

Peer Review

Review of: "Aspects of a Randomly Growing Cluster in $\mathbb{R}^d$, $d \geq 2$ "

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As already noted in other reviews, this paper introduces a model of a growing structure that is a Euclidean embedding of a random recursive tree, with the main results being estimates on the resulting cluster's diameter and the total length of the associated minimum spanning tree (with respect to the Euclidean distance). I agree it is an interesting model, though it might also be helpful to know if there is any motivation for considering this particular choice of embedding. Anyway, thinking about more detailed properties of the model and its generalisations naturally leads to plenty of interesting research questions, and thus I think it is worth publishing (after a minor revision).

Concerning the proofs, I could follow most steps, though I would prefer some additional detail in places, and I have a question about one estimate in Section 3.2 in particular (see the comment on p4, l-12 below). Some detailed comments about the content are as follows.

- There is a typo in the title; $\mathbb{R}^2$ should be $\mathbb{R}^d$.
- p1, l-9 - Should be 'set of points'. Probably better to end the next sentence with a full stop rather than a colon.
- p1, Theorem 1 - I would prefer if the statement of this did not start with a mathematical symbol. (Similarly on p2, l6, and the statement of Theorem 2.)
- p1, l-4 - Delete 'a' before 'the'.
- p1, l-2 - Maybe better to clarify that 'this collection of points' means $\mathcal{X}_\infty$ rather than $\mathcal{Y}_\infty$.
- p2, l4 - Delete 'is' before 'grows'.
- p2, Theorem 2 - Did you define the notation L_n ?
- p2, comments at the end of Section 1 - I realise this is just a discussion, but I wonder if you could formulate the final claim about possible generalisations a little more precisely.

- p2, l-12 - I suppose $\pi(i)$ is defined from the construction algorithm?
- p2, l-6 - The subscript should be $k=1$.
- p2, l-3 - Helpful to comment on where the first inequality comes from (a union bound over possible paths of length t .)
- p3, l4 - Should it be $d^{3/2}$ in the final expression?
- p3, l9 - I wonder if you need an extra $\log(k_2)$ as a factor counting the terms in the inner sum.
- p3, end of Section 2 - I would prefer a little more detail about how the left-hand side of the final displayed equation arises from applying (2).
- p3, l-2 - Please say what you mean by 'cost' here.
- p4, l2 - It would help the reader to recall where the $10(1+\log(i))$ comes from.
- p4, proof of the upper bound in the case $\alpha \geq 1/d$ - I would prefer more detail in explaining how to translate the w.h.p. bound to an expectation bound.
- p4, l-12 - It is bad notation to write two independent normals (in the same expression) with the same symbol. Moreover, I did not understand the first inequality; are not the variances of the relevant distances given by $i^{-\alpha}$ and $j^{-\alpha}$, which might be much smaller than $m^{-\alpha}$? Also, is the final bound giving the correct power of m ? In particular, I think you are using that $P(N(0, \sigma^2) < \delta) \leq \delta / \sqrt{2\pi\sigma^2} \leq \delta / \sigma$, which with the expression you are considering would give a bound of $\delta m^{\alpha/2}$. This would presumably have a knock-on effect throughout the remainder of the section, and the result itself... Am I misunderstanding something here?
- p5, l5 - Is this the correct power of log here? In particular, with the choice of delta you make, I think the lower bound should be $n^{1-1/d} (\log n)^{-1/d}$, whereas the power of log in your version is $(\log n)^{1/d-1}$, which does not seem to match...
- p5, Section 3.2.1 - Helpful to recall where this was useful in the above arguments.

Declarations

Potential competing interests: No potential competing interests to declare.