

Peer Review

Review of: "Primary Hairs May Create Echoes"

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The manuscript presents an important study on black holes with primary Proca-Gauss-Bonnet hair and their potential to generate echoes in the ringdown phase. The authors investigate the quasinormal spectrum of scalar and Dirac test fields in these backgrounds, employing both the higher-order WKB method with Pade resummation and time-domain integration. Their results convincingly demonstrate that the presence of primary hair alone can give rise to echo signals, without requiring any environmental effects or modifications at the horizon scale, thereby providing a clean theoretical mechanism for this phenomenon.

The manuscript is well-organized and mathematically rigorous, with clear explanations of the computational techniques and detailed illustrations provided in six figures. The introduction motivates the study by situating it within the broader context of black hole perturbation theory and observational interest in gravitational-wave echoes. The derivation of the quasinormal spectra and the subsequent identification of echo structures are carefully executed and well-documented, allowing readers to follow the methodology and verify the results.

The work would benefit from additional clarifications and extensions. First, including a plot of the effective potential would help the reader visually understand the mechanism behind the echo formation. Second, it would be valuable to clarify whether the derivative interaction of the Proca field alone is sufficient to produce echoes, or if the inclusion of the Horndeski scalar interaction is necessary. Finally, while the test-field analysis provides a strong indication of echo formation, a full linear perturbation study of the Einstein-Proca-Gauss-Bonnet system would be critical. Such an analysis would check for the potential existence of ghost or gradient instabilities, which are essential to ensure the hyperbolic evolution of perturbations and the theoretical consistency of the model. Including this discussion would strengthen the paper and provide guidance for future work.

In summary, the manuscript offers a novel and well-substantiated contribution to the study of black hole perturbations in extended gravity theories. It successfully demonstrates a mechanism for echo formation due to primary hair and lays the groundwork for future studies on the stability and observational signatures of such systems.

Declarations

Potential competing interests: No potential competing interests to declare.