

Review of: "Necessity of budget deficit in a growing economy where people hold money and leave a bequest"

Kcodgoh Edgeweblime¹

¹ Université de Lomé

Potential competing interests: No potential competing interests to declare.

Necessity of budget deficit in a growing economy where people hold money and leave a bequest

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Review KL Edgeweblime

1. General comments

The author explores the possibility of substitutability between budget deficits and money holding and bequests as a policy tool for achieving full employment in a growing economy where consumers derive utility not only from consumption but also from money holding and bequests. The overlapping generations model in which consumers bequeath to their descendants and hold money and the Barro-type utility function, in which people include the utility of their children in their own utility. Using algebraic equations only, the author hopes to show that the budget deficit is necessary to maintain full employment at constant prices in a growing economy. Also, he thinks that a balanced budget would not allow to reach full employment at constant prices. Similarly, the author tries to show that if the real budget deficit is lower than that which is necessary and sufficient for full employment at constant prices, a recession would occur, if not inflation in the opposite case.

Insofar as the utility function of Robert Barro (1974) is concerned, the author refers to the Ricardian equivalence principle, which presupposes that economic agents have an infinite life horizon, and does not seem to consider the case of transitory, but rather permanent, public spending. Because of the rationality of economic agents (Ricardian equivalence), fiscal policy is totally inefficient. It will leave the aggregate product and the money market unchanged. In this respect, only a Bennett Mac Callum and John Whitaker (1979) style policy surprise could have a small effect on the aggregate product and the money market. Therefore, the following equation, in particular, cannot be obtained:

$$G_t - \tau w_t L_t^f = M_t - M_{t-1} = m_t L_t^f - m_{t-1} L_{t-1}^f = n m_t L_{t-1}^f. \quad (12)$$

In the case of a political surprise, the increase in income would cause an increase in the demand for active cash holdings. The rise in interest rates would lead to a fall in the demand for idle cash (purchase of securities by households from the government to finance its budget deficit). There would be a crowding-out effect, the magnitude of which would depend on the elasticity of demand for active LQ balances to changes in income, where only a sharp increase in interest rates could restore equilibrium in the money market) and the elasticity of investment to changes in the interest rate.

1. Proposal for discussions: Growth models with consumer optimization (The Ramsey model)

The reasoning is based on infinitely long-lived households that choose consumption and saving to maximize their dynastic utility, subject to an intertemporal budget constraint, a key element of the Ramsey model (1928), refined by Cass (1965) and Koopmans (1965) being the precursors of sustainable economics or steady state. One of the most important conditions of the sustainable growth or steady state, (is that) each generation has to satisfy its own needs without mortgaging the capabilities of future generations to overcome their needs. In this model, the saving rate is no longer constant, but is determined by the per capita capital stock, k . Therefore, the average level of the saving rate is not fixed, so it can rise or fall as the economy develops. The saving rate is also determined by interest rates, tax rates, and subsidies. The Ramsey model still has a convergence property under general conditions, so that the Solow-Swan Model (SSM), with a constant saving rate, is treated as a special case. Bajona and Kehoe (2006) have developed a similar model in an infinitely long-lived consumers on the overlapping generations basis. I pointed out several problems in these kinds of analysis: the absence of technological progress in the model implies that intergenerational trade has many problems: 1) The constant returns production function at the aggregate level can reflect learning-by-doing and spillovers of technology but is not Pareto-optimal; 2) There is no attempt to internalize (within generations and countries) spillovers of technology; 3) Convergence to steady states and price equalization indicates that countries and generations are strictly identical and, therefore, intergenerational and international trade is impossible; 4) The picture of properties of a dynamic model (HO) poses the problem of dynamic inefficiency. Fundamentally, we should admit that the first generations impose externalities (positive or negative) on subsequent generations. Bajona and Kehoe model ignores intergenerational and international trade interferences. Intergenerational trade is one of the main reasons why some countries are developed and others are not. The hypothesis of fixed endowments for consumer-workers cannot be stated. Several other hypotheses of this model should be revisited./.

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Abstract

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The Qeios logo is displayed in a large, bold, blue font. The word "Qeios" is centered within a light gray rectangular background. The letter "Q" is particularly large and stylized, with a thick stroke. The dot on the "i" is a solid blue circle. The overall design is clean and modern.

The main result is that if people want to hold money, then budget deficits cannot be avoided to achieve full employment when the economy grows at a positive rate under constant prices. We examine the existence of budget deficits in a growing economy in which consumers derive utility not only from consumption but also from money holding and bequest. We use an overlapping generations model in which consumers leave the bequests to descendants and hold money, and we use a Barro-type utility function, where people include the utility of their children in their own utility. We will show that budget deficit is necessary to maintain full employment under constant prices in a growing economy. Thus, the balanced

budget cannot achieve full employment under constant prices. Moreover, we show that if the actual budget deficit is smaller (larger) than that which is necessary and sufficient for full employment under constant prices, a recession (inflation) occurs.

Keywords: Budget deficits, Growing economy, Money holding, Bequest, Overlapping generations model, J.M.Keynes

1 Introduction

Using an overlapping generations model in which consumers leave a bequest to descendants and hold money, we examine the existence of budget deficits in a growing economy. The significance of government debt and budget deficit, and intergenerational burden have been analyzed by Junpei Tanaka. References are J.Tanaka(2010,2011a,2011b,2013). Junpei Tanaka focuses on the intergenerational economic welfare gap due to the presence or absence of government debt, but his main model does not include growth and assumes that all government debt is redeemed by taxes. The interest of this paper lies elsewhere. We are interested in proving that we need budget deficits in a growing economy where consumers hold money. In our model, people save their income primarily through capital, but they are also willing to hold money other than capital for reasons such as liquidity. We use a Barro-type utility function, where people include the utility of their children in their own utility, according to Barro(1974) and Carmichael(1982).

In the next section, we will present our model and prove that a budget deficit is necessary to maintain full employment under constant prices in a growing economy. If the actual budget deficit is smaller (larger) than that which is necessary and sufficient for full employment under constant prices, a recession (inflation) occurs.

In another paper, we will analyze a similar problem in an overlapping generations endogenous growth model according to Grossman and Yanagawa(1993) and Maebayashi and Tanaka, J. (2021) incorporating money holding. Moreover, in another paper, we analyze a similar problem in a traditional neoclassical overlapping generations model with exogenous economic growth without bequest according to Diamond(1965) incorporating money holding. In a previous study about the existence of involuntary unemployment, we used another overlapping generations model without bequest according to Otaki(2007,2009,2015).

This paper is an example of an analysis using a very simple model of the following statement by J.M.Keynes.

“Unemployment develops, that is to say, because people want the moon; — men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control.” (Keynes(1936), Chap.17)

In the appendix we show that if money as well as goods are produced by capital and labor, a budget deficit is not necessary for full employment under constant prices. However, if money circulates with a value greater than the cost of production, a budget deficit will be necessary. The difference between the value of money and its production cost is the

so-called *seigniorage*. A moderate seigniorage to economic growth is necessary unless the production of money is quite costly.

The Barro-type utility function is often used to prove the neutrality of government debt or budget deficits, i.e., that increasing government spending due to budget deficits has no effect when future increases in the tax burden are taken into account (Barro(1974), Carmichael(1982)). This paper has a very different aim. Using a similar utility function, we prove that budget deficits are essential in a growing economy.

In our model, money is supplied by wage payments, which consumers use to pay taxes and to pay for consumption and investment in capital. What remains after that is money holding. Net money supply equals an increase in money holding.

In this paper, we will assume that the budget deficit will be financed by seigniorage and does not consider government bonds. In terms of liquidity, government bonds are considered to be somewhere between money and capital (stocks), and it would be possible to introduce them into the model, but it would complicate the discussion.

2 Budget deficit in a growing economy with money holding

We introduce the demand for the bequest of the next generation consumers and money demand or money holding as well as the consumption demand of consumers into an overlapping generations model.

2.1 Consumers' behavior

According to Barro(1974), Carmichael(1982), and J.Tanaka(2010), the utilities of consumers include the discounted value of the utilities of their descendants. Consumers live over two periods, younger (working) and older (retired) periods. In the older period, they leave the bequest to the next generation consumers as well as consume goods. The younger consumers in Period t maximize

$$U_t = U_t^y + \beta U_t^o, \text{ where}$$

$$U_t^y = \ln c_t^y + \ln m_t^y + (1 - \beta) \ln c_t^o, \quad \beta > 0, \quad 1 - \beta > 0.$$

U_t^o is the indirect utility function of the next generation consumers perceived by the consumers in Period t . β is the rate of population growth. $\beta > 0$ is the discount rate. c_t^y is consumption in the younger period, c_t^o is consumption in the older period, m_t^y is money holding of a younger consumer in Period t . p_t is the price in Period t . Let b_t and s_t be the bequest and savings of a consumer in Period t , then

$$U_t = (1 + \beta)(U_t^y - U_t^o) + U_t^o - U_{t+1}^y = (1 + \beta)U_t^y - U_{t+1}^y - U_{t+1}^o + U_t^o.$$

r_t is the rate of return on capital received in Period $t + 1$. p_{t+1} is the price in Period $t + 1$. From this

$$U_t = \beta U_{t+1}^y + (1 + \beta)U_t^y + \beta U_{t+1}^o. \quad (1)$$

We assume

$$i - i' > 0.$$

$i - i'$ is the investment to capital in Period .

The budget constraint for younger consumers in Period is

$$i_{t+1} = i_t + (1 - \tau) i_t + (1 + r) i_t - i_t = (1 - \tau) i_t + (1 + r) i_t - i_t = (1 - \tau) i_t + r i_t = (1 - \tau + r) i_t$$

$$i_{t+1} = (1 - \tau) i_t + (1 + r) i_t - i_t = (1 - \tau) i_t + (1 + r) i_t - i_t = (1 - \tau) i_t + r i_t = (1 - \tau + r) i_t$$

i_t is consumption in the older period of a consumer in the previous generation. i_{t+1} is the bequest left by previous generation consumers which is equally distributed to consumers in Period . i_t is the nominal wage rate, and τ is the tax rate. i_{t+1} , i_t and i_t are the values of the variables in Period - 1. If there exists unemployment in Period - 1, i_{t+1} , i_t , i_t and i_t are the average values across employed and unemployed consumers. i_t is the indicator of whether the consumer is employed or not. If he is employed (unemployed), $i_t = 1$ ($i_t = 0$). The budget constraint is rewritten as follows.

$$i_{t+1} = (1 - \tau) i_t + (1 + r) i_t - i_t = (1 - \tau) i_t + (1 + r) i_t - i_t = (1 - \tau) i_t + r i_t = (1 - \tau + r) i_t$$

The Lagrange function is

$$\mathcal{L} = \ln i_t + \ln i_{t+1} + (1 - \tau) \ln i_t + (1 + r) \ln i_t - i_t = (1 - \tau) \ln i_t + (1 + r) \ln i_t - i_t$$

λ_t is the Lagrange multiplier. The first-order conditions for utility maximization are

$$u'(c_t) = \lambda_t, \quad (2)$$

$$p_t c_t = \lambda_t, \quad (3)$$

$$1 - \lambda_t = 1 - \lambda_t, \text{ and} \quad (4)$$

$$\frac{\partial L}{\partial c_t} = 0 = \frac{1}{c_t} - \lambda_t \quad (5)$$

From the descendant's budget constraint,

$$\frac{\partial L}{\partial c_t} = \frac{1}{c_t} - \lambda_t = 0 \quad (6)$$

From (2), (3), and (4), we get

$$\lambda_t = \frac{1}{c_t} = \frac{1}{A(1 - \lambda_t) + \lambda_t} \quad (7)$$

$$\lambda_t = \frac{1}{A(1 - \lambda_t) + \lambda_t} \quad (8)$$

$$\lambda_t = (1 - \lambda_t) \quad (9)$$

From (1), the savings are

$$s_t = \lambda_t + (1 - \lambda_t) A(1 - \lambda_t) - \lambda_t \quad (10)$$

$$s_t = \lambda_t + (1 - \lambda_t) A(1 - \lambda_t) - \lambda_t \quad (11)$$

Let c_t be the employment, and N_t be the population in Period t . Assume that the population increases at the rate of $n > 0$, that is,

$$N_t = (1 + n) N_{t-1}.$$

We assume that upto Period $t - 1$ full employment is achieved and the prices are constant. The total consumption of the younger consumers in Period t is

$$\lambda_t = \frac{1}{c_t} = \frac{1}{A(1 - \lambda_t) + \lambda_t} \quad (12)$$

λ_t is the total bequest in Period t . The total money holding in Period t is

$$\lambda_t = (1 - \lambda_t) \quad (13)$$

The total consumption of the younger consumers in Period $t + 1$ is

$$c_{t+1}^y = (1 - \alpha)w_t + (1 - \beta)w_{t+1} - \alpha w_{t+1} = (1 - \alpha)w_t + (1 - \beta)w_{t+1} - \alpha w_{t+1}.$$

On the other hand, the total consumption of the older consumers in Period t is

$$c_t^o = \alpha w_t + (1 - \alpha)w_{t+1} - (1 - \beta)w_{t+1} = \alpha w_t + (1 - \alpha)w_{t+1} - (1 - \beta)w_{t+1}.$$

The total savings in Period t is

$$s_t = w_t - c_t^y - c_t^o = w_t - (1 - \alpha)w_t - (1 - \beta)w_{t+1} + \alpha w_t = \alpha w_t - (1 - \beta)w_{t+1}. \quad (7)$$

The real value of the capital in Period $t + 1$ is

$$k_{t+1} = (1 - \delta)k_t + s_t = (1 - \delta)k_t + \alpha w_t - (1 - \beta)w_{t+1}. \quad (8)$$

The savings of the consumers other than the money holding is invested to the capital. Similarly, for Period $t - 1$,	
$s_{t-1} = (1 - \alpha)w_{t-1} - (1 - \beta)w_t = (1 - \alpha)w_{t-1} - (1 - \beta)w_t.$	(9)
The total bequest and the total money holding in Period $t - 1$ are also written as	
$w_t = (1 - \alpha)w_{t-1} + s_{t-1},$	(10)
$w_t = (1 - \alpha)w_{t-1} + s_{t-1}.$	(11)

2.2 Firms' behavior

About production we suppose

$$Y_t = F(K_t, L_t) = F(K_t, 1).$$

Y_t is the real GDP in Period t . $F(K_t, L_t)$ is the production function. We assume constant returns to scale technology. The profit of a firm is

$$\pi_t = Y_t - w_t L_t - r_t K_t = F(K_t, 1) - w_t - r_t K_t.$$

We normalize the number of firms to one. The conditions for profit maximization are

$$1 = w_t / F_1(K_t, 1),$$

$$r_t = F_2(K_t, 1) = F'(K_t).$$

From them

$$1 = w_t / F_1(K_t, 1) \Rightarrow w_t = F_1(K_t, 1).$$

w_t is decreasing in K_t .

$$= (1 - \frac{1}{4}) \frac{1}{4} + \frac{1}{4} - \frac{1}{4} + \frac{1}{4} = \frac{1}{4}.$$
$$I = \frac{1}{\beta} \left(\frac{1}{1 + (1 - \beta) A(1 - \beta) \frac{1}{4} + \frac{1}{\beta} \frac{1}{4} B} \right) \left(\frac{1}{1 + \beta + (1 - \beta) T(1 - \beta) \frac{1}{4} + \frac{1}{\beta} \frac{1}{4} U} \right)$$
$$! - ! 4! = ! - !_{+ \#} = ! 4! - !_{+ \#} 4!_{+ \#} = ! 4!_{+ \#}. \quad (12)$$

Proposition *If the economy grows at a positive rate, and the consumers derive positive utility from holding money, we need a positive budget deficit to maintain full employment under constant prices.*

If the actual budget deficit is smaller than that in (12), \bar{y} is smaller than $(1 + \frac{1}{\lambda}) \bar{y}_0$. Since a consumer divides his budget into consumption, bequests and money holding, his consumption is also smaller than that when (12) is satisfied. Then, a recession (with involuntary unemployment) occurs. On the other hand, if the actual budget deficit is larger than that in (12), \bar{y} is larger than $(1 + \frac{1}{\lambda}) \bar{y}_0$, and the nominal consumption also increases. However, under full employment, production can not further increase. Thus, an inflation is triggered.

We have mainly proved the following results by incorporating consumers' desire to hold money into an overlapping generations model with Barro-type bequest motives.

1. The budget deficit is necessary and inevitable to maintain full employment under constant prices when people derive utility from money holding as well as consumption and the economy grows at a positive rate.
2. If the budget deficit is insufficient (excessive), a recession (inflation) occurs.

Although we considered only growth due to population growth, we expect to obtain similar results for growth due to technological progress.

Appendix: When Money is Produced

We suppose that money is produced by capital and labor. The production function is

$$M_t = f(K_t, L_t) = f(K_t, 1 - \alpha K_t).$$

M_t is the supply of money in Period t . It is the total cost to produce money in Period t . The market equilibrium condition for the goods is

$$M_t = R(1 - \alpha)M_t + M_t - 1 + \alpha M_t + M_t + M_t + M_t + M_t = M_t + M_t - V_t.$$

The condition for the money market equilibrium is that the supply of money equals an increase in money holding. Therefore,

$$V_t = M_t - M_{t-1}.$$

Then, in the steady state, (12) is reduced to

$$M_t - M_{t-1} = 0.$$

Therefore, if money as well as goods are produced by capital and labor, a budget deficit is not necessary for full employment under constant prices.

However, if money circulates with a value greater than the cost of production, a budget deficit will be necessary. In that case we have

$$V_t < M_t - M_{t-1}.$$

Then,

$$M_t - M_{t-1} = V_t$$

is the *seigniorage*. A moderate seigniorage to economic growth is necessary unless the production of money is quite costly.

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