Review of: "Multiplicity of solutions for nonlocal fractional equations with nonsmooth potentials"

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In the paper under review, the authors have discussed the existence of at least three weak solutions for nonlocal fractional problems involving nonsmooth potentials, applying an abstract critical point theorem for nonsmooth functionals and combining with the analytical framework on fractional Sobolev spaces developed by Servadei and Valdinoci.

In recent years, a great deal of attention has been focused on the study of fractional and nonlocal operators of elliptic type, for both pure mathematical research and concrete real-world applications. Fractional and nonlocal operators appear in many fields such as optimization, finance, phase transitions, stratifed materials, anomalous diffusion, crystal dislocation, soft thin films, semipermeable membranes, flame propagation, conservation laws, ultra-relativistic limits of quantum mechanics, quasi-geostrophic flows, multiple scattering, minimal surfaces, materials science, water waves, and L\'{e}vy processes.

Here, the authors by checking that the associated energy functional satisfies the assumptions requested by a very recent and general nonsmooth three critical points theorem derived by Yuan and Huang have obtained the existence of at least three weak solutions for the problem with no condition on the behavior of the involved nonlinearities at the origin is assumed, comparing to most of the known results in the classical Laplacian case.

The paper is technical and the results are new and nontrivial, the proofs are accurate. It is very interesting and helpful for the people working in a such direction. The paper is well written.

Comments:

1. I suggest the authors to cite the following paper in which existence results for fractional equations using variational methods and critical point theory have been established:

F. Gharehgazlouei, J.R. Graef, S. Heidarkhani, L. Kong, Existence and multiplicity of solutions to a fractional p-Laplacian elliptic Dirichlet problem, Electronic Journal of Differential Equations, Vol. 2023 (2023), No. 46, pp. 1-15.

A. Ghobadi, S. Heidarkhani, Multiple solutions for nonlocal fractional Kirchhoff type problems, Differential Equations & Applications-DEA14(4) (2022) 597-608.

S. Heidarkhani, F.-F. Liao, A. Salari, Existence results for Kirchhoff nonlocal fractional equations, Kragujevac Journal of Mathematics 49(1) (2025) 17-30.

2. I suggest the authors to cite the following paper in which existence results for fractional equations using nonsmooth variational principles have been obtained:

J. Chu, S. Heidarkhani, F. Gharehgazlouei and A. Solimaninia, Three nontrivial solutions for Kirchhoff-type variational-hemivariational inequalities, Results in Mathematics 68 (2015) 71-91.

S. Heidarkhani, F. Gharehgazlouei, A. Solimaninia, Existence of infinitely many symmetric solutions to perturbed elliptic equations with discontinuous nonlinearities in R^N, Electronic Journal of Differential Equations, Vol. 2015 (2015), No. 123, pp. 1-17.

I suggest the editor to accept the paper for publication after the authors can revise the paper based on the above comment.