3. Relation to prior work

The superfluid vacuum program of Zloshchastiev [5], [6], [21] derives emergent spacetime metrics from the logarithmic condensate but does not compute PPN parameters explicitly. The analogue gravity program of Barceló, Liberati, and Visser [22], [16] proves that fluid perturbations propagate on acoustic metrics but does not construct specific models matching solar system tests. The present work connects these programs by deriving the explicit PPN constraints and identifying the gap between the static and flowing acoustic metrics.

The flowing acoustic metric's equivalence to the Painlevé-Gullstrand form of Schwarzschild was noted by Unruh [17] and developed extensively by Visser [18] and Volovik [19]. Our contribution is to test this identification against self-consistent fluid dynamics, revealing that the velocity profile is not automatically produced.

7.4. Observational tests passed by the framework

What "light" is, the dispersion relation, and achromaticity bounds

Within the framework, light is not separate from the condensate: photons are long-wavelength excitations of the same relativistic logarithmic Klein-Gordon field whose Madelung decomposition produces the acoustic metric. The framework is a single-field theory; there is no independent electromagnetic medium "on top" of the vacuum. The same complex order parameter ψ that gives the acoustic metric through its hydrodynamic envelope also carries photonic excitations as its transverse

phonon-like modes. The U(1) gauge structure of electrodynamics is an internal symmetry of ψ, with the derivation of Maxwell's equations from the gauged log-KG field forming a separate manuscript of the program. The relevant dispersion relation follows directly from the linearized log-KG equation.

Linearizing the log-KG equation about the background ($\rho = \rho_0,\ S_0 = -m_{eff}c^2 t$) and including the Bohmian quantum potential $Q = -(\hbar_{eff}^2/2m_{eff}^2)\Box\sqrt{\rho}/\sqrt{\rho}$ at next-to-leading order yields the Bogoliubov-like dispersion relation:

$$\omega^2 = c^2 k^2 \left[1 + (k\xi)^2\right], \tag{25}$$

where $\xi = \hbar_{eff}/(2m_{eff}c)$ is the condensate healing length, derived from dimensional self-consistency of the framework's gravitational coupling to be of order the Planck length, $\xi \sim \ell_P$. At long wavelengths ($k\xi \ll 1$), $\omega = ck$ exactly; the speed of light, gravitational waves, and acoustic phonons all equal c at the background, consistent with the GW170817 constraint $|c_{gw} - c|/c < 10^{-15}$ [23]. The phase velocity is $v_{ph} = c\sqrt{1 + (k\xi)^2}$ and the group velocity is $v_g = c(1 + 2(k\xi)^2)/\sqrt{1 + (k\xi)^2}$; both reduce to c as $k\xi \to 0$, with $|v_{ph} - c|/c \approx (k\xi)^2/2$.

For all electromagnetic frequencies probed by Cassini Shapiro-delay timing [2], gravitational lensing measurements, and standard astrophysical observations, $k\xi$ is utterly negligible. For visible light at $\lambda = 550$ nm, $(k\xi)^2/2 \sim 10^{-56}$; for X-band radio at 8 GHz (Cassini), $(k\xi)^2/2 \sim 10^{-66}$; even for the highest astrophysical PeV gamma rays, $(k\xi)^2/2 \sim 10^{-27}$, far below the best experimental bound on Lorentz invariance violation in vacuum ($\sim 10^{-20}$ from gamma-ray burst time-of-flight measurements). The framework therefore predicts achromatic light deflection and Shapiro delay throughout all current and foreseeable observational regimes. Frequency-dependent deflection is a falsifiable prediction of the framework, but only at energies approaching the Planck scale.

The PG flow and preferred-frame tests

The Painlevé-Gullstrand flow $v(r) = \sqrt{2GM/r}$ takes the value $\sim 10^{-3}c$ at the solar limb, raising the question of why high-precision local experiments do not detect a Sagnac-like "ether wind." The answer is the same structural feature that delivers $\gamma = 1$. The PG metric is identically Schwarzschild — the transformation from PG coordinates to standard Schwarzschild coordinates is a passive change of variables that cannot alter any invariant observable. The PPN preferred-frame parameters $\alpha_1, \alpha_2, \alpha_3$ vanish in Schwarzschild geometry by construction [1], and they continue to vanish in any coordinate system describing the same geometry, including the PG flow representation. The framework

therefore predicts $\alpha_1 = \alpha_2 = \alpha_3 = 0$ at the same order as $\gamma = 1$: as a direct consequence of the PG-Schwarzschild equivalence, not as a separate calculation.

This resolves the apparent puzzle. The PG flow is a real macroscopic feature of the framework — the condensate genuinely supports the velocity field $v(r)$ around the Sun — but its observable consequences are precisely those of Schwarzschild gravity. Laboratory rulers and clocks situated in the solar system are in free fall in the same gravitational field; by construction, $v(r)$ is the free-fall velocity at radius r. An Earth-bound experiment cannot detect the underlying flow for the same reason a freely-falling observer in general relativity cannot detect the gravitational field they are falling through, expressed here in the language of an explicit medium picture rather than a geometric one. The framework's preferred rest frame is identified observationally with the CMB rest frame (in which the CMB dipole vanishes); this defines the global rest frame against which the macroscopic vacuum dynamics are described, while local physics in the solar system reproduces the standard PPN bounds. Refinements at higher post-Newtonian order, where the medium picture and the geometric picture could in principle diverge, are not derived in this work.

The weak equivalence principle

The MICROSCOPE measurement of the Eötvös parameter, $\eta(Ti, Pt) = [-1.5 \pm 2.3(stat) \pm 1.5(syst)] \times 10^{-15}$ [24], is the tightest current bound on equivalence-principle violation. In any theory in which gravitation arises from coupling to a substrate rather than from a postulated geometric metric, this bound constrains composition-dependent terms in the matter-substrate coupling. The framework's leading-order universality, established above through the PG-Schwarzschild equivalence, automatically passes this test. The remaining theoretical question is whether composition-dependent corrections to the soliton-condensate coupling — arising from differences in electromagnetic binding energy, nuclear binding energy, or spin-orbit structure between test materials — generate an η_{12} within the MICROSCOPE confidence interval, or whether they predict a specific non-zero value that future experiments could resolve. Computing η_{Pt-Ti} explicitly from the framework's matter-substrate coupling is deferred to dedicated work; it requires the U(1) gauging of the condensate and the soliton internal-energy decomposition, both developed in companion follow-up manuscripts of this program.

Shapiro time delay

The recovery of $\gamma = 1$ via the Painlevé-Gullstrand structure automatically reproduces the Shapiro time delay. Once the acoustic metric in the PG frame is shown to be identically Schwarzschild (Sec. 6), all weak-field tests parameterized by the same PPN coefficients $\{\gamma, \beta, \alpha_i, \ldots\}$ [1] are reproduced together as a consequence: the coordinate transformation from PG to standard Schwarzschild coordinates is a passive change of variables that cannot alter any invariant observable. The Cassini measurement of γ from Shapiro delay timing [2] is therefore reproduced without additional input, with no framework-specific dispersion in the long-wavelength limit (see also discussion of the electromagnetic sector below).

Asymptotic constraints

The GW170817 constraint ($|c_{gw} - c|/c < 10^{-15}$ [23]) requires that the asymptotic sound speed equal c exactly, fixing a combination of the equation of state parameters. The second PPN parameter β, controlling perihelion precession, takes the GR value $\beta = 1$ by the same coordinate-equivalence argument as γ. The full post-Newtonian expansion to 2PN and higher orders, which probes the framework beyond the linearized acoustic-metric regime, is left for future work.

7.5. Limitations and open problems

Conformal flatness and strong fields

The acoustic metric (14) is conformally flat ($g_{ij} \propto \delta_{ij}$). General relativity admits nontrivial Weyl curvature. The present framework reproduces the weak-field PPN limit but cannot describe strong-field phenomena: binary pulsar orbital decay [25], black hole ringdown, or nonlinear gravitational wave dynamics. Whether extensions incorporating vorticity or background flow can access the strong-field regime is an open question.

The quantum potential

The Bohm potential $Q = \Box\sqrt{\rho}/\sqrt{\rho}$ was neglected in the macroscopic limit. At the soliton coherence scale, Q becomes important and governs quantum-mechanical particle behavior. A complete theory must show a smooth transition between quantum and gravitational regimes—analogous to the phonon-to-hydrodynamic crossover in superfluid helium [19].

8. Conclusion

We have shown that the failure of scalar gravity to produce the observed light bending is a property of the wave equation representation, not of scalar fields per se. The Madelung decomposition of a relativistic logarithmic superfluid introduces two degrees of freedom—a density-dependent sound speed and a macroscopic flow velocity—that together provide the structural ingredients needed for $\gamma = 1$.

The static acoustic metric of the logarithmic superfluid is mathematically pathological: the equation of state exponent $\alpha = -1$ at the background density places the static PPN parameter $\gamma_{static} = (1-\alpha)/(1+\alpha)$ exactly at the pole. No finite weak-field expansion around the static background exists. By contrast, the Painlevé-Gullstrand flow $v = \sqrt{2G_{eff}M/r}$ regularizes this pathology: the resulting acoustic metric is identically Schwarzschild in Painlevé-Gullstrand coordinates, and $\gamma = 1$ follows for *any* barotropic equation of state. This selects the flowing solution as the unique physically admissible weak-field configuration.

The Bondi accretion calculation shows that a single spherically symmetric source does not produce the PG profile: far from the sonic radius the flow decays as r^{-2}, falling nearly nine orders of magnitude below the $r^{-1/2}$ requirement at the solar limb. The remaining open problems are therefore to derive the macroscopic PG flow as a collective mean-field effect of the microscopic soliton dynamics—analogous to the Feynman-Onsager relation in rotating superfluids—and to identify the steady-state mechanism (two-fluid counterflow or vortex-core absorption) that resolves the continuity constraint. Both are amenable to analytical treatment and form the subject of forthcoming work.

Statements and Declarations

Data and Code Availability

The Klein-Gordon wave packet simulation code (Figure 1) and the Bondi accretion integration code (Figure 4) are available at `https://github.com/kulangiev/kulangiev.github.io/tree/main/papers/paper_1` under Creative Commons Attribution 4.0 (CC BY 4.0) licence. All numerical parameters, initial conditions, and post-processing routines used to generate the figures are documented in the repository.

Acknowledgments

The author thanks K. G. Zloshchastiev for foundational work on the logarithmic superfluid vacuum and C. Barceló, S. Liberati, and M. Visser for the acoustic metric framework. The methodological stance of this paper—treating the order parameter as a real classical field whose hydrodynamic behavior underlies emergent gravitational phenomena—is indebted to the Madelung-de Broglie-Bohm tradition, particularly as developed in P. R. Holland's exposition of the quantum theory of motion. Claude (Anthropic) and Gemini (Google) were used as editorial and computational tools during manuscript preparation. All scientific content, equations, and interpretations are the author's own.

References

1. a, b, c, d, e, f *Will CM (2014). "The Confrontation between General Relativity and Experiment." Living Rev Relativ. **17**(4). doi:10.12942/lrr-2014-4.*
2. a, b, c, d *Bertotti B, Iess L, Tortora P (2003). "A Test of General Relativity Using Radio Links with the Cassini Spacecraft." Nature. **425**:374.*
3. ^ *Nordström G (1913). "Zur Theorie der Gravitation vom Standpunkt des Relativitätsprinzips" [On the Theory of Gravitation from the Standpoint of the Principle of Relativity]. Ann Phys (Leipzig) [Annals of Physics (Leipzig)]. **347**:533.*
4. ^ *Einstein A (1911). "Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes" [On the Influence of Gravity on the Propagation of Light]. Ann Phys (Leipzig) [Annals of Physics (Leipzig)]. **340**:898.*
5. a, b, c, d *Zloshchastiev KG (2023). "Derivation of Emergent Spacetime Metric, Gravitational Potential and Speed of Light in Superfluid Vacuum Theory." Universe. **9**:234.*
6. a, b, c *Zloshchastiev KG (2020). "An Alternative to Dark Matter and Dark Energy: Scale-Dependent Gravity in Superfluid Vacuum Theory." Universe. **6**:180.*
7. ^ *Brans C, Dicke RH (1961). "Mach's Principle and a Relativistic Theory of Gravitation." Phys Rev. **124**:925.*
8. ^ *de Broglie L (1927). "La mécanique ondulatoire et la structure atomique de la matière et du rayonnement" [Wave Mechanics and the Atomic Structure of Matter and Radiation]. J Phys Radium [Journal of Physics and Radium]. **8**:225.*
9. ^ *Madelung E (1927). "Quantentheorie in hydrodynamischer Form" [Quantum Theory in Hydrodynamic Form]. Z Phys [Journal of Physics]. **40**:322.*

10. ^Bohm D (1952). "A Suggested Interpretation of the Quantum Theory in Terms of “Hidden” Variables. I." Phys Rev. **85**:166.
11. ^Holland PR (1993). The Quantum Theory of Motion. Cambridge: Cambridge University Press.
12. ^Białynicki-Birula I, Mycielski J (1979). "Gaussons: Solitons of the Logarithmic Schrödinger Equation." Phys Scr. **20**:539.
13. a, bAvdeenkov AV, Zloshchastiev KG (2011). "Quantum Bose Liquids with Logarithmic Nonlinearity: Self-Sustainability and Emergence of Spatial Extent." J Phys B. **44**:195303.
14. ^Zloshchastiev KG (2012). "Volume Element Structure and Roton-Maxon-Phonon Excitations in Superfluid Helium Beyond the Gross-Pitaevskii Approximation." Eur Phys J B. **85**:273.
15. ^Babichev E, Mukhanov V, Vikman A (2008). "k-Essence, Superluminal Propagation, Causality and Emergent Geometry." JHEP. **02**:101.
16. a, b, cBarceló C, Liberati S, Visser M (2005). "Analogue Gravity." Living Rev Relativ. **8**:12.
17. a, b, cUnruh WG (1981). "Experimental Black-Hole Evaporation?" Phys Rev Lett. **46**:1351.
18. a, bVisser M (1998). "Acoustic Black Holes: Horizons, Ergospheres and Hawking Radiation." Class Quant Grav. **15**:1767.
19. a, b, c, dVolovik GE (2003). The Universe in a Helium Droplet. Oxford: Oxford.
20. ^Bondi H (1952). "On Spherically Symmetrical Accretion." Mon Not Roy Astron Soc. **112**:195.
21. ^Zloshchastiev KG (2011). "Spontaneous Symmetry Breaking and Mass Generation as Built-in Phenomena in Logarithmic Nonlinear Quantum Theory." Acta Phys Polon B. **42**:261.
22. ^Barceló C, Liberati S, Visser M (2001). "Analogue Models for FRW Cosmologies." Class Quant Grav. **18**:1137.
23. a, bAbbott BP, et al. (2017). "Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A." Astrophys J Lett. **848**:L13.
24. ^Touboul P, et al. (MICROSCOPE Collaboration) (2022). "MICROSCOPE Mission: Final Results of the Test of the Equivalence Principle." Phys Rev Lett. **129**:121102.
25. ^Taylor JH, Fowler LA, McCulloch PM (1979). "Measurements of General Relativistic Effects in the Binary Pulsar PSR 1913+16." Nature. **277**:437.

Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.