Review of: "NP on Logarithmic Space"

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Potential competing interests: No potential competing interests to declare.

Major Comments:

1. The statement $L = NL$ if and only if $\forall S(n) \forall (A) DSPACE^{A}(S(n)) = NDSPACE^{A}(S(n))$ in lines 8-9 on page 4, does not follow from the work in [7]. Actually, Theorem 12.3 in [7] states a similar result for oracles of a weaker form (see the second definition, page 126 in [7]), in which all the tapes, including the oracle tape, have the same bound. This result does not hold in the case of oracles in which the space bounds of the oracle tape is $2^{O(S(n))}$ and the queries may be constructed using nondeterminism. See also:
   J. Hartmanis, R. Chang, J. Kadin, J. Mitchell
   The structural complexity column: some observations about relativization of space-bounded computations
   Bull. EATCS, 35 (1988), pp. 82-92.
2. For the above reason, the proof of Theorem 2 is not correct. The oracle Turing machine that you use to prove that $ISET$ is in NP^{coNL} requires both non-determinism as well as an oracle tape of space linear to the size of its input (that is, logarithmic space is not sufficient). If Hypothesis 1 is true, then we can conclude that NP is a subset of NL^{NL}If $L = NL$ also holds, we can conclude that NP is a subset of NL^{L}. However, $NL^{L} = L^{L}$ cannot be obtained using Theorem 12.3 of [7].
3. The proof of Theorem 6 is also incorrect, since it is based on Theorem 2.

Some minor comments:

- page 3, Definition 1: please insert the words "and a" before the word "polynomial".
- page 4: In Hypothesis 1: "another language L_3 which is closed under logarithmic space reductions in NP-complete" should probably be replaced with "another language L_3which NP-complete with respect to logarithmic space reductions".
- pages 4,5: I suggest the following form for the equations in Hypothesis 1 and Theorem 5:
  $L_3 = \{w : \exists u \in \{0,1\}^{p(|w|)}$ such that $M(w,u) \in L_2\}$
  $ISET = \{w : \exists u$ such that $M(w,u) \in SPG\}$
- page 5: please use a unique symbol to represent the certificate in Theorem 5 and its proof (that is, use either u or A, uniformly)
- page 6: In line 17 of the algorithm, $v > n + 1$ could be replaced with $v \geq n + 1$.