

Zeno and Einstein

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Abstract

A common thread unites Zeno's denial of the reality of motion and Einstein's denial of an objective, i.e. frame-independent, present moment. Both claims privilege the intellect over experience, the abstract over the concrete. My guide – or rather foil – in this discussion is the philosopher of science Wesley Salmon.

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1. Introduction

Sometime after 500 BC the Greek philosopher Zeno presented compelling arguments that motion generates paradox and therefore cannot be real. Yet no one takes his conclusion seriously. Since things do actually move, the task is to identify Zeno's error.

Why then do philosophers and physicists take seriously Einstein's equally absurd claim that objects in relative motion occupy different times? According to the principle of the relativity of simultaneity, anything that moves relative to an observer occupies a different present moment from that of the observer. The answer, I contend, is that the relativity of simultaneity is falsely associated with the proven phenomenon of time dilation. Because time does in fact dilate for an object with a speed approaching that of light, and because Einstein successfully explains this dilation in his special theory of relativity, we naturally assume that relative simultaneity, which plays a prominent role in Einstein's theory, is also physically real.

In this article I demonstrate that the relativity of simultaneity cannot be affiliated with time dilation because (1) whereas relative simultaneity is symmetrical between observers in different frames of reference, time dilation is asymmetrical between frames, (2) whereas relative simultaneity follows from nothing more than observation, time dilation requires a physical force, (3) whereas relative simultaneity provides no means of verification, the effect of time dilation can be verified at any time in any frame of reference, (4) whereas relative simultaneity depends on the equality of all frames with respect to the timing of a given set of events, a simple mechanism reveals that the timing of events in their proper frame is privileged over the timing as viewed from other frames and (5) relative simultaneity does not even predict time dilation but

instead predicts a mythical phenomenon I have dubbed "time regression." Once it has been decoupled from time dilation, Einstein's principle of relative simultaneity is relegated to the status of Zeno's denial of the reality of motion, an intellectually stimulating but vacuous claim.

I begin with an examination of Zeno's paradoxes of motion and demonstrate that the standard response to Zeno misses the point. The failure to grasp Zeno's fundamental error follows from the same overreliance on abstraction that has perpetuated Einstein's misunderstanding of the role of time in special relativity.

2. Zeno's Denial of Motion

Is motion real or merely apparent? Taken at face value, the paradoxes attributed to Zeno of Elea demolish the commonsense belief in motion and change. In each of the three paradoxes I consider, Zeno's logic is airtight.

In a race with a tortoise which has been given a head start, Achilles cannot overtake his sluggish opponent because no matter how fast he runs, he must first arrive at the tortoise's starting point, by which time the tortoise has managed to move ahead a little. So Achilles must reach *that* point, by which time the tortoise has moved still further along the racecourse, etc. Simply by repeating endlessly the ever-shrinking interval separating Achilles from the tortoise's previous location, Zeno establishes that Achilles never quite catches up and therefore cannot pass the tortoise and win the race.

In the Dichotomy paradox, Zeno argues that simply arriving at the finish line of a racecourse is impossible since the runner must first reach the halfway point, after which he must reach the halfway point of the remaining distance, and so on. Forever bogged down in traversing ever smaller distances, the runner never finishes the course. But this is only the "progressive" form of the Dichotomy. The real kicker comes with the "negative" form. In order to go any distance at all, the runner must first reach half that distance, and before he can do that, he must reach half *that* distance, and so on. Thus he can never even start the run, much less finish it (Salmon 1980, 32-33).

The Arrow paradox yields the same conclusion from the assumption that time is composed of instants. According to Zeno, even in flight an arrow is really at rest since it occupies a particular location at a particular instant. As Salmon (1970, 10) puts it, at every instant "the arrow is where it is, occupying a portion of space equal to itself." Moreover, any movement during the instant would, in effect, divide it into parts, each of which would then be the true instant at which the arrow has a precise location and is therefore motionless. According to Russell, the only other option is for the arrow to change position *between* instants, that is, "not at any time whatever" (Salmon 1980, 33-34).

Of course, we can resolve the paradoxes simply by rejecting Zeno's assumptions. As Percy Bridgman put it, "if I literally thought of a line as consisting of an assemblage of points of zero length and of an interval of time as the sum of moments without duration, paradox would then present itself" (Grünbaum 1967, 116). The most natural way around the Arrow is to reject the actuality of zero, specifically zero-duration instants. Zeno's Arrow is paradoxical because a durationless instant is paradoxical. If an instant is to be a unit of time, it must contain some amount of time, no matter how slight. The mathematical concept of a point-instant follows entirely from its convenience in making calculations, not its real-world

applicability. Rather than based on observation, the ideal instant is just that, an ideal generated in human thought. Granted, the actual state of the world is not necessarily what our senses tell us, and nowhere is this better illustrated than quantum mechanics and the indeterminacy that lies beneath the appearance of objects with definite properties. Yet quantum mechanics also reveals the limits of spatial and temporal divisibility. As expressed in Planck length and Planck time, any division of space beyond 10^{-33} centimeters or of time beyond 10^{-43} seconds is physically meaningless. Unrestricted divisibility in classical physics, according to Niels Bohr, is incompatible with the quantum jump (1958, 99).

In regard to the Dichotomy and Achilles paradoxes, we simply reject Zeno's belief in the divisibility of motion. As Bergson (1911, 250-252) points out, to say that Achilles cannot take the lead without first arriving at the previous location of the tortoise is to divide his motion into two motions, one leading up to that point and another continuing beyond it. The fact that we can "disarticulate at will the movement of Achilles" – that is, freeze him in our imagination at a particular point – in no way means his motion actually stops at any point in the race. That we can represent his motion with a line and then divide that line does not render the motion itself divisible. Moreover, the points comprising the line "are not *in* the movement [but] are simply projected by us under the movement, as so many places where a moving body, which by hypothesis does not stop, would be if it were to stop" (1949, 42). Though we may divide Achilles' trajectory in our imagination, only Achilles, by actually stopping, can do so in reality.

Suppose Achilles does indeed stop at each prior point occupied by the tortoise before resuming his run. Though the distance between him and the tortoise is always less than the previous time he stopped, it never drops away entirely. Because the process repeats endlessly, he would have to complete an infinite number of movements to overtake the tortoise. By definition, infinity cannot be completed. If it could be completed, it would not be infinite but would turn out, in retrospect, to have been finite. But Achilles has no need to complete an infinite process. Because he keeps running – happily oblivious to the divisions of his motion generated by the intellect of the observer – he overtakes the tortoise in a single fluid movement.

But distinguishing the actual from the imaginary, the concrete from the abstract, is not at all how the paradoxes are generally believed to be resolved. According to the prevailing view, as Wesley Salmon explains, the reality of motion is restored by way of nineteenth-century calculus. As defined by Cauchy, an infinite sequence is "convergent" if it has a limit, meaning that the terms of the sequence approach and remain arbitrarily close to a given number. So, for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc., the limit is 0. For $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, etc., the limit is 1. By replacing the commas with plus signs we arrive at an infinite series. Each term in the series is a "partial sum" of all the preceding terms. If a sequence of partial sums has a limit, the infinite series is convergent. According to Salmon, "the sum of a convergent infinite series is a number that can be approximated arbitrarily closely by adding up a sufficient (finite!) number of terms." Because the distance traveled by Achilles sums to 1, he reaches the tortoise and therefore can pass it (1980, 36-38).

Rather than accept that the analysis of movement into an infinity of steps is a purely abstract product of the intellect, Salmon attempts to restore reality by way of a second abstraction as ungrounded as the first. That we can cap off an infinite series with a "1"

has no bearing on the world as it exists beyond our imagination.

The same defect characterizes Salmon's use of calculus with respect to Zeno's claim that an arrow is at rest at every point in its trajectory. In accord with a derivative, "the rate of change of position with respect to time... is defined as the limit of the average velocity during decreasing nonzero intervals of time." An arrow traveling inertially 10 feet in 1 second and 1 foot in .1 seconds and 1.2 inches in .01 seconds, etc., has a limit of 10 feet per second, which applies at every instant. "If Zeno felt that the only intelligible instantaneous velocity is zero," says Salmon triumphantly, "nineteenth-century mathematics proved him wrong" (1980, 38-39).

That we can mentally average out velocity over time and then assign that value to a durationless instant in no way establishes movement without duration. Calculus also allows the multiplication of zero by infinity to equal any finite value (Capek 1991, 47). Does this mean a given interval of time is *actually* constructed from an infinity of zero-duration instants? More broadly, that a conceptual procedure is useful for making calculations does not mean nature itself abides by this procedure. Zeno's paradoxes rely on the abstract extremes of zero and infinity to capture the concrete middle. Though intellectually convenient, this procedure creates a problem if taken literally. The standard response to Zeno, as articulated by Salmon, thus amounts to addressing an imaginary problem with an imaginary solution. Why not simply reject the substitution of reality with image in the first place?

By capping off, in our minds, an infinite series with a "1" we are in effect saying that a procedure with an infinite number of steps can be completed in a finite amount of time. Yet even where each step takes only half as long as the previous step, the amount of time in each step never falls to zero. To answer Zeno with calculus is therefore to deny objective meaning to time, in effect to airbrush it out of the picture. Unlike Bergson, who gave time its due regard, Salmon reduces it to a plaything of the intellect. Where it becomes conceptually inconvenient we simply dismiss it.

To his credit Salmon concedes that defining a mathematical operation is no guarantee of its applicability to the physical world (1980, 45). Such caution, however, is nowhere evident in his discussion of special relativity. This is perhaps not surprising given that Einstein's concept of the relativity of simultaneity seems to undermine the objectivity of time. Just as Zeno generated an apparent problem of motion by privileging abstraction over actuality, Einstein conjured from thin air the problem of an objective present moment among bodies in relative motion.

3. The Relativity of Simultaneity

Prior to Einstein theorists assumed that light, like anything else, would appear to travel more slowly to an object pursuing it. In reality, a rocket chasing a photon at one-half the speed of light, $.5c$, would still measure the speed of the photon at exactly c . Special relativity explains this result according to space-time distortion brought on by relative motion in the context of the absolute speed of light. Due to the ratio of the speed of the rocket to c – which far exceeds that of Earth – the fact that the photon continues to outrun the rocket at c means that each second that passes for the rocket is stretched out or dilated relative to a second on Earth. The higher the rocket's speed across space – that is, the more closely it approximates the speed of light – the more sluggish its relative rate of time.

Oddly enough, the concept of time dilation originated with Joseph Larmor and H.A. Lorentz as a way of justifying absolute space and time despite evidence for their pliability with respect to motion in the context of c (Craig and Smith 2008, 2, 31). Only Einstein considered the possibility that the speed of light is a law of nature and therefore, according to the long-established principle of relativity, uniform in all inertial frames of reference (Lorentz, et al, 1923, 38). Unlike the conservation laws or the laws of motion, the absolute speed of light necessitates frame-dependent variation in the rate of time, whether time dilation in the faster-moving frame relative to the slower-moving frame or time gain in the opposite case. Relativistic time distortion has been experimentally verified many times over in a range of contexts. Hafele and Keating's 1971 experiment with atomic clocks flown on commercial aircraft demonstrated both time dilation (time loss) and time gain with respect to not only the absolute speed of light but the gravitational influence of Earth, bringing to bear not only special relativity but Einstein's generalization of it with respect to accelerated frames (Hafele and Keating 1972, 168-70).

Though Einstein's breakthrough 1905 paper establishing modern relativity does mention divergent rates of time among different reference-bodies (Lorentz, et al, 1923, 49-50), his primary focus was the intuitive notion of a single present moment across space, nowadays referred to as a plane of simultaneity (Goldberg 1984, 176). To demonstrate that the timing of events has no objective basis but varies according to one's frame of reference, he began by proposing – after Poincaré – a method for establishing simultaneity across space with a pair of clocks at rest relative to each other (Jammer 2006, 104). To ensure that the clocks are synchronized, the time of clock A is transmitted to clock B. Taking into account the elapsed time during the transmission at the speed of light, the time of clock B is set to match that of clock A. To verify the synchronization, the time of clock B is transmitted back to A. The basis of Einstein's principle of the relativity of simultaneity is that this procedure fails when viewed from a different reference frame (Lorentz, et al, 1923, 39-42). To illustrate his principle he proposed a thought experiment in the 1905 paper and another in his 1920 book, *Relativity: The Special and the General Theory*. Following Hawking and Mlodinow (2010, 96-98), I present a streamlined combination of these thought experiments.

Suppose light is transmitted from the rear to the front of a train in motion relative to an embankment. Because clock A, at the back of the train, has been synchronized at a distance with clock B at the front, both clocks read the same time on the train, that is, in the frame of reference in which the train is at rest. In the embankment frame, however, the motion of the train increases the distance the light must travel between clocks. Since speed is distance over time, and the speed of light is the same in all frames, the increased distance can only mean increased time. When the beam of light reaches the front of the train, clock B therefore reveals a later time to the embankment observer than it does to a passenger on the train. Because the clocks are not synchronized from the standpoint of the embankment, what constitutes a single present moment for the train is a pair of successive moments for the embankment. If the events in question are sufficiently distant from each other, their timing is relative to the reference frame of the observer.

As Bohm points out, as long as we must take into account the limited speed of light in judging the timing of distant events, and as long as all observers measure the same speed of light, "the relativity of simultaneity will be an inescapable necessity" (1996, 58). Also counting in its favor is the fact that the relativity of simultaneity follows from the Lorentz transformation, which replaces the Galilean transformation in the context of electrodynamics.

Despite its logical and mathematical soundness, the physical meaning of Einstein's principle is unclear. Whereas time dilation is an objective effect in which one body undergoes measurably less temporal progression than another, the relativity of simultaneity seems to be a mere clash of viewpoints. How does the frame of reference of an external observer call into question the objectivity of what constitutes the present moment on the train? Surely the projection of the observer's plane of simultaneity onto the train is subjective, in which case Einstein's principle belongs not to physics but to psychology. Yet Salmon (1980, 76) expresses the prevailing view when he denies that there is anything "subjective or illusory about the results" since they "would be the same if inanimate instruments of observation replaced the human observers."

Rather than establish objectivity, however, the fact that a camera on the embankment registers a different timing of events from a camera on the train tells us the illusion follows not from the visual image itself but from our misinterpretation of it. For only a person, not a camera, would demand that an external view of events taking place on the train be as valid as the view from within the train itself. I return to this point in the following section.

As a physical effect, time dilation requires a physical cause, in this case the force that accelerates one reference-body but not another. Once the force is withdrawn, the previously accelerated body travels inertially with a higher ratio of c relative to the other body. The relativity of simultaneity, on the other hand, relies only on the speeds of the bodies relative to each other and therefore cannot account for time dilation. Overlooking this key point, Salmon conflates the physical reasoning of time dilation with the purely conceptual reasoning of relative simultaneity. Having omitted the physical fact of a force generating a higher ratio of c for one object relative to another, Salmon concludes that time dilation is "completely symmetrical." Among a pair of objects in relative motion, he writes, "the time of each is dilated with respect to the other" (1980, 85). Setting aside the intuitive appeal of symmetry, this outcome defies not only logic but the experimentally-established fact that time dilation for one body corresponds to relative time *gain* for the other. Choosing the train as our reference frame in no way grants time dilation to the embankment. Nor does choosing the embankment as our frame grant time dilation to the train. Attributing physical effects to the shifting perspectives of an observer could hardly be more removed from physics as an objective science.

Not surprisingly, a physically meaningless principle offers no means of verification. Suppose we have two synchronized clocks and leave one at rest while accelerating the other to $0.5c$. We then return the traveling clock to the location and frame of the stationary clock and compare their readings. The result is that the traveling clock reveals an earlier reading than the stationary clock, indicating time dilation for the traveling clock. This means the stationary clock has undergone relative time gain and therefore reveals a later time than the traveling clock. The reciprocal difference in the times displayed by the clocks can be viewed by anyone at any time in any frame of reference. By contrast, the effect of relative simultaneity is discernible only so long as the clocks remain in relative motion, that is, in different frames. Thus the effect itself, as it takes place, is treated as its own confirmation.

Suppose we insist on objective verification and later bring the clocks together into a single frame. Obviously it will not be the case that each clock is running behind the other. Only one clock will display an earlier time than the other, which by

logical necessity will display a later time, a discrepancy that results not from relative simultaneity but from time dilation of whichever clock happened to be traveling at a higher ratio of c . In the Hafele-Keating experiment, the eastbound clock, whose motion added to the Earth's rotation, underwent time dilation relative to the ground clock while the westbound clock, subtracting from Earth's rotation, naturally underwent time gain. Though intellectually satisfying, the symmetry of relative simultaneity – with observers in both frames attributing a phantom time dilation to the other frame – negates it as a verifiable principle of physics.

Salmon's conflation of time dilation and relative simultaneity is common in the literature on special relativity (Goldberg 1984, 119, Kogut 2001, 28-31, Takeuchi 2010, 136). Stanley Goldberg, for instance, notes that "observers in one inertial frame of reference think that clocks in another inertial frame run slow... 'Time dilation' is the term applied to this phenomenon." When Goldberg gives an experimentally derived example of time dilation, however, relative simultaneity plays no role. Instead he describes "energy accelerators" that "accelerate radioactive materials almost to the speed of light," at which point "the half-life of the species [of atom] seems inordinately long" (1984, 124). Granted, a clock at rest with the atoms would run slow relative to a clock in the laboratory frame, but Goldberg neglects to mention that a clock in the laboratory frame would run *fast* in the frame of the atoms.

John Kogut (2001, 31) denies any problem in a pair of observers both claiming that the clock in the other frame is running slow. "There is no contradiction in this statement because the two observers do not occupy the same time axis." But this explains only why each clock *appears* to be running slower than the other, not whether this is somehow actually this case. As an abstract representation of relative motion, a Minkowski space-time diagram – which depicts each frame of reference with a different time axis – cannot bequeath reality to the impossible.

Prior to experimental verification, many critics dismissed time dilation on the basis of the so-called twin paradox. Suppose an astronaut, Clara, pilots a spaceship for five years at 97% the speed of light. Because this extreme speed carries a time dilation factor of four, 20 years elapse on Earth during the journey. As it happens, Clara has a twin, Delia, who remains on Earth. When Clara returns, Delia is now 15 years older than her twin. Though this certainly seems odd, in the context of time dilation it poses no actual problem. It gets tricky when we consider that everyone is at rest in their own frame of reference. From Clara's standpoint her rocket remains in place while Earth recedes from her at $0.97c$, causing *Delia* to dilate in time. When the twins are reunited, it should be Delia who has aged only five years while Clara has aged 20. So which is it? The paradox is easily resolved by taking into account the role of acceleration. At no point does Earth accelerate to high speed relative to c . Only Clara's spaceship accelerates and later decelerates. This breaks the inertial symmetry between frames and guarantees that Clara dilates in time while Delia – along with the rest of Earth's contents – reciprocally gains in time.

Though Einstein himself – along with a host of other luminaries – endorsed the resolution of the twin paradox on the basis of the asymmetrical impact of acceleration (Capek 1991, 305), Salmon rejects it on the grounds that acceleration is treated in general relativity, which is far more complex than special relativity and therefore involves more premises. It is, he says, "a general and fundamental principle of logic" that "contradictions in a set of premises can never be eradicated by adding new premises" (1980, 96). Once again he prefers the pure workings of the intellect to the messy business of the

real world. What he overlooks is that general relativity is the *fundamental* theory. Just as classical physics approximates special relativity where the speed of light can be ignored, special relativity approximates general relativity where the curvature of space-time, i.e. gravity, is minute enough to be disregarded. More to the point, why insist on trying to resolve the twin paradox without the appeal to acceleration when the force that causes time dilation is precisely what renders the resolution physical and not merely an intellectual exercise?

Salmon's attempt to overcome the twin paradox without resorting to physics calls to mind his attempt to resolve the paradoxes of motion on the basis of the conceptual apparatus of calculus rather than simply recognize that Zeno's assumptions of the actuality of zero and the divisibility of motion have no tangible basis. In both cases, he would rather fight one abstraction with another than grant that the world contains contingent facts beyond the reach of pure reason.

4. Special Relativity in Light of Quantum Mechanics

Einstein opposed quantum theory over its defiance of not only determinism and localism but especially realism, that is, a one-to-one correspondence between the elements of a theory and the elements of reality (Folse 1985, 145, Whitaker 2006, 223). What all these objections share in common is resistance to the idea that a contingent fact, the quantum of action, could necessitate a radical reorganization of our concepts of the world.

The quantum of action is the unavoidable consequence of measuring a quantum system. Determining the position of an atom requires an interaction between the atom and a measuring device, which causes the atom to "jump" to a well-defined location, not from another well-defined location but from a wavelike superposition of *possible* locations (Jammer 1974, 58, 106). The quantum jump would be unnecessary if the master equation of quantum mechanics, the Schrödinger wave equation, determined precise values of the properties of quantum systems. In the investigation of matter and light at the smallest scales, however, deterministic mathematics decouples from empirical outcomes (Whitaker 2006, 143). Quantum mechanics establishes a barrier beyond which pure reason cannot penetrate. Bohr referred to the quantum of action as "a basic fact that cannot be derived from ordinary mechanical physics," an irreducible element of the world that must be accepted as is (Katsumori 2011, 20).

In accord with the Schrödinger equation, quantum states evolve *continuously* from one set of superposed possibilities to another. In accord with the Born rule, these possibilities are quantified as probable outcomes of a measurement of the quantum system. Only upon external interaction, e.g. measurement, does the probability wave abruptly "collapse," breaking the temporal continuity of the system, which then settles on a determinate state for an instant consisting of at least 10^{-43} seconds (Planck time). After the temporal rupture, the system returns to the smooth evolution of probable values of its variable properties in the event of a subsequent interaction (Norsen 2017, 59-63, Jammer 1974, 5). Given that sensorial existence depends on objects with precise properties, we may surmise that atoms and their constituents frequently emerge from the wave-mechanical cloud as well-defined material entities at well-defined moments. Only upon the rapid succession of these extremely brief moments can atoms provide the building blocks of the sensorial world of classical physics.

But what about the rest of the time? What is the time of a quantum system *during* wave evolution? How can superposed potential values of properties evolve except over time? Just as the quantum system is the foundation of both matter and light, we may conjecture that the time of wave-mechanical evolution is the foundation of classical time. In that case the discrete instants that underlie the tangible world are *instantiations* of the temporal continuum underlying the evolving probability wave. Perhaps Russell was wrong. Rather than containing it, the instant only *expresses* time.

What defines a continuum, according to Cantor, is that between any two points is another point. Between points 1 and 2 is 1.5. Between 1 and 1.5 is 1.25. Between 1 and 1.25 is 1.125, and so on to infinity (Frigerio 2023, 236). No matter how infinitesimal the region of a continuum, it contains an infinite number of points. To get rid of the discreteness of a finite number of units of nonzero interval, Cantor extends their number to infinity while shrinking their interval to zero. The mathematical continuum achieves its continuity via an infinity of zeroes. As Schrödinger observed, echoing Hilbert, a "continuous range" composed of an infinity of zeroes is "quite exorbitant, an enormous extrapolation of what is really accessible to us" (1952, 30-31, Capek 1991, 49).

Bergson, Poincaré, Weyl and Cassirer all regarded mathematical continuity as an alternative term for infinite divisibility, that is, a "disguised discontinuity" as Milic Capek put it (1991, 248). If, on the other hand, the temporal continuum is *simply* continuous – meaning it has no subunits in the first place, much less an infinite number of zero-duration subunits – Cantor's elaborate conceptual apparatus fails to capture reality. We have no need to sum over infinity to arrive at 1 when the true continuum is nothing but a single "movement" of wave evolution. Thus the mathematical continuum of Galileo, Descartes, Leibniz and Kant falls short of the simple continuity of time as evident in the wave-mechanical continuum while also misrepresenting the discontinuous succession of a finite number of instants of nonzero duration (1991, 45). This is not to say that classical time is strictly discontinuous. The simple continuum is evident also at the classical level – as Bergson would insist – in the unbroken flux of kinetic energy that causally links successive moments.

Newton's "absolute, true, and mathematical time" fails not just because – in violation of special relativity – it "flows equably without regard to anything external" but because it relies on a mathematical conception of time as a series of points on a line, in Capek's words a "one-dimensional diagram associated with the word 'time'" (1991, 11). If absolute time is instead the simple continuum from which every discrete moment emerges upon interaction-induced collapse of the probability wave, we have an objective basis for a locally-defined present. Not every quantum system, upon instantiation, resolves into a single particle. What gives a system its singularity is that its wave-mechanical evolution is determined by a single probability wave. When external interaction collapses the wave, thereby instantiating the system, any number of particles can appear, all at precise locations. The expanse of the many-particle system defines a plane of simultaneity, a shared present moment from one end of the system to the other. As a scaled-up cinematic effect of rapidly recurring quantum instantiations, a train constitutes a classical approximation of the absolute plane of simultaneity of each of the quantum systems comprising it.

In a variation of Einstein's famous paired-lightning thought experiment, Michael Lockwood (2005, 29-32) supposes that lightning strikes the window of a car on a train in motion relative to an embankment. A passenger, Alice, stands before the window. A mirror is attached to each end of the car, one to her left and one to her right. Though for Alice the flash is

reflected in the mirrors simultaneously, for an embankment observer, Bob, the flash reflects in the mirrors successively, first in the mirror approaching the location of the flash and then in the one receding from it. For Lockwood, in keeping with Einstein's interpretation of special relativity, Bob's frame of reference is just as valid as Alice's, and therefore no definitive order can be assigned to the events. Once we recognize, however, that the train approximates the objectively present moments established by the successive instantiations of its quantum components, we find that the train's motion defines the frame in which to establish the objective timing. Conversely, if the events in question took place on the embankment, the perspective from the train would distort the actual timing. Special relativity tells us not only that *no* frame is privileged with respect to the laws of nature but that *every* frame is privileged with respect to events that take place in the context of that frame.

This is not only what we learn from quantum mechanics but exactly what is implied in the Lockwood thought experiment illustrating the relativity of simultaneity. Reflecting light off a mirror requires duration, specifically the time needed for atoms to be excited by the absorption of a photon and then relax as a new photon is emitted. Likewise, in Einstein's paired-lightning thought experiment (1920, 25-26), a certain duration is required for the flashes emitted by the lightning strikes to reach the midpoint between them. Only during this time does the motion of the train distinguish its frame from that of the embankment. Without this motion the difference between frames is nullified. Both Einstein and Lockwood assume duration – however slight – of events which are treated mathematically as mere points on a time-axis.

We have seen that the relativity of simultaneity depends on the equal validity of both frames of reference with respect to events that occur in only one of the frames. This does indeed make sense if events consist of durationless points. In this case events cannot be said to "occur" or "happen" or "take place" in a particular frame, which after all is defined against all other frames by motion over time. Only by occupying a certain duration can an event participate in the motion of a given frame and thereby be identified with that frame. The conceptual utility of the mathematical continuum in no way undermines the physical reality of events as frame-specific happenings, not mere space-time points.

Frame-specificity of events can be put to the test in a variation on Lockwood's thought experiment. Suppose the mirrors are connected to a mechanism that triggers a jack-in-the-box but only if the mirrors light up simultaneously. Does the jack-in-the-box pop open or not? Clearly, because the mirrors occupy the frame of reference defined by the train and not the one defined by the embankment, it does pop open. Upon being startled by it, Alice does not care that Bob thinks the jack-in-the-box should not have activated. Since the relevant frame for understanding the events in question is Alice's, Bob's account is merely subjective.

Just as nonzero duration is implicit in thought experiments intended to illustrate relative simultaneity, the flow of time is implicit in time dilation. How can the rate of time vary from one frame to another unless time flows? But the reality of temporal succession generates a new version of the twin paradox. If Einstein (1920, 26) is correct that each frame "has its own particular time," we cannot equate the five years that elapse in Clara's frame with the 20 years that elapse in Delia's frame, as the relativity of simultaneity has shattered the universal timeline by which to compare frame-dependent "proper" times. Since her rate of time has slowed relative to Delia's – and every frame has its own time – rather than dilate in time, she ought to *regress* into Delia's past by 15 years. Instead of 15 years younger than her twin in the year 2050, Clara ought

to be 15 years *to the past* of not only Delia but everyone on Earth. Rather than watch her return in 2050, we ought to remember the event from 15 years earlier. If that were the case, however, by 2050 Clara would have aged another 15 years and thus would still be the same age as Delia, defeating the verified effect of time dilation.

To resolve this variant of the twin paradox, we need only reject the relativity of simultaneity as a physical principle and grant frame-independence to the present moment. All frames are equal with respect to physical law, as Einstein says, but additionally all frames are equal with respect to the present. Time dilation not only falsifies relative simultaneity but confirms the fundamental basis of temporal presence. This is in keeping with the conjecture that each moment of experiential time instantiates the fundamental time implicit in continuous wave evolution.

Though every frame generates a different plane of simultaneity across space, a frame of reference is only a perspective onto the world, not the world itself. So long as each frame approximates an objective present moment established – and continually reestablished – at the quantum level, physics provides a means of arriving at the objective timing of events even if the clashing perspectives of frame-dependent macroscopic observers cannot.

5. Conclusion

Abstraction is essential to scientific inquiry. For example, to identify the causal basis of emergent phenomena such as low-temperature magnetic states or ethnic divisions between urban neighborhoods, extraneous elements must be abstracted out (Jensen 2023, 78-80). So powerful are the abstract concepts that enable us to systematically investigate the world that we readily confuse the two, treating the menu as the meal, so to speak.

Concepts are useful insofar as their meanings remain stable over time. Whereas a concept, like anything, must originate in a particular mind at a particular moment, its content is timeless, removed from the flux. If we think according to the logic of thought itself, we cannot help but favor static being over dynamic becoming. This is perhaps why Hegel reversed the natural order by defining becoming as the synthesis of being and nonbeing. Instead of starting with the inherent temporality of experience and attempting to capture it conceptually, he began with a pair of abstract concepts in order to arrive at the immediately given (Capek 1991, 3).

Yet Hegel, despite his starting point in pure abstraction, recognized the need to rise above "the thinking that belongs to the understanding alone" and therefore accepted the reality of becoming (Papa-Grimaldi 1996, 313). By contrast, Russell, at least in his early years, succumbed to the lure of timeless abstraction. In his 1903 book, *Principles of Mathematics*, he agreed that Zeno's arrow is "truly at rest at every moment of its flight" since it occupies at each moment a static point on a line. Nor, he wrote, can there be any transition from point to point, for its trajectory is laid out in static eternity, and the relations of the parts of the trajectory are immutable (Robinson 2018, 194).

In aligning the content of his thought with its innate logic, Zeno was merely being faithful to his teacher, Parmenides, who denied not just motion but more fundamentally plurality. Instead of many, there is only the One. By logical necessity the One is identical to itself. Yet to become is to cease, at that moment, to be identical to oneself. Hence all change – and

therefore motion – is impossible (Papa-Grimaldi 1996, 305-6).

This is what happens when thought folds in on itself and becomes its own "reality." When he devised his paradoxes in defense of his teacher, Zeno treated thought as an independent principle apart from the experience that gives rise to it. Salmon did the same when he sought to overcome Zeno's challenge by way of an abstract procedure developed in the confines of mathematical reasoning, thereby resolving the paradoxes without having to concede that the world is shaped, in part, by wholly contingent and "unreasonable" facts.

By promoting the relativity of simultaneity, Einstein not only denied the objectivity of the present moment – a brute fact verified literally every moment of our lives – but contradicted the evidence for time dilation, the chief experimental confirmation of his own theory. Salmon was so devoted to Einstein's conceptual construct that he extended its innate symmetry to time dilation, as if both of a pair of reference frames could run slow in time relative to the other. Clearly such a belief can hold only when time is abstracted out of the analysis at the outset. Like Zeno's paradoxes of motion, Einstein's principle of relative simultaneity is an exercise in pure thought divorced from real-world content.

As Bohr put it, "we are both onlookers and actors in the great drama of existence" (1987, 119). The fundamental meaning of quantum mechanics is that we cannot simply step outside the world and understand it in abstraction from its moment-to-moment content. The desire to detach from the stream of existence and attain perfect knowledge is a stumbling block in the attainment of genuine knowledge.

References

- Bergson, H. 1911. *Matter and Memory*. London: Swan Sonnenschein.
- Bergson, H. 1949. *Introduction to Metaphysics*. Indianapolis: The Liberal Arts Press.
- Bohm, D. 1996. *The Special Theory of Relativity*. London: Routledge.
- Bohr, N. 1958. *Atomic Physics and Human Knowledge*. New York: John Wiley & Sons.
- Bohr, N. 1987. *Atomic Theory and the Description of Nature*. Woodbridge, Connecticut: Ox Bow Press.
- Capek, M. 1991. *The New Aspects of Time: Its Continuities and Novelities*. Dordrecht, Holland: Kluwer.
- Craig, W.L., Smith, Q. (eds). 2008. *Einstein, Relativity and Absolute Simultaneity*. London: Routledge.
- Einstein, A. 1920. *Relativity: The Special and the General Theory*. New York: Crown Publishers.
- Folse, H.J. 1985. *The Philosophy of Niels Bohr: The Framework of Complementarity*. Amsterdam: Elsevier.
- Frigerio, C. 2022. Josiah Royce's "Flat Absolutism." *Cosmos and History*. **18**, 2. December, 2022.
<https://cosmosandhistory.org/index.php/journal/article/view/1050/1696>
- Goldberg, S. 1984. *Understanding Relativity*. Boston: Birkhäuser.
- Grünbaum, A. 1967. *Modern Science and Zeno's Paradoxes*. Middletown, Connecticut: Wesleyan University Press.
- Hawking, S., Mlodinow, L. 2010. *The Grand Design*. New York: Bantam Books.
- Hafele, J.C., Keating, R.E. 1972. Around the world atomic clocks: observed relativistic time gains. *Science* **177**. 14 July, 1972.

- Jammer, M. 1974. *The Philosophy of Quantum Mechanics*. New York: John Wiley & Sons.
- Jammer, M. 2006. *Concepts of Simultaneity*. Baltimore: Johns Hopkins University Press.
- Jensen, H.J. 2023. *Complexity Science: The Study of Emergence*. Cambridge: Cambridge University Press.
- Katsumori, M. 2011. *Niels Bohr's Complementarity*. Dordrecht, Holland: Springer.
- Kogut, J.B. 2001. *Introduction to Relativity*. San Diego: Harcourt/Academic Press.
- Lockwood, M. 2005. *The Labyrinth of Time*. Oxford: Oxford University Press.
- Lorentz, H.A., Einstein, A., Minkowski, H., Weyl, H. 1923. *The Principle of Relativity*. New York: Dover.
- Norsen, T. 2017. *Foundations of Quantum Mechanics*. Cham, Switzerland: Springer.
- Papa-Grimaldi, A. 1996. Why Mathematical Solutions to Zeno's Paradoxes Miss the Point. *Review of Metaphysics* 50. December, 1996.
- Robinson, K.A. 2018. Becoming and Continuity in Bergson, Whitehead and Zeno. *Lo Sguardo*, N. 26. 28 January, 2018.
- Salmon, W.C. (ed). 1970. *Zeno's Paradoxes*. Indianapolis: The Liberal Arts Press.
- Salmon, W.C. 1980. *Space, Time and Motion*. Minneapolis: University of Minnesota Press.
- Takeuchi, T. 2010. *An Illustrated Guide to Relativity*. Cambridge: Cambridge University Press.
- Whitaker, A. 2006. *Einstein, Bohr and the Quantum Dilemma*. Second Edition. Cambridge: Cambridge University Press.