On Bell Experiments and Quantum Entanglement

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Abstract

Bell inequalities are intended to formulate mathematically the argument of the thought-experiment proposed by Einstein, Podolsky and Rosen (EPR) in the Einstein-Bohr Debate. Devised to test Bell inequalities against quantum mechanics for resolving the Einstein-Bohr debate, Bell experiments are purportedly modern versions of the EPR experiment. However, there exists a fatal logical flaw in Bell experiments. Because of the logical flaw, Bell experiments differ essentially from the EPR experiment, and quantum correlation in the EPR experiment is not "quantum entanglement" in Bell experiments; the latter is not physically meaningful. In addition to the logical flaw in Bell experiments, a physical constraint imposed on measuring individual quantum objects is violated by all Bell inequalities, which makes Bell inequalities physically meaningless. Unlike Bell experiments, the EPR experiment will not violate any physical constraint, and the EPR argument has no logical flaw. Consequently, the experimental invalidation of Bell inequalities is not evidence for falsifying the two fundamental hypotheses for scientific research, i.e., locality and realism, underlying the EPR experiment, and by no means can Bell experiments resolve the Einstein-Bohr debate. Stemming from abandoning either locality or realism or both, the so-called new applications of quantum mechanics, including various quantum information technologies, are all ineligible and doomed to failure, for they attempt to exploit physically meaningless "quantum superposition" or "quantum entanglement", treating them as physical resources "out there" in the real world.

Keywords: Quantum entanglement, Bell experiment, Quantum randomness, Unattainability of space and time coordinates

1 Introduction

With quantum-mechanical predictions having been confirmed by numerous experiments, needless to say, quantum mechanics is a supremely successful scientific theory, and its correctness is already well-established and widely recognized. However, in sharp contrast to the above fact, the conceptual foundations of quantum theory have been worrisome ever since the inception of quantum mechanics in the 1920s. Following Einstein, scientists and researchers in the minority of the scientific community still consider some important issues in the celebrated Einstein-Bohr debate [1, 2, 3, 4] remaining open nowadays.

For example, one of the well-known issues debated by Einstein and Bohr is whether quantum mechanics can provide a complete description of the physical world [1, 3]. Based on quantum correlation between two systems, which had previously interacted, then spatially separated, and do not interact anymore, Einstein, Podolsy, and Rosen (EPR) proposed the famous EPR thought-experiment, showing the notion of "quantum superposition" and Heisenberg's uncertainty relation implying a contradiction, and demonstrating physical reality existing independently of human consciousness. The assumptions behind the EPR experiment are mainly locality and realism, which are fundamental hypotheses for scientific research. Because elements of physical reality characterized by EPR are missing in the quantum-mechanical description of the physical world, Einstein considered such a description incomplete.

Moreover, Einstein was not satisfied with the standard interpretation of quantum randomness, i.e., randomness exhibited in outcomes obtained by measuring quantum objects. Because of quantum randomness, predictions of quantum mechanics are probabilistic rather than deterministic. How to interpret quantum randomness is the essence of the Einstein-Bohr debate. According to the standard interpretation, quantum randomness is an intrinsic property of the physical world, and hence quantum mechanics is inherently probabilistic. Einstein considered this interpretation totally unacceptable, although quantum-mechanical predictions are always correct. As one of the greatest masters of statistical physics, Einstein did not object to the use of probability in description of the physical world; he objected the standard interpretation. Rather than believing in a dice-playing God, Einstein believed in complete law and order of the physical world that exists objectively [5]. Locality and reality are not necessarily in conflict with quantum randomness.

Motivated by the Einstein-Bohr debate, David Bohm introduced the notion of "hidden variable" for interpreting quantum mechanics [6]. Inspired by Bohm's work, John S. Bell derived an inequality, attempting to express the argument of the EPR experiment mathematically [7, 8]. Bell hoped that the hidden variable in his inequality could represent the missing elements of physical reality, and hence quantum randomness might be interpreted in a way just like randomness in classical physics is interpreted by statistical mechanics [9]. Following Bell's work, similar inequalities named after Bell are also derived [10, 11, 12]. By using the notion of "hidden variable" to model the elements of physical reality, Bell inequalities are all derived purportedly under the same assumptions behind the EPR experiment, and intended to find a better description of the physical world, more complete than that given by quantum mechanics.

Based on the inequality he derived, Bell proved a theorem, which is now well-known and referred to as Bell's theorem in the literature [7]. According to this theorem, predictions of Bell inequalities differ significantly from quantum-mechanical predictions. Unlike the EPR argument, Bell inequalities can be tested against quantum mechanics by performing real experiments [9, 10, 11, 12, 13]. For this reason, physicists believe that the only way to resolve the Einstein-Bohr debate is to test Bell inequalities by experiment, and treat experiments for testing Bell inequalities, referred to as Bell experiments in the following, as modern versions of the EPR experiment [13]. After three Bell experiments attempting to close all the relevant loopholes at the same time [14, 15, 16], the Einstein-Bohr debate is purportedly settled ultimately [17]. According to the literature, the implications of Bell experiments are philosophically startling: the purported settlement of the Einstein-Bohr debate leads to either denying the existence of the physical world independent of human consciousness, or revising dramatically the space-time concept widely accepted in the scientific community. In addition to the above philosophical implications, by closing the door on the Einstein-Bohr debate and renouncing one or both of locality and realism underlying the EPR experiment, the door to the so-called new applications of quantum mechanics, including various quantum information technologies [18], is opened.

However, the implications of Bell experiments are questionable. For the following two reasons, Bell experiments and the EPR experiment are essentially different.

(1) In Bell experiments, the validity of the quantum-mechanical description of the physical world and the legitimacy of the standard interpretation of quantum randomness are taken for granted, which is a fatal logical flaw.

(2) In Bell inequalities, a physical constraint imposed on measuring individual quantum objects, i.e., *the same single quantum object can at most be measured only once*, is violated.

The fatal logical flaw cannot be explained away as a loophole to be closed. The physical constraint is crucial; violating this constraint makes Bell inequalities physically meaningless. Unlike Bell experiments, the EPR experiment will not violate any physical constraint, and the EPR argument has no logical flaw. Therefore, the implications of Bell experiments must be reconsidered, although Bell's theorem is correct in the following sense, i.e., predictions of Bell inequalities and quantum-mechanical predictions are significantly different.

In the rest of this article, Section 2 reveals the logical flaw in Bell experiments and shows some of its serious consequences. Section 3 explains in detail how the physical constraint imposed on measuring individual quantum objects is violated by Bell inequalities. Section 4 discusses some of the so-called new applications of quantum mechanics stemming from the implications of Bell experiments, such as quantum computation and quantum communication. Section 5 concludes with a brief summary of the findings reported in the article.

2 Logical Flaw in Bell Experiments

Devised based on the notion of "quantum entanglement", Bell experiments aim to test Bell inequalities against quantum mechanics for resolving the Einstein-Bohr debate. The entangled states in Bell experiments are actually quantum superpositions. The quantum-mechanical description of the physical world is typically based on the notion of "quantum superposition", which implies the standard interpretation of quantum randomness as shown below, see also [19]. Thus, not only the validity of the quantum-mechanical description of the physical world but also the legitimacy of the standard interpretation of quantum randomness is already taken for granted in Bell experiments. In other words, Bell experiments rely on the following two assumptions:

(a) The quantum-mechanical description of the physical world is valid.

(b) The standard interpretation of quantum randomness is legitimate.

However, the above assumptions must not be taken for granted without verification. Needless to say, the quantum-mechanical description of the physical world and the standard interpretation of quantum randomness are both important issues in the Einstein-Bohr debate to be resolved. Taking the assumptions for granted is a fatal logical flaw in Bell experiments, which are devised to settle the Einstein-Bohr debate. Clearly, even before Bell experiments are performed, the experimental results are already determined by the two assumptions!

To be specific, consider a popular optical Bell experiment [9]. Let a source be located at the origin on an arbitrarily fixed z axis. The source emits a sequence of ordered pairs (ν_1, ν_2) of correlated photons, one pair at a time. The photons in each pair counter-propagate along the z axis. All pairs are identically prepared, and quantum-mechanically described by an entangled state given below [9].

$$|\Psi(\nu_1, \nu_2)\rangle = \frac{1}{\sqrt{2}}[|x, x\rangle + |y, y\rangle],$$

where $|x\rangle$ and $|y\rangle$ are linear polarization states. Such an entangled state is a quantum superposition. The superposed states are $|x, x\rangle$ and $|y, y\rangle$.

Actually, because the entangled state in the optical Bell experiment is a quantum superposition, the validity of the quantum-mechanical description (i.e., the validity of the entangled state) to be tested has already been taken for granted. As shown above, this is a fatal logical flaw in the optical Bell experiment. The flaw can also be found in all other Bell experiments, and explains why Bell experiments cannot refute current quantum theory. What Bell experiments can refute are just Bell inequalities, the alternatives supposed to provide a better description of the physical world more complete than that given by quantum mechanics. In other words, the fate of Bell inequalities is already determined in advance by the logical flaw in Bell experiments. The flaw has serious consequences concerning the quantum foundations. In particular, as shown below, because the legitimacy of the standard interpretation of quantum randomness is taken for granted, quantum correlation in the EPR experiment is mistaken for "quantum entanglement" in Bell experiments.

In the optical Bell experiment, the polarizations of the photons ν_1 and ν_2, after they are spatially separated, are analyzed by linear polarizers I and II in arbitrarily chosen orientations $\mathbf{a}$ and $\mathbf{b}$ perpendicular to the z axis, respectively. Depending on whether the linear polarization of a photon is parallel or perpendicular to the orientation of the corresponding polarizer, the photon has two distinguishable measurement outcomes, denoted by $+$ and $-$. For $k = 1, 2, \cdots$ and $\ell = 1, 2$, let $h_k(\nu_\ell)$ represent the outcome of measuring ν_ℓ in the k-th pair. According to Einstein, because the two photons in the same pair are spatially separated, measuring one photon will not disturb the other photon in any way. As we shall see, Einstein was correct.

Let the orientations of the two polarizers be parallel, i.e., $\mathbf{a} = \mathbf{b}$, and use $\mathbf{a}$ to represent this common orientation. A perfect correlation between the measurement outcomes then manifests itself for all k.

$$h_k(\nu_1) = h_k(\nu_2),\ k = 1, 2, \cdots ,$$

where the meaning of "=" is "measurement outcomes on the two sides of '=' are identical." Because each pair is prepared and tested in the same way in a symmetrical configuration,

$$h_k(\nu_1) = h_k(\nu_2) = +$$

or

$$h_k(\nu_1) = h_k(\nu_2) = -$$

with equal probability 1/2. The probability of finding $h_k(\nu_1) \neq h_k(\nu_2)$ is zero.

According to the quantum-mechanical description (i.e., the description given by the entangled state), for an arbitrarily given k, if no measurement is performed on either photon in the k-th pair, then neither ν_1 nor ν_2 in the pair has a definite polarization state; however, once a measurement is performed, say, on ν_1, then immediately ν_2 in the same pair attains a definite polarization state identical to the measurement outcome of ν_1, even though the two photons are spatially separated. Einstein considered such quantum-mechanical description implying a "spooky action at a distance", and hence contradicting relativity. By interpreting the sudden state change of ν_2 as a result implied by the so-called "non-locality", which is purportedly a character of quantum mechanics, most physicists believe that Einstein's criticism can be explained away. However, this interpretation is also problematic because interpreting the sudden state change as a result implied by "non-locality" cannot tell us why the pairs of correlated photons behave randomly, although the probabilistic prediction of quantum theory, i.e.,

$$\mathbb{P}[h_k(\nu_1), h_k(\nu_2)] = \mathbb{P}(+, +) = \mathbb{P}(-, -) = \frac{1}{2}$$

is correct for all k.

For example, although the pairs are all identically prepared and tested under the same condition, if $i \neq j$, the measurement outcomes $[h_i(\nu_1), h_i(\nu_2)]$ may or may not be identical to $[h_j(\nu_1), h_j(\nu_2)]$; however, there seems to be no way to distinguish any one pair from any other pair. According to the standard interpretation, such quantum randomness is an inherent character of the physical world. Actually, as we can readily see, by taking the validity of the entangled state for granted, the quantum-mechanical description has already implied the standard interpretation even before the optical Bell experiment is performed. Clearly, represented by the entangled state, the quantum-mechanical description is the subject to be tested against Bell inequalities in the optical Bell experiment, and hence the validity of the entangled state must not be assumed in any way. Consequently, conclusions drawn from the optical Bell experiment are all questionable. In particular, the standard interpretation of quantum randomness is incorrect.

Let $\mathbb{N}$ represent the set of positive integers. Denote by Ω the set of the measurement outcomes obtained in the optical Bell experiment.

$$\Omega = \{[h_k(\nu_1)], h_k(\nu_2)] : k \in \mathbb{N}\},$$

where $[h_k(\nu_1)], h_k(\nu_2)] \in \{(+,+), (-,-)\}$ for any k. Just like all other physical quantities, the polarizations of photons are measured in space usually modeled by the Euclidean space $\mathbb{R}^3$. Any direction in $\mathbb{R}^3$ corresponds to one and only one point in the set given below.

$$D = \{\mathbf{r} \in \mathbb{R}^3 : d(\mathbf{r}, 0) = 1\},$$

where d is the usual distance function between any two points in $\mathbb{R}^3$. Consequently, directions or orientations in space can be represented by coordinates of the points in D.

Because of the unattainability of precise space coordinates [19], quantum randomness exhibited in Ω is actually due to subjective ignorance of knowledge about precise coordinates of the points in D representing directions or orientations in space. The measurement outcomes in Ω are random rather than deterministic, because they are the results of measuring the polarizations for *different* pairs of photons in *almost surely different, unknown orientations* $\mathbf{a}_k, k = 1, 2, \cdots$. First, the outcomes in Ω represent measurement results of *different* pairs of photons, because either photon in each pair can at most be measured only once as required by the constraint imposed on measuring individual quantum objects. Indeed, once a photon is registered at a detector, it cannot be detected anymore. Secondly, the outcomes are obtained by measuring the polarizations of photons along *almost surely different, unknown orientations*, because the precise coordinates of the orientations are unattainable. The precise coordinates of $\mathbf{a}_k$ and $\mathbf{a}$ are all contained in an infinitesimal volume. Therefore, the standard interpretation of quantum randomness is wrong. Unfortunately, because of the logical flaw, the validity of the entangled state is taken for granted, and the infinitesimal volume is mistaken for the desired direction $\mathbf{a}$.

Now we can see clearly that the notion of "quantum entanglement" is not physically meaningful; the standard interpretation of quantum randomness just attaches some physical meaning such as "non-locality" to it. In fact, there is no "quantum entanglement" in the physical world. The phenomena purportedly described by "quantum entanglement" in Bell experiments are actually quantum correlations. Because the unattainability of precise space coordinates is ignored, quantum randomness is interpreted, incorrectly, as an intrinsic property of the physical world. Consequently, quantum correlation is mistaken for physically meaningless "quantum entanglement". Unlike "quantum entanglement" represented by entangled states in Bell experiments, quantum correlation between spatially separated systems is due to physically explainable reasons. For example, the two systems in the EPR experiment are correlated, because they were interacted before separation; photons in the same pair in the optical Bell experiment are correlated, because they are created by the same source. Different from the misleading notion of "quantum entanglement" in Bell experiments, quantum correlation in the EPR experiment does not need "non-locality" to explain away "spooky action at a distance". So long as the unattainability of precise coordinates is taken into account, we can get rid of the inexplicable "quantum entanglement" while avoiding any "spooky action at a distance" disguised as "non-locality".

3 Flaw in Bell Inequalities

As shown in the preceding section, the failure of Bell inequalities is mainly due to the logical flaw in Bell experiments. In other words, the fate of Bell inequalities is already predetermined even before Bell experiments are performed. Besides the logical flaw in Bell experiments, Bell inequalities are also fundamentally flawed, because they all violate the physical constraint imposed on measuring individual quantum objects. The violation makes Bell inequalities physically meaningless, and explains why predictions of Bell inequalities differ significantly from quantum-mechanical predictions. To see this, consider the CHSH inequality, which is a generalized form of Bell's original inequality and directly applicable to the optical Bell experiment [9, 10].

Let F and λ be the distribution and value of a hidden variable, respectively. Consider a given pair (ν_1, ν_2) of correlated photons purportedly characterized by a given value λ of the hidden variable. The results obtained by measuring ν_1 and ν_2 are given by functions A and B corresponding to polarizers I and II, respectively; the functions take either $+1$ or -1 as their values. The measurement results are purportedly determined by the given value λ and the orientations of the polarizers, each of which can have two different, arbitrarily chosen orientations. Hence

$$A(\lambda, \mathbf{a}) = \pm 1,\ A(\lambda, \mathbf{a}') = \pm 1,\ B(\lambda, \mathbf{b}) = \pm 1,\ \text{and } B(\lambda, \mathbf{b}') = \pm 1.$$

Define a quantity S as follows.

$$
\begin{aligned}
&S(\lambda, \mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') \\
&= A(\lambda, \mathbf{a})B(\lambda, \mathbf{b}) - A(\lambda, \mathbf{a})B(\lambda, \mathbf{b}') \\
&+A(\lambda, \mathbf{a}')B(\lambda, \mathbf{b}) + A(\lambda, \mathbf{a}')B(\lambda, \mathbf{b}').
\end{aligned}
$$

After a simple inspection, we see

$$S(\lambda, \mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') = A(\lambda, \mathbf{a})[B(\lambda, \mathbf{b}) - B(\lambda, \mathbf{b}')] + A(\lambda, \mathbf{a}')[B(\lambda, \mathbf{b}) + B(\lambda, \mathbf{b}')] = \pm 2.$$

Denote by Λ the set of all values of the hidden variable. Integrating S over Λ yields

$$\int_{\lambda \in \Lambda} dF(\lambda) S(\lambda, \mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') = \pm 2.$$

When the polarizers are in orientations, say, $\mathbf{a}$ and $\mathbf{b}$, the corresponding correlation function of A and B is

$$\mathbb{E}(\mathbf{a}, \mathbf{b}) = \int_{\lambda \in \Lambda} dF(\lambda) A(\lambda, \mathbf{a}) B(\lambda, \mathbf{b}).$$

Consequently, the CHSH inequality takes the form given below.

$$-2 \leq \mathbb{E}(\mathbf{a}, \mathbf{b}) - \mathbb{E}(\mathbf{a}, \mathbf{b}') + \mathbb{E}(\mathbf{a}', \mathbf{b}) + \mathbb{E}(\mathbf{a}', \mathbf{b}') \leq 2.$$

As we can readily see, allowing each component of the given pair (ν_1, ν_2) to be measured in two different orientations, the CHSH inequality violates the constraint imposed on measuring individual quantum objects, i.e., *the same single quantum object can at most be detected only once*. To see how the CHSH inequality violates the constraint in detail, we can explicitly label S with the given pair (ν_1, ν_2), and label A and B with the corresponding components ν_1 and ν_2. Because each polarizer has two different orientations, rewrite $A(\lambda, \mathbf{a}), A(\lambda, \mathbf{a}'), B(\lambda, \mathbf{b})$, and $B(\lambda, \mathbf{b}')$ as $A_{\nu_1}(\lambda, \mathbf{a}), A_{\nu_1}(\lambda, \mathbf{a}'), B_{\nu_2}(\lambda, \mathbf{b})$, and $B_{\nu_2}(\lambda, \mathbf{b}')$. Accordingly, S takes the following form.

$$
\begin{aligned}
&S_{(\nu_1, \nu_2)}(\lambda, \mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}') \\
&= A_{\nu_1}(\lambda, \mathbf{a})B_{\nu_2}(\lambda, \mathbf{b}) - A_{\nu_1}(\lambda, \mathbf{a})B_{\nu_2}(\lambda, \mathbf{b}') \\
&+A_{\nu_1}(\lambda, \mathbf{a}')B_{\nu_2}(\lambda, \mathbf{b}) + A_{\nu_1}(\lambda, \mathbf{a}')B_{\nu_2}(\lambda, \mathbf{b}').
\end{aligned}
$$

As shown clearly in the above expression, each component of the pair (ν_1, ν_2) is measured twice in two different orientations, i.e., ν_1 is measured along $\mathbf{a}$ and $\mathbf{a}'$, and ν_2 is measured along $\mathbf{b}$ and $\mathbf{b}'$. However, after ν_1 and ν_2 are measured in whatever directions, they will not be available for detection anymore. For instance, if ν_1 and ν_2 have been detected when polarizer I is in orientation $\mathbf{a}$ and polarizers II is in orientation $\mathbf{b}$, then it is no longer possible to measure either ν_1 or ν_2, and talking about the so-called counter-factual outcomes obtained by measuring ν_1 and ν_2 in any other direction is meaningless, because the same single quantum object cannot be detected more than once, not only in actually performed measurements but also in counter-actual measurements. Of course, actually performed measurements will not violate the constraint; violation of such constraint occurs only in mathematical descriptions or explanations of the measurement outcomes.

The violation of the constraint can also be found in other Bell inequalities. Because Bell inequalities violate the constraint, the disagreement between predictions of Bell inequalities and quantum-mechanical predictions is not surprising at all, and in no sense have Bell experiments resolved the Einstein-Bohr debate. By advocating abandoning either locality or reality or both [17], the implications of Bell experiments given in the literature have not only led to various far-fetched conclusions about quantum randomness and quantum correlations [20], but also opened the door to various so-called new applications of quantum mechanics, such as quantum computation, quantum communication, and quantum cryptography [18]. However, the eligibility of the so-called new applications is questionable, which will be briefly discussed in the next section.

4 Discussion

The elementary units for the so-called quantum information processing, such as quantum computation and quantum communication, are quantum bits (qubits for short). Mathematically, a qubit is a quantum superposition. Because a quantum superposition is merely an abstract mathematical entity, treating it as a physical object to be realized as an actual physical system is physically meaningless. Treated as a physical object, the quantum superposition implies two assumptions, i.e., the validity of the quantum-mechanical description of the physical world and the legitimacy of the standard interpretation of quantum randomness. The assumptions are both false, as shown in the preceding sections. See also [19].

Furthermore, individual quantum objects are supposed to carry the so-called quantum information. According to the physical constraint imposed on measuring individual quantum objects, the same single quantum object can at most be measured only once. Consequently, after a given quantum object is detected, the same quantum object is no longer available for the so-called quantum information processing.

In fact, a single quantum object carries no information to be processed by the so-called quantum information technologies. Because the unattainability of precise space or time coordinates is ignored, different quantum objects of the same kind are mistaken for a single quantum object with its state purportedly represented by a qubit, which is a physically meaningless quantum superposition. Because qubits are not physical objects, the so-called quantum information technologies supposed to process qubits cannot be anything physically realizable. Stemming from the implications of Bell experiments given in the literature, the so-called new applications of quantum mechanics, including quantum computation, quantum communication, quantum teleportation, and quantum cryptography, are all ineligible and doomed to failure.

5 Conclusion

By introducing the notion of "hidden variable" to represent elements of physical reality characterized by EPR, which are missing in the quantum-mechanical description of the physical world, Bell inequalities aim to formulate the argument of the EPR experiment mathematically, and are intended to find a better description of the physical world more complete than that given by quantum mechanics. Bell experiments are devised to test Bell inequalities against quantum mechanics for resolving the Einstein-Bohr debate. Because of the logical flaw in Bell experiments, and because of Bell inequalities violating the physical constraint imposed on measuring individual quantum objects, Bell inequalities are all experimentally invalidated. Unfortunately, because Bell experiments are treated as modern versions of the EPR experiment, the experimental invalidation of Bell inequalities is interpreted, incorrectly, as evidence for falsifying locality and realism, the two fundamental hypotheses for scientific research behind the EPR experiment. The incorrect interpretation of the invalidation of Bell inequalities advocates abandoning either locality or realism or both, and opens the door to ineligible applications of quantum mechanics, including various so-called quantum information technologies. By treating physically meaningless notions of "quantum superposition" and "quantum entanglement" as resources "out there" in the physical world, such ineligible applications are doomed to failure.

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