Qeios

TWO-BODY PROBLEM WITH FINITE SPEED OF THE INTERACTION PROPAGATION

V. ONOOCHIN

Abstract. Using one result of general relativity, namely the finiteness of the speed of gravity, it is considered how the finiteness of this speed can change behavior of two-body system, for example, wide binary stars. For such a system, when bodies move in circular orbits, one property of these orbits makes it possible to calculate the exact positions of the bodies at retarded time and, therefore, the forces acting on the bodies at the present instant, but taking into account the effect of gravitational retardation, namely, that these forces were created by sources located in retarded positions.

For Keplerian orbits, the effect of the gravitational retardation leads to an increase in the angular momentum of the two-body system. As applied to binary stars, the increase in the angular momentum over time can explain a disruption of wide binary stars into two single ones.

This effect has not previously been described in the scientific literature.

1. Introduction

Let us consider one aspect of the two-body problem, namely, how the Keplerian solution for this system changes if interaction between the bodies propagates at a finite speed. A similar problem is well known in astronomy, it is the problem of binary stars. When the stars rotate around each other, their average linear velocity is not small (average velocity of stars of pulsar PSR B1913+16 is estimated at $2 \cdot 10^5$ meters per sec.). Thus, it would be reasonable to expect that the effects caused by the finiteness of the speed of gravity are taken into account when calculating the motion of such stars, especially, calculating the periastron shift. However, the retardation effects, when a source of gravitation force acting on the star is not in the instant position of the source but in its retarded position, is not considered in the general relativity. Instead, when considering many-body system, the Lagrangian is expanded in series over the parameters v_i/c, where c is the speed of gravity and v_i the linear velocities of massive bodies of the system. This expansion is obtained in original paper of Einstein, Infeld and Hoffmann, 1938. More detailed and scrupulous derivation of this expansion is given in the textbook of Fock [1]. With this approach, the coordinates of retarded positions of massive bodies are not included in the equations of motion.

A factor of the speed of gravity finiteness gives one effect that affects the motion of cosmic bodies. If the velocity of a moving massive body creating some gravitational field is comparable to the speed of gravity, then this leads to the so-called "gravitational force aberration", since the gravitational force acting on the test body is directed not from the current location of this moving massive body, but from its retarded location. Some discussion of "gravitational force aberration" appeared after work of Van Flandern [2], who stated that a possible aberration or transformation of the Newtonian gravitational force of the central type into a non-central type should ensure the instability of the solar system. This paradox was resolved by Carlip [3]. Meanwhile, all the authors who were involved in this discussion analyzed only the forms of the gravitational potential, but not the effect of the retarded position of the source of this potential. The latter is caused by impossibility to determine the exact position of a massive body at a retarded time t_r, since this requires solving the transcendental equation for this quantity. However, if two bodies revolve in perfect circular orbits, it is possible to find the exact position of these bodies in their orbits. In this paper, we will analyze how the retardation effect changes the motion of bodies in the two-body problem.

In order to separate the influence of retardation on motion of binary stars from other effects, it is advisable to consider behavior of a two-body system in pseudo-Newtonian model, *i.e.* when the

https://doi.org/10.32388/R1KXRJ

metric of the space is flat but the speed of the gravitational interaction propagation is finite and equal to the speed of light [4]. Since the Einstein field equations are non-linear there is no general, analytic solution of them available and one has to rely on approximative methods or simple solutions [5]. So to obtain the most precise solution of two-body problem, post-Newtonian approximation is used.

As will be shown below, the effect of retardation on the motion of binary stars is of the order of v/c or c^{-1} so the terms of order c^{-2} in Lagrangian can be omitted in post-Newtonian approximation. But up to v/c the Lagrangians of Newtonian gravity and of post-Newtonian approximation of general relativity coincide. A similar model is also used in [6].

It is necessary to clarify one aspect of the problem considered. Both in classical electrodynamics - the Darwinian approximation, and in the theory of gravitation - the post-Newtonian approximation - all expansions of quantities in powers of c^{-1} begin with terms of order $(v/c)^2$ (excluding the zero term in c^{-1}). In the presented consideration of the problem, the expansion of force factors begins with terms of order (v/c). This is a significant difference from other expansions. The appearance of a term of order (v/c) is due to the fact that the exact position of gravitating bodies at retarded moments of time will be determined.

Also, the use of the flat metric makes it possible to avoid cumbersome calculations caused by gravitational space curvature and to estimate only effect of the finiteness of the speed of gravity. For such an estimation, it is necessary to determine retarded positions of bodies depending on the observation time. Let us do that in the next section.

2. Determination of retarded position of the bodies in their orbits

It should be noted that any factor caused by the effect of gravitational retardation cannot be greater v'/c, where v' is the average velocity of the moving bodies. Because for cosmic bodies this ratio is less 0.001 (Sec. I), a force caused by retardation should be treated as perturbation in this system. It allows to consider the effects of retardation as corrections of order of small parameter v'/c to unperturbed motion of rotating bodies.

Therefore, as first step in analyzing the problem it should be considered a behavior of the system when the retardation effects are absent. For this purpose, let us consider the classical problem of two bodies, heavy and light, with masses m_h and m_l orbiting each other under the force of gravity. The indexes 'h' and 'l' correspond to the heavy and light bodies. The Lagrangian of this system is

$$\mathcal{L} = \frac{1}{2} m_h \dot{\mathbf{r}}_h^2 + \frac{1}{2} m_l \dot{\mathbf{r}}_l^2 - \gamma \frac{m_h m_l}{|\mathbf{r}_h + \mathbf{r}_l|} \,, \tag{1}$$

where $|\mathbf{r}_h + \mathbf{r}_l|$ is the distance between bodies that is much greater the sizes of the bodies so that the latter can be considered as concentrated at two points. γ is the gravitational constant. Equations of motion of these bodies are

$$\begin{aligned} m_h \frac{\mathrm{d}^2 \mathbf{r}_h}{\mathrm{d}t^2} - \gamma \frac{m_h m_l (\mathbf{r}_h + \mathbf{r}_l)}{|\mathbf{r}_h + \mathbf{r}_l|^3} &= 0\,; \\ m_l \frac{\mathrm{d}^2 \mathbf{r}_l}{\mathrm{d}t^2} - \gamma \frac{m_h m_l (\mathbf{r}_h + \mathbf{r}_l)}{|\mathbf{r}_h + \mathbf{r}_l|^3} &= 0\,. \end{aligned} \tag{2}$$

It is known that both bodies rotate around the common center O of their orbits in such a way that at any instant the bodies are being in opposite (each other) points of the orbits, and therefore, $|\mathbf{r}_h + \mathbf{r}_l| = r_h + r_l = r$.

By introducing,

$$r_h = \frac{m_l}{m_h + m_l}\, r\,; \quad r_l = \frac{m_h}{m_h + m_l}\, r\,,$$

the equations of motion, Eqs (2) are reduced to

$$m_h \frac{\mathrm{d}^2 r_h}{\mathrm{d}t^2} - \gamma \frac{m_h m_l}{r_h + r_l} = 0 \ \rightarrow \ \frac{m_h m_l}{m_h + m_l} \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - \gamma \frac{m_h m_l}{r^2} = 0 \ \rightarrow$$
$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - \gamma \frac{m_h + m_l}{r^2} = 0 \,,$$
$$m_l \frac{\mathrm{d}^2 r_l}{\mathrm{d}t^2} - \gamma \frac{m_h m_l}{r_h + r_l} = 0 \ \rightarrow \ \frac{m_h m_l}{m_h + m_l} \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - \gamma \frac{m_h m_l}{r^2} = 0 \ \rightarrow$$
$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - \gamma \frac{m_h + m_l}{r^2} = 0 \,. \tag{3}$$

One can see from Eqs. (3) that both equations of motion of the massive bodies are reduced to one equation. Thus, a solution of the same form describes the motion of bodies in two-body system. Physically it means that the bodies move in similar orbits.

Solution of Eq. (3) describes the motion of a fictitious body along a circular orbit, as well along any Keplerian orbit [7]. For simplicity, let us assume that an orbit of the fictitious body is perfect circle (the outer orbit in Fig. 1). Then the heavy body moves along the inner (purple) orbit and the light body moves along the intermediate (red) orbit in such a way that the heavy and light bodies are always being in opposite points to each other one other points relative to the center O of the orbits. Let us assume that the fictitious body is always opposite the light body, but in an outer orbit. If at the moment $t = 0$ the bodies are at the points A (heavy body), B (light body) and C (fictitious body), then one geometric property of the ideal circle allows one to determine the positions of the bodies at some retarded time t'. Suppose that the fictitious body has a velocity v and at t' it is at the point C'. Also assuming that this body emits a light signal in the direction p. O and when this signal approaches the center of the orbits, the fictitious body will be at the point C. Since the times of passage of the light signal and motion of the fictitious body are equal, one can write

$$\frac{r}{c} = \frac{\ell_{CC'}}{v} \ \rightarrow \ \frac{\ell_{CC'}}{r} = \frac{v}{c} \,,$$

where r is the radius of the external orbit and $\ell_{CC'}$ the length of the arc CC'. Since the angle (in radians) $\angle COC'$ is determined as a ratio of the length of the arc to the radius of the circle, if one knows the ratio v/c one is able to determine the exact position of the body at the retarded time $t_{ret} = r/c$, it is at the crossing the circle and the ray drawn from p. O under the angle $\alpha = v/c$.

Correspondingly the retarded positions of the heavy and light bodies are exactly determined. Since

$$\alpha = \frac{v}{r} = \frac{v_h}{r_h} = \frac{v_l}{r_l} \,,$$

one has

$$\ell_{AA'} = r_h \cdot \alpha \,; \ \ell_{BB'} = r_l \cdot \alpha \,.$$

Now let us calculate the force as a gradient of the gravitational potential. According to Jefimenko, a factor of finiteness of c gives 'effective prolongation' of the volume of moving massive body creating the gravitational potential (Eq. (4-1.8) of [4]). Therefore, Newtonian gravitational potential V_N changes to

$$V_N(r) = \frac{\gamma m M}{r} \ \rightarrow \ V(\mathbf{r}_r) = \frac{\gamma m M}{r_r - \dfrac{\mathbf{v}_r \cdot \mathbf{r}_r}{c}} \,,$$

where m the mass of the test particle, M the mass of the massive body, and v_r is its velocity calculated at the retarded time. Therefore r_r is the distance between the retarded position of the body and the point of the test particle location. Since for the circular orbit $\mathbf{v} \perp \mathbf{r}$ and $(\mathbf{v} \cdot \mathbf{r}) = 0$, one is able to use the form of the potential 'at rest', $V(\mathbf{r}_r) = (\gamma m M)/r_r$, with the only difference, *namely* the distance r_r is the distance between the retarded position of the body creating the potential $V(\mathbf{r}_r)$ and the present position of the body experiencing the action of this potential. So when the heavy body, being at the retarded position, p.A', creates some potential, a 'wave' of this potential reaches the light body when the latter is at p.B. Since the gravitational force acts along the line $B'A$ one finds that due to the

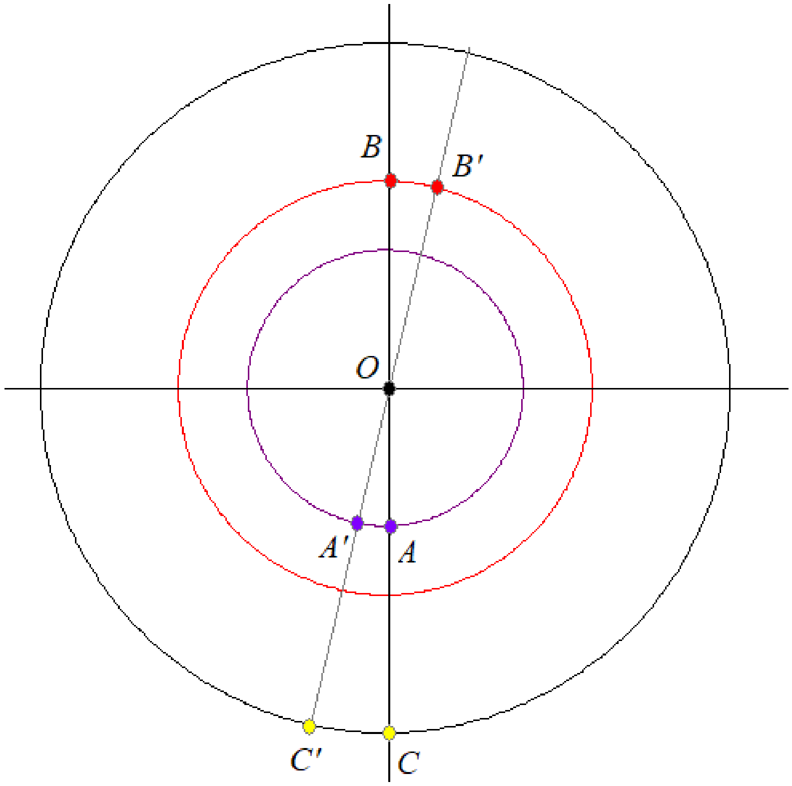

FIGURE 1. Positions of the rotating bodies at the present and retarded times.

effect of retardation, the gravitational force is now not of central type. Despite both bodies continue to rotate around the common center of the orbits, a small tangential component appears in the system. A presence of this tangential component changes properties of two-body system. Let us consider this change. First, it is necessary to determine the radial and tangential components of the gravitational force acting along the line AB' (A is the present position of the heavy body, B the regarded position of the light body), and the line $A'B$.

The distances $|AB'|$ and $|AB|$ are

$$|AB'| = \sqrt{(r_h + r_l\cos\alpha)^2 + r_l^2\sin^2\alpha} \approx (r_h + r_l) - \frac{r_h r_l}{2(r_h + r_l)}\alpha^2\,,$$

$$|A'B| = \sqrt{(r_l + r_h\cos\alpha)^2 + r_h^2\sin^2\alpha} \approx (r_h + r_l) - \frac{r_h r_l}{2(r_h + r_l)}\alpha^2\,.$$

In the first order of α these distances are equal to r. Then the forces acting between the bodies are

$$\mathbf{F}_{l\to h} = \gamma\frac{m_h m_l}{|AB'|^2}\mathbf{n}_{AB'}\,; \quad \mathbf{F}_{h\to l} = \gamma\frac{m_h m_l}{|A'B|^2}\mathbf{n}_{A'B}\,.$$

where $\mathbf{F}_{l\to h}$ is the attractive gravitational force acting by the light body to the heavy one (and wise verca for $\mathbf{F}_{h\to l}$), $\mathbf{n}_{AB'}$. $\mathbf{n}_{A'B}$ are the unit vectors along these lines. With accuracy to α, the radial components of the forces are

$$F_{r,l\to h} = \gamma\frac{m_h m_l}{|AB|^2} = \gamma\frac{m_h m_l}{r^2}\,; \quad F_{r,h\to l} = \gamma\frac{m_h m_l}{|AB|^2} = \gamma\frac{m_h m_l}{r^2}\,.$$

But different tangential components act on the heavy and light bodies,

$$F_{\theta,l\to h} = \gamma\frac{m_h m_l}{r^2}\cdot\sin\angle BAB' \approx \gamma\frac{m_h m_l}{r^2}\frac{vr_l}{cr} = \alpha\gamma\frac{m_h m_l}{r^2}\frac{m_h}{m_h+m_l}\,,$$
$$F_{\theta,h\to l} = \gamma\frac{m_h m_l}{r^2}\cdot\sin\angle A'BA \approx \gamma\frac{m_h m_l}{r^2}\frac{vr_h}{cr} = \alpha\gamma\frac{m_h m_l}{r^2}\frac{m_l}{m_h+m_l}\,. \tag{4}$$

The equations of motion for the bodies are now

$$m_h\frac{\mathrm{d}^2\mathbf{r}_h}{\mathrm{d}t^2} - \gamma\frac{m_h m_l\mathbf{n}_r}{r^2} - \gamma\alpha\frac{m_h m_l\mathbf{n}_\theta}{r^2}\frac{m_h}{m_h+m_l} = 0 \;\to$$
$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} - \gamma\frac{(m_h+m_l)\mathbf{n}_r}{r^2} - \gamma\alpha\frac{m_h\mathbf{n}_\theta}{r^2} = 0\,,$$
$$m_l\frac{\mathrm{d}^2\mathbf{r}_l}{\mathrm{d}t^2} - \gamma\frac{m_h m_l\mathbf{n}_r}{r^2} - \gamma\alpha\frac{m_h m_l\mathbf{n}_\theta}{r^2}\frac{m_l}{m_h+m_l} = 0 \;\to$$
$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} - \gamma\frac{(m_h+m_l)\mathbf{n}_r}{r^2} - \gamma\alpha\frac{m_l\mathbf{n}_\theta}{r^2} = 0\,. \tag{5}$$

It can be seen from Eqs. (5) that - in contrast to the classical two-body problem - changing the coordinates Eq. (2) does not reduce the equations of motion to identical form. This means that there may be no similarity of orbits. Moreover, according to Eq. (4) non-zero internal force

$$\Delta F = F_{\theta,l\to h} - F_{\theta,h\to l} = \alpha\gamma\frac{m_h m_l}{r^2}\frac{m_h-m_l}{m_h+m_l}\,, \tag{6}$$

appears in the closed two-body system which contradicts the law of the total momentum conservation. In this connection, it is appropriate to note that similar example in classical electrodynamics is analyzed by McDonald [8] who shows that nonzero internal force due to asymmetry of the electromagnetic interaction gives very small oscillations of the center-of-mass of the system but does not give 'perpetual motion'.

To avoid consideration of such possible small oscillations of the center-of-mass of two-body system, let us assume $m_h = m_l$ for further analysis. This assumption yields identical form for equations of motion of both bodies,

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} - \frac{\mu\mathbf{n}_r}{r^2} - \alpha\frac{\mu\mathbf{n}_\theta}{2r^2} = 0\,, \tag{7}$$

where $\mu = \gamma(m_h + m_l)$.

Up to the first order $\alpha = v/c$, this equation, written separately for the radial and tangential components, has the form

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 = -\frac{\mu}{r^2}\,; \tag{8}$$
$$r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + 2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mu\alpha}{2r^2}\,. \tag{9}$$

Using $\dot\theta = \omega$ the above system can be written as

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - r\omega^2 = -\frac{\mu}{r^2}\,; \tag{10}$$
$$r\frac{\mathrm{d}\omega}{\mathrm{d}t} + 2\omega\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mu\alpha}{2r^2}\,. \tag{11}$$

By introducing new unknown, $L = r^2\omega$, Eq. (11) becomes,

$$\frac{1}{r}\frac{\mathrm{d}r^2\omega}{\mathrm{d}t} = \frac{\mu\alpha}{2r^2} \;\to\; \frac{\mathrm{d}L}{\mathrm{d}t} = \frac{\mu\alpha}{2r}\,. \tag{12}$$

A system of equations similar to Eqs. (10) and (11) had been actively investigated in celestial mechanics. It was found (including experimentally) that the most effective transfer of a spacecraft from one planet of the solar system to another planet is carried out by applying a small tangential (to the circumplanetary orbit of the spacecraft) force to the apparatus [9, 10].

The presence of the small parameter α means that a thrust in tangential direction is very low and if the zero approximation ($\alpha = 0$) correspond to the circular orbit, a very low thrust gives the spiral orbit of low pitch.

It should be noted that a problem of the body motion in a near-circular orbit due to a small tangential force parallel to the linear velocity vector of the body has a very close analogue with so the called 'Satellite paradox' [11]. In this paradox, the satellite falls to the Earth due to the drag force created by the friction of the satellite's surface in very rarefied air at high altitudes. Despite the drag force acts opposite to the velocity v_o, the latter increases. This is due to the fact that the prevail condition for the motion of the satellite is the equality of the centrifugal force to the gravitational force [11],

$$\frac{m_{sat} v_o^2}{r} = \gamma \frac{m_{sat} M}{r^2} ,$$

where r is the radius of the orbit, m_{sat} is the mass of the satellite and M the mass of the Earth. So when r decreases (the satellite falls), v_o increases.

Since the tangential force per mass of the rotating body $F_\theta = \dfrac{\mu\alpha}{2r^2}$ acts in opposite direction to the drag force in the satellite paradox, one can expect that in the considered system the velocity of rotating bodies will decrease but the distance between them increases. Let us assume that the linear velocity decreases with time as $v(t) = v_0 - C\alpha t$, where v_0 is the linear velocity of the fictitious body in the orbit at $t = 0$, C is some dimensional constant (meter/sec^2), which will be determined below. Then from the condition to the instantaneous equilibrium for the fictitious body

$$a_r = \frac{v(t)^2}{r(t)} = \frac{\mu}{r(t)^2} \;\rightarrow\; \frac{[v_0 - C\alpha t]^2}{r(t)} = \frac{\mu}{r(t)^2} , \tag{13}$$

where a_r is denoted as a radial (centrifugal) component of the body acceleration.

One can find the time dependence of the radius–vector $r(t)$ on time from the above equation

$$r(t) = \frac{\mu}{[v_0 - C\alpha t]^2} , \tag{14}$$

Let us find the expression for the angular momentum using Eq. (14),

$$L(t) = r(t)v(t) = \frac{\mu}{[v_0 - C\alpha t]} , \tag{15}$$

This expression will be used to determine C from Eq. (7). Since α is small parameter, let us expand $L(t)$ in series over α,

$$L(t) \approx \frac{\mu}{v_0} + \frac{\mu C\alpha t}{v_0^2} \;\rightarrow\; \frac{\mathrm{d}L}{\mathrm{d}t} \approx \frac{\mu C\alpha}{v_0^2} .$$

Inserting it to Eq. (7), one obtains

$$\frac{\mu C\alpha}{v_0^2} = \frac{\mu\alpha}{r(t)} .$$

In the zeroth order over α, $r(t) \approx \mu/v_0^2$. Then

$$\frac{\mu C\alpha}{v_0^2} = \frac{\mu v_0^2 \alpha}{\mu} \;\rightarrow\; C = \frac{v_0^4}{\mu} . \tag{16}$$

The value of this parameter is small. For example, for the system 'the Sum – the Earth', $C \approx 0.012$.

Now let us find equation of the body trajectory. Its angular speed is

$$\omega = \frac{v(t)}{r(t)} = \frac{[v_0 - C\alpha t]^3}{\mu}$$

and, therefore, the angle $\theta(t)$

$$\theta = \int\limits_0^t \frac{[v_0 - C\alpha t]^3}{\mu}\mathrm{d}t = \frac{v_0^3}{\mu}t - \frac{3Cv_0^2}{2\mu}\alpha t^2 + \frac{C^2 v_0}{\mu}\alpha^2 t^4 - \frac{C^3}{4\mu}\alpha^3 t^4 \,.$$

With accuracy to α^1 terms,

$$\theta = \frac{v_0^3}{\mu}t - \frac{3Cv_0^2}{2\mu}\alpha t^2 \,. \tag{17}$$

Expressing the time variable via the angle from the above equation and inserting to Eq. (14), one obtains the equation of the body trajectory

$$r(\theta) = \frac{9\mu v_0^2}{\left(2v_0^2 + \sqrt{v_0^4 - 6C\mu\alpha\theta}\right)^2} = \frac{\mu}{v_0^2}\frac{9}{(2 + \sqrt{1 - 6\alpha\theta})^2} \,.$$

Since $r(t = 0) = r_0 = \mu/v_0^2$, the equation of the trajectory is

$$r(t) = \frac{9}{\left[2 + \sqrt{1 - 6\alpha\theta(t)}\right]^2}\, r_0 \,. \tag{18}$$

Because of increase of θ, the radii of the of the orbits and therefore the distance between the bodies increates with time. But Eq. (18) correctly describes behavior of the bodies only for $6\alpha\theta(t) \ll 1$.

3. Conclusions

In this work, it is considered how retardation of a gravitational force affects the behavior of two-body system. This effect can be estimated because, due to one property of a circular orbit, the retarded positions of rotating bodies are exactly determined.

It is shown that retardation of the gravitational interaction gives tangential component of the gravitational force and since the bodies when rotating always move in opposite directions, the tangential component leads to acceleration of the bodies so their circular orbits should be transformed into spiral orbits with a very small pitch (Eqs. (17) and (18)).

When the bodies move in spiral orbits, the angular momenta of these bodies increase. Formally, this contradicts one of the basic principles of classical mechanics - Noether's principle. The gravitational force, in Newton's interpretation, is conservative and the interaction of bodies in a closed system by means of this force cannot give any change in the system parameters defined by spatial symmetry, including the angular momentum. However, gravitation interaction propagates with finite speed equal to the speed of light. Therefore, the gravitational force is a retarded force. Noether's theorem is formulated for forces of instantaneous type. Thus, the fact that the angular momentum of the system increases due to the retarded force does not contradict Noether's theorem.

The effect of such an increasing angular momentum can explain one aspect of the evolution of binary stars. According to Raghavan and co-authors [12], the overall observed fractions of single, double, triple, and higher order systems are $56\% \pm 2\%$, $33\% \pm 2\%$, $8\% \pm 1\%$, and $3 \pm 1\%$, respectively, counting all confirmed stellar and brown dwarf companions. So the number of stars forming two-body systems is greater than the number of single stars ($33\% \times 2 > 56\%$). Meanwhile, there is an assumption (not confirmed reliably) that at an early stage of star formation, these cosmic bodies form many-body systems of double, triple etc. stars. Single stars are formed during the evolution of wide binary stars, when the latter are disrupted into two single stars [13]. One of factors which can provide disruption of wide binary stars into two single ones is an increase in the angular momentum of these stars by analogue with Eq. (15). For the evolution of wide binary stars, this mechanism can be dominant since it is of order of v/c in contrast to the mechanism of energy dissipation due to radiation of the gravitational waves that is of order of $(v/c)^5$.

However, the effect of increasing the angular momentum of typical binary stars is quite small and can be detected over large periods of time - several centuries. Therefore, this effect cannot be determined by direct observation of stars, but only by the statistics of the distribution of single and double stars.

Regarding 'aberration of the gravitational force', it should be noted that in the stationary mode of two-body rotation, this aberration cannot be detected experimentally for the bodies of the solar system since velocities of any body in this system are too small comparatively to the speed of gravity and the mass of the Sun is much greater the mass of the planets. Therefore, motion of the planets is like a motion of bodies around a very massive body fixed in space and retardation can be neglected when considering behavior of planets in the solar system [14].

In addition, consideration of this problem provides a fascinating example of unusual behavior of a mechanical system if parts of this system are connected by a retarded interaction. To the author's knowledge, consideration of similar example is absent in scientific literature.

References

[1] Fock, V., The theory of space, time and gravitation. (Pergamon Press. 1959). Ch. VI.

[2] Van Flandern, T., The speed of gravity - what the experiments say. Phys. Lett. A. **250**, 1–12 (1998)

[3] Carlip, S., Aberration and the speed of gravity. Phys. Lett. A. **267**, 81-90 (2000)

[4] Jefimenko, O.D., Gravitation and cogravitation. Electret Scientific company, Star city. (2004)

[5] Blanchet, L., Gravitational radiation from Post-Newtonian Sources and Inspiring Compact Binary Systems. Living Rev. Relativity. **17**, 2-188 (2014)

[6] Hilborn, R. C., Gravitational waves from orbiting binaries without general relativity. Am. J. Phys. **86**(3), 186-197 (2018)

[7] Landau, L.D. and Lifshitz, E.M., Mechanics, Course of Theoretical Physics, 1, Pergamon Press, Oxford; New York, (1969), 2nd edition. (Cited on page 21.)

[8] K.T. McDonald, Onoochin's paradox.
http://kirkmcd.princeton.edu/examples/onoochin.pdf

[9] Bacon, R.H., Logarithmic spiral: An ideal trajectory for the interplanetary vehicle with engines of low sustained thrust. Am. J. Phys. **27**, 164-167 (1959)

[10] Benney, D., Escape from a circular orbit using tangential thrust. Jet Propul. **28**(3), 167-169 (1958)

[11] Mills, B. D. , Satellite paradox. Am. J. Phys. **27**, 15-117 (1959)

[12] Raghavan, D., McAlister, H.A., Henry T.J., et al., A Survey of Stellar Families: Multiplicity of Solar-Type Stars. Astrophysical Journal Supplement Series. **190**, 1-42 (2010)

[13] Jiang, Y., Tremaine, S., The evolution of wide binary stars. Monthly Notices of the Royal Astronomical Society, **401**(2), 977-994 (2010)

[14] K.T. McDonald, Laplace and the Speed of Gravity. http://kirkmcd.princeton.edu/examples/laplace.pdf