

Research Article

An Empirical Constraint on the Hubble Constant from the Relation $\Omega_\gamma = \alpha^2$

Boris Kulangiev¹

1. Independent researcher

We examine the identity $\Omega_\gamma = \alpha^2$ relating two independently measured dimensionless quantities of distinct physical origin: the CMB photon density parameter and the squared fine structure constant. Because the photon density parameter depends on the Hubble constant while α^2 does not, the identity is not an approximate numerical coincidence but an exact constraint: it maps the measured photon temperature onto a single value of the Hubble constant, $H_0 = 68.15 \text{ km s}^{-1} \text{ Mpc}^{-1}$, with a $\pm 0.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$ uncertainty propagated from the FIRAS temperature. The significance of the relation rests not on how closely it holds—at its predicted H_0 it holds exactly, by construction—but on whether that independently selected value agrees with Hubble-constant determinations that use no distance ladder. The predicted value is consistent with the Planck 2018 determination (TT,TE,EE+lowE+lensing, $H_0 = 67.36 \pm 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1}$) at 1.5σ , lies between early-universe and intermediate-distance determinations, and is in 4.7σ tension with the SHOES distance-ladder value. We make no theoretical claim regarding the origin of the identity; we treat it as an empirical hypothesis predicting a definite H_0 , quantify the prediction's sensitivity to input parameters, discuss interpretive considerations including the look-elsewhere effect, and identify the gravitational-wave-standard-siren and strong-lensing measurements that will confirm or exclude the prediction within the decade.

Correspondence: papers@team.qeios.com — Qeios will forward to the authors

1. Introduction

The Cosmic Microwave Background (CMB) temperature $T_{\text{CMB}} = 2.72548 \pm 0.00057 \text{ K}$ is among the most precisely measured quantities in cosmology ^[1]. In the standard Λ CDM reading this temperature is interpreted as a relic of the early-universe thermal history (set by the baryon-to-photon ratio η from

baryogenesis), with no fundamental principle in the Standard Model or General Relativity that would predict η or T_{CMB} at the percent level from first principles.

The fine structure constant, defined (in SI) as $\alpha = e^2/(4\pi\epsilon_0\hbar c) \approx 1/137.036$, quantifies the strength of the electromagnetic interaction. Like T_{CMB} , it has no accepted derivation from more fundamental principles.

In this note we examine the identity $\Omega_\gamma = \alpha^2$ relating these two quantities. We present the arithmetic, test its sensitivity to cosmological input, derive the Hubble constant predicted by the identity, and discuss its interpretation — geometric motivation, the question of whether the CMB temperature is a relic or a present-epoch quantity, and arguments for and against physical significance.

2. Statement of the Relation

The energy density of blackbody radiation at temperature T is

$$u_{CMB} = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T_{CMB}^4. \quad (1)$$

The critical density of the universe is

$$\rho_c = \frac{3H_0^2}{8\pi G}. \quad (2)$$

The photon density parameter is $\Omega_\gamma \equiv u_{CMB}/(\rho_c c^2)$. The empirical observation is

$$\boxed{\Omega_\gamma = \alpha^2} \quad (3)$$

Equivalently, solving for the temperature,

$$T_{CMB} = \sqrt{\alpha} \left[\frac{15\hbar^3 c^5 \rho_c}{\pi^2 k_B^4} \right]^{1/4}, \quad (4)$$

where the bracketed quantity is the temperature whose blackbody radiation energy density equals $\rho_c c^2$.

Physical photon density and H_0 dependence.

Combining Eqs. (1) and (2) gives the physical photon density,

$$\Omega_\gamma h^2 = \frac{8\pi^3 G k_B^4}{45\hbar^3 c^5 H_{100}^2} T_{CMB}^4 = 2.473 \times 10^{-5}, \quad (5)$$

where $h \equiv H_0/H_{100}$ and $H_{100} \equiv 100 \text{ km s}^{-1} \text{ Mpc}^{-1} = 3.2408 \times 10^{-18} \text{ s}^{-1}$. Because the explicit H_0 cancels from numerator and denominator, the quantity $\Omega_\gamma h^2$ is determined uniquely by T_{CMB} and fundamental

constants; in particular, $\Omega_\gamma h^2$ can be computed without any prior knowledge of H_0 .

In contrast, the right-hand side of Eq. (3) multiplied by h^2 ,

$$\alpha^2 h^2 = 2.416 \times 10^{-5}, \text{ (using Planck } h = 0.6736), \quad (6)$$

retains an explicit dependence on h . The identity $\Omega_\gamma h^2 = \alpha^2 h^2$ is therefore a constraint on h : it predicts a specific numerical value for the Hubble constant, derived in Sec. 3.

3. Numerical Verification

3.1. Forward calculation

Using the Planck 2018 best-estimate cosmological parameters ^[2] (TT,TE,EE+lowE+lensing combination, $H_0 = 67.36 \text{ km s}^{-1} \text{ Mpc}^{-1}$) and CODATA 2018 fundamental constants, Table 1 summarizes the test of the identity (3) and the inferred Hubble constant.

Quantity	Value
α^2 (CODATA 2018)	5.325×10^{-5}
Ω_γ (Planck $H_0 = 67.36$)	5.450×10^{-5}
Ω_γ / α^2	1.024
H_0 implied by exact equality	$68.15 \text{ km s}^{-1} \text{ Mpc}^{-1}$

Table 1. Numerical test of $\Omega_\gamma = \alpha^2$ using observed quantities.

3.2. Inverse prediction: determining H_0

Combining the Stefan–Boltzmann law (1) with the critical density (2), the photon density parameter as an explicit function of the Hubble constant is

$$\Omega_\gamma(H_0) \equiv \frac{u_{CMB}}{\rho_c c^2} = \frac{8\pi^3 G k_B^4}{45 \hbar^3 c^5} \frac{T_{CMB}^4}{H_0^2}, \quad (7)$$

which makes the $\Omega_\gamma \propto H_0^{-2}$ scaling manifest. Imposing the identity (3), $\Omega_\gamma(H_0) = \alpha^2$, and solving for H_0 yields a closed-form expression in terms of T_{CMB} , α , and fundamental constants alone:

$$H_0 = \frac{T_{CMB}^2}{\alpha} \sqrt{\frac{8\pi^3 G k_B^4}{45 \hbar^3 c^5}}. \quad (8)$$

No measured value of H_0 enters the right-hand side: the value is extracted from the photon temperature and the electromagnetic coupling only.

Substituting $T_{CMB} = 2.72548$ K and $\alpha = 1/137.036$ yields

$$H_0^{pred} = 68.15 km s^{-1} Mpc^{-1}. \quad (9)$$

Because $H_0^{pred} \propto T_{CMB}^2/\alpha$ and the fractional uncertainty in α is $\sim 10^{-10}$, the uncertainty is dominated by the FIRAS temperature error:

$$\frac{\sigma_{H_0}}{H_0} = 2 \frac{\sigma_T}{T} = 4.2 \times 10^{-4} \implies \sigma_{H_0} \approx 0.03 km s^{-1} Mpc^{-1}. \quad (10)$$

The predicted value lies between the two major determinations of the Hubble constant (Table 2).

Measurement	H_0 (km s ⁻¹ Mpc ⁻¹)	Tension
Planck 2018 (CMB) ^[2]	67.36 ± 0.54	1.5σ
ACT DR4 (ACT+WMAP) ^[3]	67.6 ± 1.1	0.5σ
This relation	68.15 ± 0.03	—
TRGB (Freedman) ^[4]	69.8 ± 1.7	1.0σ
SH0ES (Cepheids) ^[5]	73.04 ± 1.04	4.7σ

Table 2. Predicted H_0 from Eq. (8) compared with current measurements. The tension column gives the deviation from the predicted value.

3.3. Sensitivity to H_0

Since $\Omega_\gamma \propto H_0^{-2}$ (at fixed T_{CMB}), the relation is sensitive to the assumed Hubble constant. The relation holds exactly for $H_0 = 68.15 km s^{-1} Mpc^{-1}$ and degrades smoothly away from this value (Table 3).

H_0 (km/s/Mpc)	Ω_γ/α^2	Deviation
65.00	1.099	+9.9%
67.00	1.035	+3.5%
67.36	1.024	+2.4%
68.00	1.004	+0.4%
68.15	1.000	0.0%
69.00	0.975	−2.5%
70.00	0.948	−5.2%
73.00	0.871	−13.0%

Table 3. Sensitivity of Ω_γ/α^2 to the assumed Hubble constant. Exact equality requires

$$H_0 = 68.15 \text{ km s}^{-1} \text{ Mpc}^{-1}.$$

4. Discussion

4.1. Character of the relation

Equation (3) connects a cosmological quantity (Ω_γ , determined by T_{CMB} and H_0) with an atomic quantity (α , determined by the elementary charge, Planck’s constant, and the speed of light). No known mechanism in the Standard Model or general relativity ties these quantities together; in the standard Λ CDM reading, T_{CMB} is interpreted as a relic of the early-universe thermal history (with η from baryogenesis), while α is a fundamental coupling of quantum electrodynamics.

The identity is not of the Dirac large-number type. Those typically arise from ratios of Planck-scale and Hubble-scale quantities, producing numbers of order 10^{60} . Here $\alpha \approx 1/137$ is of order 10^{-2} , and the relation equates $\alpha^2 \approx 5.3 \times 10^{-5}$ to Ω_γ . Both quantities are small but not astronomically so, and neither is a Planck-to-Hubble ratio.

The identity $\Omega_\gamma = \alpha^2$ as stated is a relation among quantities determined by present-day observation: Ω_γ from the measured T_{CMB} and the cosmological parameters, α from atomic physics. The physical

combination $\Omega_\gamma h^2$ is set by T_{CMB} alone, independent of H_0 , and the identity equates this combination, via Eq. (5), with $\alpha^2 h^2$ — thereby predicting the present-day h (Sec. 3.2).

Whether the relation holds also at earlier epochs is a separate question whose answer depends on the underlying cosmological model. We do not address this question here; we examine the identity as stated and treat its predictive content for H_0 as the testable consequence.

4.2. Geometric and physical motivation

If the identity reflects underlying physics, two interpretive readings are suggestive. First, Ω_γ is the fraction of cosmic energy density carried by photons, while $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ is the dimensionless coupling that controls how strongly photons interact with electric charge. The relation $\Omega_\gamma = \alpha^2$ then equates the *abundance* of photons in the cosmic energy budget to the *squared coupling strength* of the photon sector. Read this way, the identity expresses a correspondence between the macroscopic photon content of the universe and the microscopic coupling that governs photon-matter interaction.

Second, both quantities are dimensionless ratios constructed from independent inputs: Ω_γ from thermal-radiation energy density relative to the cosmic critical density (set by H_0 and G), and α^2 from the electromagnetic coupling. Their equality, if real, would constitute a relation between gravitational and electromagnetic scales not anticipated by the Standard Model or by general relativity alone.

These readings are interpretive, not derivations: the identity itself is empirical, and no specific mechanism is required for the prediction $H_0 = 68.15 \text{ km s}^{-1} \text{ Mpc}^{-1}$ to be tested. We note them as candidate motivations for why the relation might reflect underlying physics rather than coincidence.

4.3. CMB temperature as a present-epoch quantity

The identity $\Omega_\gamma = \alpha^2$ involves the present-day value of Ω_γ and is silent about earlier epochs — the relation at $z \gtrsim 0$ depends on the cosmological model, while at $z = 0$ it reduces to a constraint among the present-epoch observables T_{CMB} , α , and H_0 . Its status therefore hinges on the interpretation of T_{CMB} itself.

The standard reading is that T_{CMB} is a fading relic of the early universe, set by thermal-history parameters such as the baryon-to-photon ratio and the post-annihilation photon-to-neutrino temperature ratio. Under this reading, the identity at $z = 0$ is a coincidence of timing: the time-evolving Ω_γ happens to coincide with α^2 at the present epoch, and would not have held at recombination ($\Omega_\gamma \approx 0.15$) or at most other times.

An alternative reading is that T_{CMB} reflects a present-epoch property of the cosmic vacuum — for instance, an equilibrium condition between the radiation sector and underlying physics that fixes T_{CMB} as a structural quantity rather than as a relic temperature. Under this reading, the identity is structural, and T_{CMB} is determined now (by the present configuration of fundamental constants) rather than then (by initial conditions of the early universe). Any framework in which the CMB is a stationary or quasi-stationary equilibrium quantity, rather than a fading relic, would naturally predict relations between Ω_γ and other coupling constants.

We do not attempt to discriminate between these readings here. The empirical content of the identity — the prediction $H_0 = 68.15 \text{ km s}^{-1} \text{ Mpc}^{-1}$ — is independent of which interpretation is correct, and the prediction will be tested by improved H_0 measurements regardless of the underlying mechanism.

4.4. A possible standard-cosmology pathway

Within the standard Λ CDM reading, one possible physical pathway for the identity is the following. The photon density at early times is fixed by thermal equilibrium at temperature T_γ , and the subsequent ratio of photon and neutrino temperatures is set by e^+e^- annihilation [6]. The e^+e^- annihilation cross section is proportional to α^2 . The baryon-to-photon ratio $\eta \approx 6 \times 10^{-10}$ is set by baryogenesis and the subsequent thermal history [7]. Whether these dependencies combine to produce $\Omega_\gamma = \alpha^2$ as an exact result within standard Big Bang nucleosynthesis is an open theoretical question; we note the pathway without claiming a derivation. This pathway is offered as one possibility within Λ CDM cosmology; alternative interpretations are discussed in Sec. 4.3.

4.5. Arguments for physical significance

Several features argue against pure coincidence. First, the predicted $H_0 = 68.15 \text{ km s}^{-1} \text{ Mpc}^{-1}$ lies 1.5σ from the Planck 2018 determination despite the prediction using only observational inputs independent of any Planck cosmology fit (the FIRAS temperature and CODATA α). Second, the predicted value falls between the Planck and TRGB determinations [4] and well below the SH0ES determination [5], raising the possibility that it captures the true global expansion rate more accurately than either endpoint of the H_0 tension. Third, the selected value is corroborated by determinations that share none of its inputs. The sound-horizon standard ruler, calibrated on pre-recombination physics and measured through baryon acoustic oscillations, yields expansion rates in the $67\text{--}68 \text{ km s}^{-1} \text{ Mpc}^{-1}$ band under a Λ CDM-like history, independent of both the photon temperature and the local distance ladder. That an

atomic-and-thermal relation with no cosmological fit selects an H_0 coincident with the sound-horizon and CMB-acoustic determinations, while departing from the distance-ladder value, is the empirical content that distinguishes the relation from an arbitrary pairing.

The relation is silent on the origin of the higher distance-ladder values; whether these reflect a local environmental effect or an unresolved systematic is beyond the scope of this note, which concerns only the value selected by the identity and its comparison with ladder-independent determinations.

4.6. Arguments against physical significance

The principal objection is the look-elsewhere effect. Given the many dimensionless constants in physics (α , α_s , $\sin^2 \theta_W$, η , the Yukawa couplings) and the several cosmological density parameters (Ω_b , Ω_c , Ω_γ , Ω_Λ , Ω_m), the number of pairwise combinations with various integer and half-integer powers is easily of order 10^2 . A coincidence at the few-percent level among ~ 100 trials has a non-negligible probability of occurring by chance, even if (as here) exact equality is recovered at the implied H_0 . A rigorous statistical treatment would require specifying the prior over possible relations; we do not attempt this here. We note, however, that the pairing examined here is not drawn arbitrarily from this space: it relates the photon share of the cosmic energy budget to the coupling constant of the photon sector, a physically motivated correspondence rather than a match sought among unrelated quantities. The look-elsewhere penalty applies to searches over an unstructured space of pairings; it is weaker for a relation whose two sides belong to the same physical sector and could have been written down in advance. We do not claim this removes the concern, only that it bounds it.

The relation selects H_0 sharply: Ω_γ/α^2 departs from unity by 2.4% at Planck's H_0 and by 13% at SH0ES's, so a future consensus far from $68.15 \text{ km s}^{-1} \text{ Mpc}^{-1}$ would falsify it. This is a strength rather than a weakness: the claim is testable. But it also means that the relation's viability is tied to the outcome of the Hubble tension.

4.7. Falsifiability and future tests

The relation makes a definite, testable prediction: $H_0 = 68.15 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The accompanying $\pm 0.03 \text{ km s}^{-1} \text{ Mpc}^{-1}$ propagates the FIRAS temperature error through Eq. (8) and reflects how sharply T_{CMB} fixes H_0 under the identity, not confidence that the identity holds. The systematic uncertainty in any test is set by the precision of the independent H_0 determination against which the prediction is compared. Upcoming measurements from gravitational-wave standard sirens [8], strong-lensing time

delays [9], and further refinements of the distance ladder will sharpen the comparison within the next several years.

A second consistency check is available. The relation $\Omega_\gamma = \alpha^2$ is specific to the photon sector by construction, since Ω_γ is determined by T_{CMB} alone. The total radiation density Ω_r includes a neutrino contribution proportional to N_{eff} , and Ω_r/Ω_γ is set by the photon-neutrino temperature ratio established at electron-positron annihilation. If a future measurement of Ω_r were found to bear a similarly clean relation to α , this would suggest a thermal-history origin involving both species; the photon-only relation studied here would then be a constituent of a broader pattern. Continued agreement of $\Omega_\gamma = \alpha^2$ without a corresponding Ω_r relation would point instead to physics specific to the photon sector.

5. Conclusion

We have examined the identity $\Omega_\gamma = \alpha^2$ connecting the CMB photon density parameter to the square of the fine structure constant. Combined with the FIRAS temperature measurement and the CODATA value of α , the identity predicts a Hubble constant $H_0 = 68.15 km s^{-1} Mpc^{-1}$ (Sec. 3.2), consistent with Planck at 1.5σ and in 4.7σ tension with the local SHOES determination. We present the observation as an empirical relation, not as a derivation from first principles. Whether it reflects an unrecognized constraint from primordial thermodynamics, a feature of underlying physics, or a numerical accident awaits both theoretical investigation and convergence of independent H_0 measurements. Two questions are open. First, can the relation be derived from any specific physical mechanism, in Λ CDM or in extensions thereof? Second, will the predicted $H_0 = 68.15 km s^{-1} Mpc^{-1}$ be confirmed or excluded by gravitational-wave standard sirens, strong-lensing time delays, and refined distance ladders, all expected to reach percent-level precision within the decade?

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