Research Article

Structure-Preserving Model Order Reduction of second-order systems using Iterative Rational Krylov Algorithm

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In this work, we are going to discuss the Structure-Preserving Model Order Reduction (SPMOR) techniques of second-order linear time-invariant (LTI) continuous-time systems using the Iterative Rational Krylov Algorithm (IRKA). IRKA is well established for the first-order standard and/or generalized systems. Recently, the idea of this model reduction technique is generalized for second-order systems by converting the system into a first-order form. In this case, one can't return back to the original second-order system since the structure of the system is already demolished. Sometimes preservation of second-order structure is essential to perform the further simulations of the system. Also, Structure-preserving MOR allows meaningful physical interpretation and provides a more accurate approximation to the full model. We mainly focus on the SPMOR of the second-order systems using IRKA without converting the system into first-order forms. We have applied and numerically investigated the applicability and efficiency of the proposed techniques to some practical data derived from real-world models.

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Introduction

We consider second-order linear time-invariant (LTI) continuous-time system of the form:

$$M\ddot{z}\left(t\right)+D\dot{z}\left(t\right)+Kz\left(t\right)=Hu\left(t\right),$$

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$$y(t) = Lz(t) \qquad (1)$$

where the matrices $M, D, K \in \mathbb{R}^{n\times n}$, $H \in \mathbb{R}^{n\times m}$ is the input matrix describing the external access to the system and $L \in \mathbb{R}^{p\times n}$ represents the measurement output matrix of the system. If $M = I$ then the system is called a standard state-space system or if M is invertible then the system can also be converted into a standard state-space system. Again, if M is not invertible then the system is called descriptor system and it has several indices according to the structure of the system. The dimension of the system (1) is n while $x(t) \in \mathbb{R}^n$ is the vector of states, $u(t) \in \mathbb{R}^n$ is the vector of control input and $y(t) \in \mathbb{R}^n$ is the measurement outputs of the system. This second-order form (1) of the systems, usually arises in the analysis and modelling of structural vibration, multibody dynamics, electrical circuit and many other disciplines of science and engineering. In many applications, system (1) contains a large number of equations which leads to the system very complex [1][2][3][4].

If the model becomes very large, performing the simulation with it requires prohibitively high computational effort or is simply infeasible due to the limited computer memory. Therefore, reducing the size of the system is unavoidable for fast simulation. In general, to find a reduced-order models (ROM) of second-order systems is to first rewrite (1) into a first-order form and then apply suitable MOR techniques to find a reduced first-order state-space system. Since the structure of the original model is destroyed in the reduced model, one can't return back to the second-order representation if it is required. Therefore, second-order-to-second-order model order reduction (SPMOR) of second-order systems has received a lot of attention in recent years. See, e.g., [5][6][7] and the references therein.

Currently, two methods namely, the Gramian based method Balanced Truncation (BT) and the interpolation-based technique Iterative Rational Krylov Algorithm (IRKA) are frequently used methods for the model reduction of large-scale dynamical systems. Both these methods are well established for the first-order linear time-invariant (LTI) dynamical systems. Most of the literature discusses the Balanced Truncation (BT) for the structure-preserving model order reduction [8][9][10][11]. One of the drawbacks of the balanced truncation is that to perform the method two continuous-time algebraic Lyapunov equations have to be solved which is computationally expensive.

On the other hand, IRKA is popular with the model reduction community as it is computationally cheap. This method is established to the second-order system by converting the system into a first-order form. To the best of the authors' knowledge, no investigation of the IRKA for SPMOR has been performed yet. That's why we mainly focus on the SPMOR of the second-order system (1) using IRKA without converting the system into a first-order form. Using IRKA we develop some algorithms for the second-order systems to obtain the substantially lower-dimensional systems of the form

$$M_r \ddot{z}_r(t) + D_r \dot{z}_r(t) + K_r z_r(t) = H_r u(t),$$

$$y_r(t) = L_r z_r(t) \qquad (2)$$

where $M_r, D_r, K_r \in \mathbb{R}^{r\times r}$, $H_r \in \mathbb{R}^{r\times m}$, $L_r \in \mathbb{R}^{p\times r}$so that $y_r(t)$ approximates $y(t)$ for a wide range of inputs $u(t)$. It has been shown that the developed algorithms are capable to reduce the second-order systems efficiently without demolishing the structure of the systems.

Numerical results

To investigate the efficiency of the proposed algorithms, we subjected some real-world models arising from engineering applications, namely; Scalable Oscillator Model (SOM), Butterfly Gyro Model (BGM), and Adaptive Spindle Support Model (ASSM) [12][13].

The numerical results are observed in tabular and graphical form as below.

Model	Full model (n)	ROM (r)
SOM	9001	30
BGM	17361	50
ASSM	290281	60

Table 1. Comparisons of full models and ROMs

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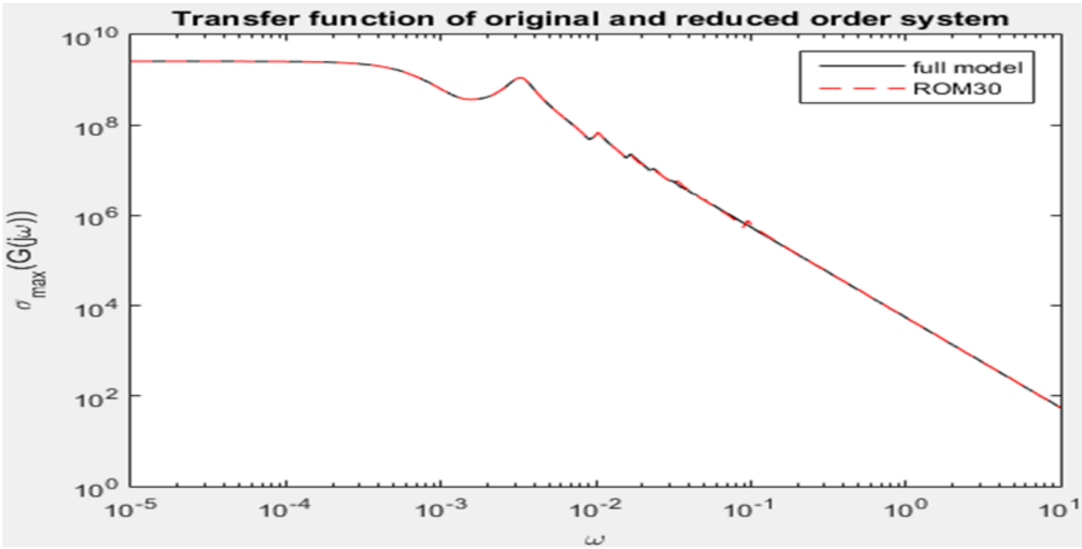
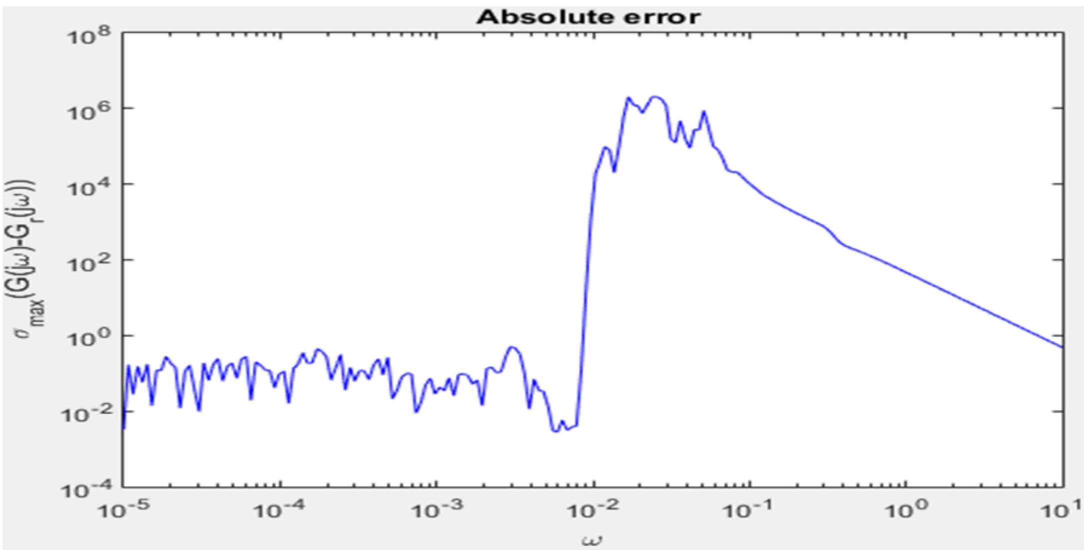

Transfer function of original and reduced order system
full model
ROM30
σmax(G(jω))
ω
10^10
10^8
10^6
10^4
10^2
10^0
10^-5
10^-4
10^-3
10^-2
10^-1
10^0
10^1


Absolute error
σmax(G(jω)-Gr(jω))
ω
10^8
10^6
10^4
10^2
10^0
10^-2
10^-4
10^-5
10^-4
10^-3
10^-2
10^-1
10^0
10^1

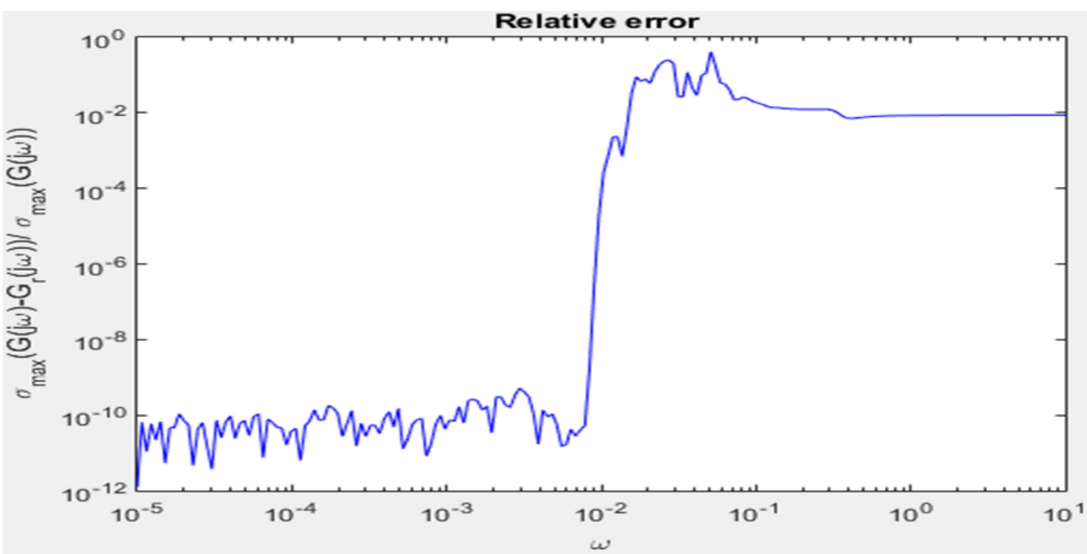

Relative error
σmax(G(jω)-Gr(jω))/ σmax(G(jω))
ω
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
10^-12
10^-5
10^-4
10^-3
10^-2
10^-1
10^0
10^1

Figure 1. The comparison of the full model and ROM of the SOM

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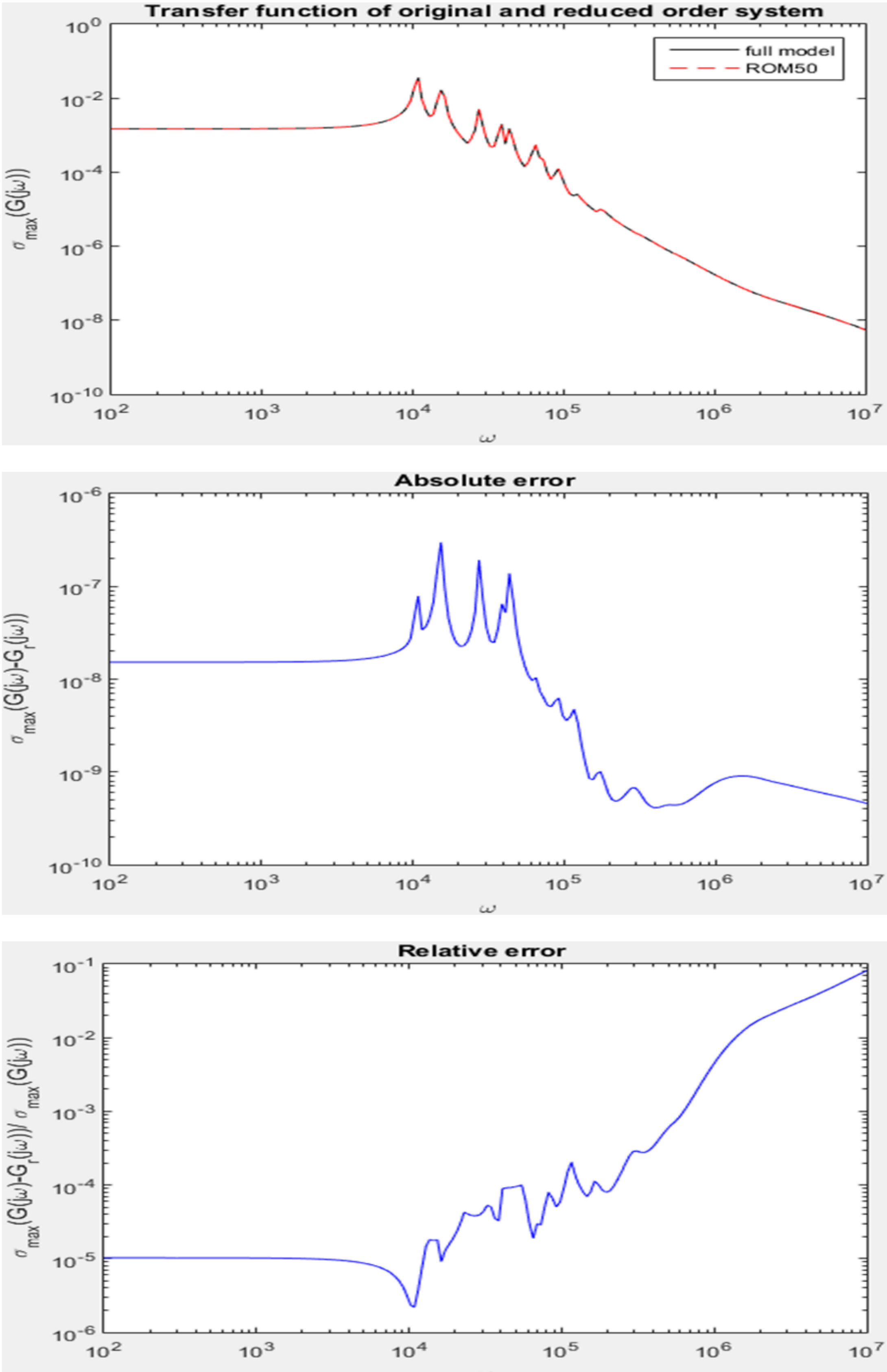
Transfer function of original and reduced order system
full model
ROM50
σmax(G(jω))
ω
Absolute error
σmax(G(jω)-Gr(jω))
ω
Relative error
σmax(G(jω)-Gr(jω))/ σmax(G(jω))
ω

Figure 2. The comparison of the full model and ROM of the BGM

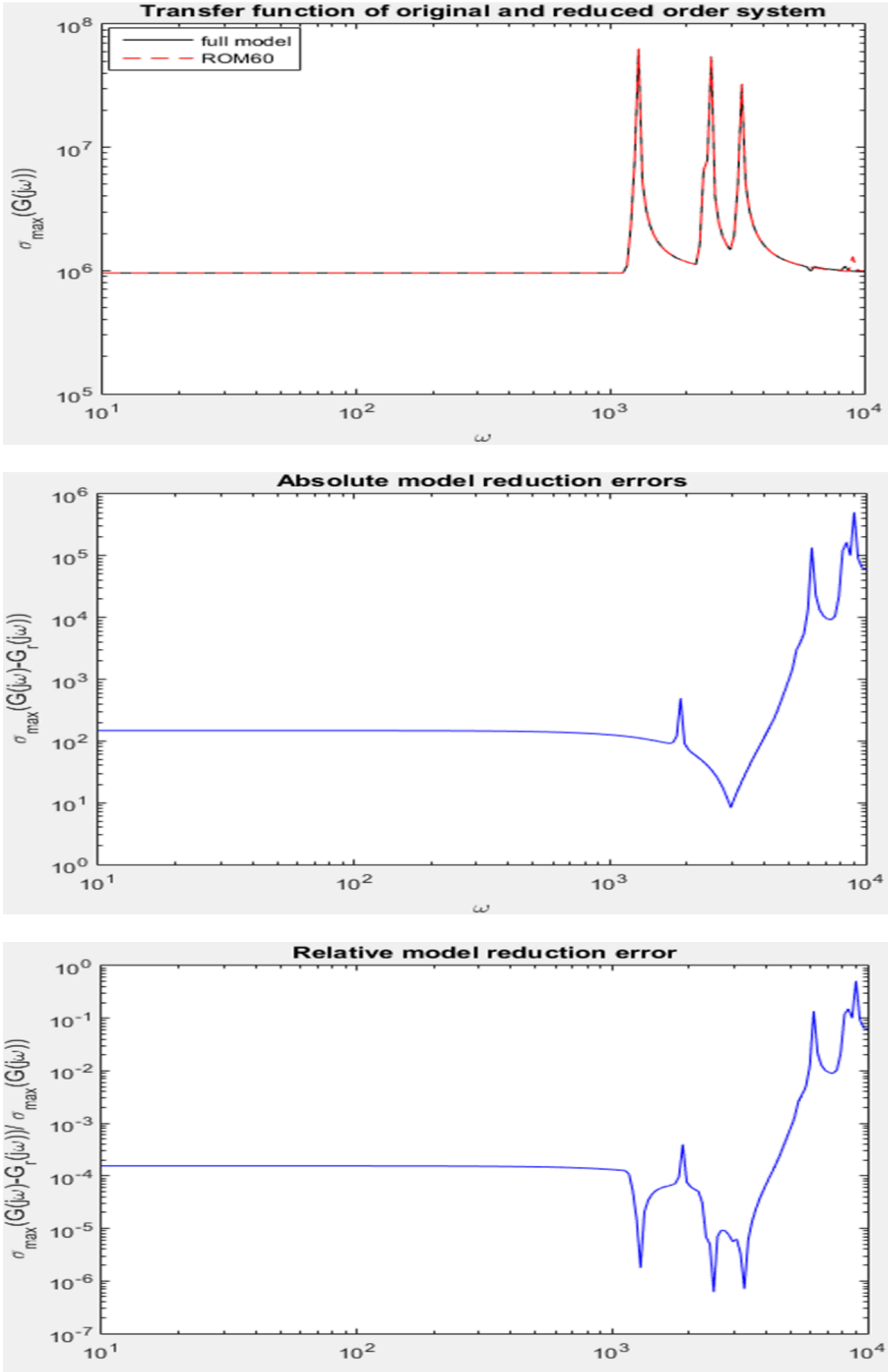

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Figure 3. The comparison of the full model and ROM of the ASSM

Conclusions

We have applied and numerically investigated the applicability and efficiency of the proposed techniques to some practical data derived from real-world models. It has been observed that the proposed techniques provide Reduced Order Models (ROM) of the target models with suitable norm of accuracy.

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Declarations

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Potential competing interests: No potential competing interests to declare.