Review of: "Taylor Series Based Domain Collocation Meshless Method for Problems with Multiple Boundary Conditions including Point Boundary Conditions"

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Potential competing interests: No potential competing interests to declare.

Regarding “Taylor Series Based Domain Collocation Meshless Method for Problems with Multiple Boundary Conditions including Point Boundary Conditions”

This paper deals with handling boundary conditions for the so-called Taylor Meshfree Method (TMM, not to be confounded with Transport Meshfree method, a kernel particle method) and proposes a method to enhance TMM with handling particular multiple boundary conditions. This paper then illustrates numerically the proposed method. It discusses technical choices to tune the model to Helmholtz and Poisson equations on some domains (rectangles, circles, and an exotic amoeba shape), illustrating the capacity of meshfree methods to handle complex geometries.

The overall impression of this paper is that the proposed method is quite interesting, but I would not recommend to publish this paper without modifications.

A first recommendation to the authors would be to reconsider their notations, that are hardly readable. For instance, the first equations (1)-(2) could be written more concisely, as well as with more generality as $u(z) = \langle U(x),K(x,z)\rangle$, $\langle . \rangle$ being the standard scalar product, $u(x) = (u(x_i))_{i=1\dots n}$, $K(x,z)= (k(x_i,z))_{i=1\dots n}$ a vector, k being a kernel, induced by the Lagrange interpolation that the authors are studying.

This suggestion is a little bit more than a notation suggestion. Writing under the form $u(z) = \langle U(x),K(x,z)\rangle$ allows to consider the proposed method for any kernels, not only Lagrange based one, as is done in this paper. To support this assessment, note that the matrix [A] just after equation (9), is indeed a Gram matrix, that is a general tool for kernel methods. In particular, the method proposed by the authors is already used within this more general formulation to handle multiple mixed-type boundary conditions for kernel PDE methods, but has never been studied in the particular case of Lagrange interpolation that the authors suggest to my knowledge. Thus the paper keeps its interest.

The second recommendation to the authors would be to be more precise concerning the range of applications that the proposed method can address. The proposed method addresses problems for which a Lagrange interpolation can give satisfactory results, that are likely to be PDE problems having analytical, smooth solutions, as well as boundary conditions expressed for points for which the Lagrange interpolation is stable. This is the case of all numerical results presented by the authors, but a warning to readers should be done: the method might not work for solutions of PDEs having heavy non linear behavior, or for boundary conditions expressed on points where a Lagrange interpolation is unstable (typically if high-order Lagrange interpolation is used at boundaries). Finally, concerning the numerical tests, we would suggest the authors to be more synthetical : maybe one or two examples, with a broader set of comparing methods, well chosen and discussed, would be enough to support their view.

The third recommendation would be to try tackling error and convergence analysis for the proposed method, without which the study remains entirely experimental. There exists approaches that should work to perform this task.