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A New Approach Towards Quantum Foundations and Some Consequences

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Abstract

A general theory based upon 6 postulates is introduced. The basical notions are theoretical variables that are associated with an observer or with a group of communicating observers. These variables may be accessible or inaccessible. From these postulates, the ordinary formalism of quantum theory is derived. The mathematical derivations are not given in this article, but I refer to the recent articles [1][2]. Three possible applications of the general theory can be given; 1) The variables may be decision variables connected to the decisions of a person or of a group of persons. 2) The variables may be statistical parameters or future data, but most importantly here: 3) The variables are physical variables in some context. This last application gives a completely new foundation for quantum mechanics, a foundation which, in my opinion, is much more easy to understand than the ordinary formalism. The other applications seem also to give interesting consequences of the approach. So-called paradoxes like that of Schrödinger's cat can be clarified under the theory. Explanations of the outcomes of David Bohm's version of the EPR experiment and of the Bell experiment are provided. Finally, references and links to relativity theory and to quantum field theory are given.

Keywords: Accessible theoretical variables; Bell experiment; Born's formula; complementarity; inaccessible theoretical variables; quantum foundation.

Introduction

In two books [3][4] and in a series of articles [1][2][5][6][7][8][9][10] this author has proposed a completely new foundation of quantum theory, and also, in this connection, discussed the interpretation of the theory. This foundation is based upon the general notion of theoretical variables connected to some person or jointly to a communicating group of persons in some given situation. These variables may be accessible or inaccessible, again primitive notions. From a mathematical point of view, it is only required that if λ is a theoretical variable and $\theta = f(\lambda)$ for some function f, then θ is a theoretical variable. And if λ is accessible to

https://doi.org/10.32388/R7S5F7

the(se) person(s), then θ is accessible. Some postulates on these variables are stated, and as consequences, several quantum-like conclusions are derived.

Depending upon the situation, several different applications of this theory may be given. In a physical context, the theoretical variables may be physical variables like time, space, momentum, spin component, energy, mass, and charge. A variable is seen to be accessible if it can be measured with arbitrary accuracy. In a statistical context, the variables may be statistical parameters that are seen to be accessible if they can be estimated invariantly with respect to a certain group. Finally, in connection to human decisions, the variables may be decision variables, which are seen to be accessible if the corresponding decision really can be made.

The postulates of the theory will be formulated in the next section, but I will here mention three of them.

First, it is assumed that there exists an inaccessible variable ϕ such that all the accessible ones can be seen as functions of ϕ. Group actions are defined on the space upon which ϕ varies. In simple physical and statistical situations, this can easily be made concrete.

The first consequence of my postulates is given in Theorem 1 of the next section. Roughly speaking, this theorem goes as follows:

Make some symmetry assumptions; in the discrete case, these seem to be unnecessary. Then assume that there, in some context and relative to a person or to a communicating group of persons, exist two really different maximal accessible theoretical variables. (Using the words of Niels Bohr, these variables may be called complementary.) The term 'really different' means that there is no one-to-one function between the variables.

Then there exists a Hilbert space $\mathcal{H}$ connected to the situation, and every accessible variable is connected to a self-adjoint operator in $\mathcal{H}$. It is crucial that the essential assumption behind this is the existence of two complementary variables.

In the discrete case, the eigenvalues of the operator are the possible values of the associated variable. An accessible variable is maximal if and only if all the eigenspaces of the operator are one-dimensional.

The Born rule is, in the physical context, a part of the conclusion. For a detailed derivation of this rule from a few postulates, see [3][2]. In the case where the actual accessible variables are maximal as accessible variables, two postulates seem to be necessary in my derivation.

First, the likelihood principle from statistics is assumed (see any textbook on theoretical statistics, or for the origin of this principle, see Berger and Wolpert [11]). To be applicable in a quantum theory situation, a focused version of this principle is derived in [3][2].

The next postulate for the Born rule seems to be important. It is assumed that the relevant observer(s) has/have ideals, ideals that can be modeled in terms of some abstract being which is seen to be perfectly rational. The rationality is made precise in terms of the Dutch book principle. This postulate may be interpreted in many different directions.

As an interpretation of the whole theory in its physical application, a general epistemic interpretation of quantum mechanics is proposed: Quantum mechanics is not a theory of the world as such, but a theory of an observer's or a communicating group of observers' knowledge about this world. The term 'knowledge' is made precise in terms of accessible theoretical variables. In the discrete case, a pure state represents knowledge of the form $\theta = u$ for some maximal accessible variable θ, and, in general, a mixed state represents knowledge in terms of a probability distribution over such a variable.

One aspect of the theory should be mentioned: Since the theory begins with constructing operators connected to variables, these are seen as the fundamental building blocks. Pure states, as vectors in $\mathcal{H}$, are only introduced as eigenvectors of physically meaningful operators. This implies a restriction of the superposition principle, but note that in the simplest case, at least, a qubit, linear combinations of two different state vectors interpreted as spin components, describe new spin components in some directions. Also, it is crucial that certain entangled state vectors also can be given as eigenvectors of some meaningful operator.

The plan of the article is as follows: In Section 2, the theory is formulated and made precise. As a first consequence, a discussion of Schrödinger's cat paradox is given in Section 3. Then the Bell experiment is briefly discussed in Section 4. Connections to relativity theory and to field theory are mentioned in Section 5. Sections 6 and 7 give two applications outside quantum foundations, and Section 8 gives some concluding remarks.

The basic theory

A completely new approach towards quantum foundations is proposed in Helland [3][4][5][6][7][8][9][10][1][2]; see in particular the most recent articles [1] and [2]. The basis can be taken to be an observer who is in some physical situation. In this situation, there are theoretical variables, and some of these variables, say $\theta, \lambda, \eta, \cdots$, are related to the observer C. Some of these variables are *accessible* to him, which means roughly that it is, in some future, in principle, possible to obtain as accurate values as he wishes on the relevant variable. Other variables are *inaccessible*. Two examples of the latter are, first, the vector (position, momentum) of a particle at some time, and, secondly, the full spin vector of a spin particle, an imagined vector whose discretized projection in the direction a is the spin component in that direction.

The above characterization of accessible and inaccessible variables is related to a purely physical implication of the theory. The theory is purely mathematical, can be made precise in different directions, and the terms 'accessible' and 'inaccessible' are just primitive notions of the theory. Two other ways that the theory can be made precise are 1) quantum decision theory, where the variables are decision variables, and 2) statistical inference theory, where the variables either are statistical parameters or future data. From a mathematical point of view, it is only assumed that if λ is a theoretical variable and $\theta = f(\lambda)$ for some function f, then θ is a theoretical variable. And if λ is accessible, then θ is accessible.

But the main application here is that these variables are theoretical variables coupled to some physical situation. If necessary, an observed variable can be modeled as a theoretical variable plus some random error. Following Zwirn [12][13], every description of reality must be seen from the point of view of some observer. Hence we can assume that the variables also exist relative to C.

Also, note that observers may communicate. The mathematical model developed in the articles mentioned above is equally valid relative to a group of people that can communicate about the physics and about the various theoretical variables. This gives a new version of the theory, a version where all theoretical variables are defined jointly for such a group. The only difference here is that, for the variables to function during the communication, they must always be possible to define in words.

In the two examples mentioned above, there are also maximal accessible variables: In the first example, this can be either position or momentum; in the second example, it can be the spin component θ^a in some direction a. From a mathematical point of view, an accessible variable θ is called maximal if there is no other accessible variable λ such that $\theta = f(\lambda)$ for some non-invertible function f. In other words, the term 'maximal' will then be seen to be maximal with respect to the partial ordering of variables given by $\alpha \leq \beta$ iff $\alpha = f(\beta)$ for some function f.

Variables that are maximal, different, and not in one-to-one correspondence are my interpretation of what Niels Bohr called complementary variables.

A basic assumption in my theory is that there exists an inaccessible variable ϕ such that all the accessible variables can be seen as functions of ϕ. In the two mentioned examples, ϕ can be taken as the vector (position, momentum), respectively the imagined full spin vector.

Two different accessible variables θ and η are defined to be *related* if there is a transformation k in ϕ-space and a function f such that $\theta = f(\phi)$ and $\eta = f(k\phi)$.

Two spin components θ^a and θ^b are related, and position and momentum are related theoretical variables. In the first case, ϕ-space can be taken as the plane spanned by the directions a and b, and k can be taken as a 180^o rotation around the midline between a and b. In the last case, k is constructed by a Fourier transform.

As a summary of the above discussion, here are the first 3 postulates of the theory:

Postulate 1: *If η is a theoretical variable and $\gamma = f(\eta)$ for some function f, then γ is also a theoretical variable.*

Postulate 2: *If θ is accessible to A and $\lambda = f(\theta)$ for some function f , then λ is also accessible to A.*

Postulate 3: *In the given context, there exists an inaccessible variable ϕ such that all the accessible ones can be seen as functions of ϕ. There is a transitive group K acting upon ϕ.*

As mentioned in the two examples above, we may take $\phi = (\mathrm{position},\ \mathrm{momentum})$, and then let K be the Heisenberg-Weyl group, and $\phi =$ the full spin vector, where K now is taken to be the rotation group. In the last example, say in the spin 1/2 case, one can model the discrete spin component θ^a in direction a as $f(\phi) = \mathrm{sign}(\ \cos(a, \phi))$. Giving ϕ a reasonable distribution here results in a correct distribution of each θ^a.

A definition is now needed for the fourth postulate:

Definition 1. *The accessible variable θ is called maximal if θ is maximal as an accessible variable under the partial ordering defined by $\alpha \leq \beta$ iff $\alpha = f(\beta)$ for some function f.*

Note that this partial ordering is consistent with accessibility: If β is accessible and $\alpha = f(\beta)$, then α is accessible. Also, ϕ from Postulate 3 is an upper bound under this partial ordering.

Postulate 4: *There exist maximal accessible variables relative to this partial ordering. For every accessible variable θ there exists a maximal accessible variable λ such that θ is a function of λ.*

Then, in my opinion, two different maximal accessible variables come very close to what Bohr called complementary variables; see Plotnitsky [14] for a thorough discussion.

It is crucial what is meant by 'different' here. If $\theta = f(\eta)$ where f is a bijective function, there is a one-to-one correspondence between θ and η, they contain the same information, and they must be considered 'equal' in this sense. θ and η are said to be 'different' if they are not 'equal' in this meaning. This is consistent with the partial ordering in Definition 1. The word 'different' is used in the same meaning in the Theorems below.

Postulate 4 can be motivated by using Zorn's lemma - if this lemma, which is equivalent to the axiom of choice, is assumed to hold - and Postulate 3, but such a motivation is not necessary if Postulate 4 is accepted. Physical examples of maximal accessible variables are the position or the momentum of some particle, or the spin component in some direction. In a more general situation, the maximal accessible variable may be a vector, whose components are simultaneously measurable.

Assuming these postulates, the main result of Helland [5][1] is as follows:

Theorem 1 *Consider a context where there are two different related maximal accessible variables θ and η. Assume that both θ and η are real-valued or real vectors, taking at least two values. Make the following additional assumptions:*

(i) On one of these variables, θ, there can be defined a transitive group of actions G with a trivial isotropy group and with a left-invariant measure ρ on the space Ω_θ.

(ii) There exists a unitary multi-dimensional representation $U(\cdot)$ of the group behind the group actions G such that for some fixed $|\theta_0\rangle$ the coherent states $U(g)|\theta_0\rangle$ are in one-to-one correspondence with the values of g and hence with the values of θ.

Then there exists a Hilbert space $\mathcal{H}$ connected to the situation, and to every (real-valued or vector-valued) accessible variable there can be associated a symmetric operator on $\mathcal{H}$.

The Hilbert space $\mathcal{H}$ is, of course, related to the representation given in point (ii). The main result is that each accessible variable ξ is associated with an operator A^ξ. The proof goes by first constructing A^θ and A^η, then operators associated with other accessible variables are found by using the spectral theorem. For conditions under which a symmetric operator is self-adjoint or Hermitian, see Hall [15].

An important special case is when the accessible variables take a finite number of values, say r. For this case, it is proved in Helland [1] that a group G and a transformation k with the above properties always can be constructed. The following Corollary then follows:

Corollary 1 *Assume that there exist two different maximal accessible variables θ and η, each taking r values, and not in one-to-one correspondence. Then, there exists an r-dimensional Hilbert space $\mathcal{H}$ describing the situation, and every accessible variable in this situation will have an associated self-adjoint operator in $\mathcal{H}$.*

Theorem 1 and its Corollary constitute the first step in a newly proposed foundation of quantum theory.

The second step now is to prove the following: If k is the transformation connecting two related maximal accessible variables θ and η, and A^θ and A^η are the associated operators, then there is a unitary matrix $W(k)$ such that $A^\eta = W(k)^{-1}A^\theta W(k)$. This, and a more general related result, is proved as Theorem 5 in Helland [1].

Given these results, a rich theory follows. The set of eigenvalues of the operator A^θ is identical to the set of possible values of θ. The variable θ is maximal if and only if all eigenvalues of the corresponding operator are simple. In general, the eigenspaces of A^θ are in one-to-one correspondence with the questions 'What is θ'/ 'What will θ be if we measure it?' together with sharp answers $\theta = u$.

If θ is maximal as an accessible variable, the eigenvectors of A^θ have a similar interpretation. In my opinion, such eigenvectors, where θ is some meaningful variable, should be tried to be taken as the only possible state vectors. These have straightforward interpretations, and from this version of the theory, also a number of so-called 'quantum paradoxes' can be illuminated, see Helland [4][1].

This version of quantum theory implies a restriction of the superposition principle. But note the following: At least in the simplest case, a qubit, where the states can be interpreted as spin components of a particle, a linear combination of two pure states also can be given the same interpretation, i.e., is an eigenvector of some spin operator. Also, the singlet state constructed from two accessible variables, a state that is important both in David Bohm's version of the EPR discussion and in any discussion of the Bell experiment, is an entangled state that is included in my theory. This will be further discussed below.

What is lacking in the above theory is a foundation for Born's formula, the means for calculating quantum probabilities. Several versions of Born's formula are derived from two new postulates in Helland [2]. The first postulate is as follows:

Postulate 5: *The likelihood principle from statistical theory holds.*

The likelihood principle is a principle that nearly all statisticians base their inference on, at least when the principle is restricted to a specific context. For a basic discussion, see Berger and Wolpert [11]. It is closely connected to the notion of a statistical model: In an experimental setting, there is a probability model of the data z, given some parameter θ. Note that a statistical parameter, in an important application of my theory, can be seen as an accessible theoretical variable. The statistical model can be expressed by a point probability p for discrete data or a probability density p for continuous data: in both cases, we have $p = p(z|\theta)$.

The *likelihood* is defined as p, seen as a function of the parameter θ:

$$L(\theta; z) = p(z|\theta), \qquad (1)$$

and the likelihood principle runs as follows: *Relative to any experiment, the experimental evidence is always a function of the likelihood.* Here, the term 'experimental evidence' is left undefined and can be made precise in several directions.

In a quantum mechanical setting, a potential or actual experiment is seen in relation to an observer C or to a communicating group of observers. Concentrate here on the first scenario. In the simplest case, we assume that C knows the state $|a; i\rangle$ of a physical system and that this state can be interpreted as $\theta^a = u_i$ for some maximal accessible variable θ^a. Then assume that C has focused upon a new maximal accessible variable θ^b, and we are interested in the probability distribution of this variable.

The last postulate is connected to the scientific ideals of C, ideals that either are given by certain conscious or unconscious principles or are connected to some concrete persons. These ideals are then modeled by some higher being D that C considers to be perfectly rational.

Postulate 6: *Consider in the context τ an epistemic setting where the likelihood principle from statistics is satisfied, and the whole situation is observed by an experimentalist C whose decisions can be modeled to be influenced by a superior being D. Assume that one of D's probabilities for the situation is q, and that D can be seen to be perfectly rational in agreement with the Dutch Book Principle.*

The Dutch Book Principle says as follows: No choice of payoffs in a series of bets shall lead to a sure loss for the bettor.

A situation where Postulate 6 holds will be called a *rational epistemic setting*. It will be seen to imply essential aspects of quantum mechanics. As shown in [7], it also gives a foundation for probabilities in quantum decision theory.

In Helland [3][2], a generalized likelihood principle is proved from the ordinary likelihood principle: Given some experiment, or more generally, some context τ connected to an experiment, any experimental evidence will under the above assumptions be a function of the so-called likelihood effect F; a definition and a discussion is given in [3][2]. In particular, the probability q is a function of F: $q(F|\tau)$.

Using these postulates and a version of Gleason's Theorem due to Busch [16], the following variant of Born's formula is proved:

Theorem 2 [Born's formula, simple version] *Assume a rational epistemic setting. In the above situation, we have:*

$$P(\theta^b = v_j|\theta^a = u_i) = |\langle a; i|b; j\rangle|^2. \qquad (2)$$

Here, $|b;j\rangle$ is the state given by $\theta^b = v_j$. In this version of the Born formula, I have assumed *perfect measurements:* There is no experimental noise, so the experiment gives a direct value of the relevant theoretical variable.

An advantage of using the version of Gleason's theorem due to Busch [16] in the derivation of Born's formula is that this version is valid also in dimension 2. Other derivations using the same point of departure are Caves et al. [17] and Wright and Weigert [18]. In Wright and Weigert [19], the class of general probabilistic theories that also admit Gleason-type theorems is identified. But, for instance, Auffeves and Granger [20] derive the Born formula from other postulates.

The simple Born formula can now be generalized to the case where the accessible variables θ^a and θ^b are not necessarily maximal. There is also a variant for a mixed state involving θ^a.

For completeness, define first the mixed state associated with any accessible variable θ. If θ is not maximal, we need, for the Born formula, the assumption that there exists a maximal accessible variable η such that $\theta = f(\eta)$ and such that each distribution of η, given some $\theta = u$, is uniform. Furthermore, some probability distribution of θ is assumed. Let Π_u be the projection of the operator of θ upon the eigenspace associated with $\theta = u$. Then define in the discrete case the mixed state operator

$$\rho = \sum_j P(\theta = u_j)\Pi_{u_j} = \sum_i \sum_j P(\eta = v_i|\theta = u_j = f(v_i))P(\theta = u_j)|\psi_i\rangle\langle\psi_i|. \qquad (3)$$

Here, $|\psi_i\rangle$ is the state vector associated with the event $\eta = v_i$ for the maximal variable η.

From this definition, we can show from ([Born]), assuming that the maximal η^a corresponding to θ^a also is a function of ϕ, that in general

$$P(\theta^b = v|\rho^a) = \mathrm{trace}(\rho^a \Pi_v^b), \qquad (4)$$

with the projection Π_v^b projecting upon the subspace of $\mathcal{H}$ given by $\theta^b = v$.

This result is not necessarily associated with a microscopic situation. A macroscopic application of Born's formula is given in [3].

The result can also be generalized to continuous theoretical variables by first approximating them by discrete ones. For continuous variables, Born's formula is most easily stated in the form

$$E(\theta^b|\rho^a) = \mathrm{trace}(\rho^a A^{\theta^b}). \qquad (5)$$

In this formula, do not assume that the accessible variable θ^b is maximal. Hence a corresponding formula is also valid for any function of θ^b, for instance $\exp(i\theta^b x)$ for some fixed x. The operator corresponding to a function of θ^b can be found from A^{θ^b} by using the spectral theorem. From this, the probability distribution of θ^b, given the information in ρ^a, can in principle be recovered.

Finally, one can generalize to the case where the final measurement is not necessarily perfect. Let us assume future data z^b instead of a perfect theoretical variable θ^b. Strictly speaking, for this case, the focused likelihood principle is only valid under the following condition: $p(z^b|\theta^b = u_j) = p(z^b|\theta^b = u_k)$ implies $u_j = u_k$; see [3]. But this is not needed here; we only need the focused likelihood principle for perfect experiments in order to prove that ([Born3]) is valid. Then we can define an operator corresponding to z^b by

$$A^{z^b} = \sum_j z^b p(z^b|\theta^b = u_j)\Pi^b_{u_j}, \qquad (6)$$

and, from version ([Born3]) of the Born formula, we obtain

$$E(z^b|\rho^a) = \mathrm{trace}(\rho^a A^{z^b}). \qquad (7)$$

All this not only points to a new foundation of quantum theory, but it also suggests a general epistemic interpretation of the theory: Quantum theory is not directly a theory about the world, but a theory about an actor's knowledge of the world. Versions of such an interpretation already exist, and they are among the very many suggested interpretations of quantum mechanics. Further discussions of this interpretation are given in [1][2][3].

Schrödinger's cat

Schrödinger's cat is a well-known so-called paradox in quantum theory, and there is a large literature on this paradox. It assumes as a thought experiment that a cat is closed into a sealed box and that there is also some deadly poison that can be released by the decay of some radioactive material. The question that troubled the originator, Schrödinger, and many other physicists was: At a time t, to an outside observer, is the cat dead or alive? The origin of the difficulty is the superposition principle. In a quantum-mechanical state connected to this observer, in a state which is a superposition of a 'dead' state and an 'alive' state, the cat seemingly is both dead and alive.

My theory does not assume a general validity of the superposition principle. A pure state vector is included in the theory only if it is an eigenvector of some meaningful physical operator.

It is enlightening to see this in connection with a recent article by Maccone [21]. There, an observator S is introduced which is complementary to the property dead/alive associated with the cat. In my terminology, there should be a theoretical variable θ connected to S. The crucial question is: Can θ be measured? According to a long discussion in Skitiniotis et al. [22], this can in principle be done, but it requires a tremendous measurement device. In practical terms, I would say that θ is inaccessible to any given human observer.

To put it simply, a state is always connected to the mind of some observer, and in my theory, I allow this observer to answer 'I don't know' to certain questions, in this case, the question 'Is the cat alive at time t?' Once the box is opened, the observer can give a precise answer to the corresponding question.

The Bell experiment

The Bell experiment is a well-known experiment whose outcome has caused much discussion among theoretical physicists. In 2022, the Nobel Prize in Physics was given to three physicists who had performed so-called loophole-free variants of this experiment. The conclusion, which has astonished both scientists and laymen, stands firm. It seems like there either is a violation of the principle of locality, or the ordinary notion of reality has to be abandoned.

Very briefly, the Bell experiment has to do with two observers, Alice and Bob, that cannot communicate. Midway between the two, a pair of entangled particles is sent out, one towards each observer. Particles with spin 1/2 are assumed. Entanglement means that the spin components of the pair are in a singlet state given by

$$|\psi\rangle = \frac{|1+,2-\rangle - |1-,2+\rangle}{\sqrt{2}}. \qquad (8)$$

Here, the spin components of the particles are specified in some direction, say the vertical; 1 and 2 refer to the two particles, and - means downwards and + means upwards. (In the most recent experiments, polarization of photons is measured, but the discussion is similar.)

Alice is given two directions a and a' to choose from, and measures spin components A or A' in these directions. Bob is given two directions b and b' to choose from, and measures spin components B or B' in these directions. The measured spin components are coded as -1 and +1.

The experiment is repeated many times, and each time, Alice and Bob make their choices. As the measured spin components can be seen as random variables in some sense, expectations of pairs presumably exist. The so-called CHSH inequality is

$$E(AB) + E(A'B) + E(AB') - E(A'B') \leq 2. \qquad (9)$$

This inequality may be derived, seemingly using only the assumption that A, B, A' and B' exist simultaneously as random variables taking the values ± 1, by a very simple argument. But, by suitable choices of a, b, a' and b', it can be shown that ([CHSH]) is violated by quantum mechanics.

Now to the conclusion of very many Bell experiments: Again, by choosing a, b, a' and b' in a suitable way, ([CHSH]) is also violated in practice. Somehow, the simple argument or its assumption must be wrong. In the physical literature, the assumption behind ([CHSH]) is called local realism.

In Helland [6][23], a very specific explanation of the outcome of the Bell experiment is given.

First, it is crucial to show that the state ([singlet]) is allowed by my theory. It should be an eigenstate for some operator which is physically meaningful, which in my terminology means that the operator is connected to a theoretical variable that is accessible to some observer.

Let the observer Charlie be some person who, after the experiments, is given all the data from Alice and Bob and then tries to model the results of the experiments. During an experiment, he may be able to observe the whole experiment and record settings and responses. To him, both the spin vectors ϕ_A and ϕ_B for Alice and Bob are inaccessible, but it turns out that the dot product $\xi = \phi_A \cdot \phi_B$ is accessible to him. In fact, he is forced to be in the state given by $\xi = -3$. This can be seen as follows: The eigenvalues of the operator corresponding to ξ are -3 and +1, and the eigenstate corresponding to $\xi = -3$ is just the singlet state ([singlet]). (Exercise 6.9, page 181 in Susskind and Freedman [24].) Thus, being in this singlet state as an observer of the whole experiment, he is forced to have $\xi = -3$.

What does it mean that the dot product of the two vectors is -3? Expressed by the cartesian components that are ±1, we must necessarily have $\phi_A^x \phi_B^x = \phi_A^y \phi_B^y = \phi_A^z \phi_B^z = -1$, that is $\phi_B^x = -\phi_A^x$ etc., which implies $\phi_B^a = -\phi_A^a$ in any direction a. This is an important conclusion, and it is true for any observer Charlie. This explains the result of David Bohm's version of the EPR experiment, an experiment proposed by Einstein et al [25], and a result that has caused much confusion.

But go back to the Bell experiment. Here my discussion relies on a general mathematical result of my postulates, a result which in the physical application must be of relevance to the mind of any observer, any person. Recall the definition of related accessible variables given in Section 2.

Theorem 3 *Assume two related maximal accessible variables θ and η. Then any other variable λ which is related to θ, but unrelated to η cannot be a maximal accessible variable.*

The proof is given in [6] and [23].

With the physical/psychological application of my theory described in Section 2, this means that λ cannot be maximally accessible to the mind(s) of any observer(s) (at the same time; if we let time vary, the observer may have many variables in his mind, also unrelated ones.)

Apply this to the argument around the CHSH inequality. Consider the observer Charlie, and assume that the variables $\theta = (A, B)$ and $\eta = (A, B')$ are accessible to him. One can show that, at a fixed time, each of these is maximal as an accessible variable: Charlie is only able to observe one run at a time. Also, they are related relative to a ϕ containing all of A, B, A' and B'. (They have the outcome A in common; we can let the transformation k rotate b onto b' and hence B onto B'.) Then look in addition to the variable $\lambda = (A', B)$. This is related to θ , but unrelated to η. If it were accessible to Charlie, it would again be maximal as an accessible variable, but this is impossible by Theorem 3.

This, any observer, Charlie cannot have the three variables θ, η, and λ in his mind at the same time. *As a consequence of this, he is not able to follow the simple argument leading to the CHSH inequality.* More concretely, if he were a statistician, he is not able to put up a valid joint probability model for the variables A, A', B, and B'. Note that Charlie can be any observer.

Thus, the result of the Bell experiment can, in my opinion, be explained by the fact that we all have a limited mind. This has consequences for this experiment, but as a general observation, it also has other consequences.

This explanation is not related to non-locality, and the same can be said about my explanation of Bohm's version of the EPR experiment discussed above. Neither can it be directly related to non-reality, but this is a deep issue that can only be discussed in relation to interpretations of quantum theory.

Some consequences for relativity theory and quantum field theory

The issues of this section are discussed in the book [4]. The notions of accessible and inaccessible variables are also applicable in special and general relativity theory, and much of the theory generalizes. In particular, it is natural to assume that variables related to the interior of a black hole are inaccessible to any observer. Finally, one may extend the notions to accessible and inaccessible fields. Details are given in the book, and some of the discussion is given in [8].

Applications to statistical inference

As stated above, statistical inference is based upon a statistical model specified by a probability function $p(z|\theta)$ for data z and parameter θ. There is a large literature on statistical inference, but in my opinion, something related to possible structures in the parameter space is missing. For instance, there may be groups acting on the parameters, and one should try to work towards a systematic theory of model reduction.

My approach involving accessible and inaccessible theoretical variables may have something to contribute here. In Theorem 1, I need a transitive group acting on θ, and in Postulate 3, I assume a transitive group acting upon the large and inaccessible ϕ. If one can find nontransitive groups acting upon parameters, a reasonable policy for model reduction is *to reduce to a suitable orbit of the group.* At least in the case of partial least squares regression [26], this works well. A systematic application of this principle to quantum mechanics has not been discussed yet.

The case of partial least squares (PLS) regression and its possible relation to the theory of Section 2 here has been systematically treated in Helland [27]. The total model parameter is called ϕ, and θ is the reduced regression parameter corresponding to a PLS model with m steps. The conditions of Theorem 1 are shown to hold in the case where η is another reduced parameter, so, in particular, a Hilbert space exists with an operator A^{θ} corresponding to θ. Optimality of the PLS model compared to other model reductions is discussed.

Another application is the following: Suppose that we have just done a PLS regression based on some data. Let the estimated regression parameter be $\widehat{\theta_1}$. Assume then that we will do a similar investigation later and call the new estimator $\widehat{\theta_2}$. Then we may look upon these two estimators as maximal accessible variables, use Theorem 1 again, and by the Born formula ([Born5]) in principle find a distribution of $\widehat{\theta_2}$, given a posterior distribution of $\widehat{\theta_1}$, taking into account that the same parametric model is used in the two cases.

Similar applications should be investigated.

A foundation of quantum decision theory

The simplest model of a decision process connected to some person C or to a group of communicating persons is as follows: Assume that the choice is between n possible actions $a_1, a_2, ..., a_n$. Define a decision variable θ as being equal to u if action a_u is chosen. As stated in the Introduction, such a decision variable can be taken as a theoretical variable, accessible if the decision really can be done. If there are two complementary decision processes at the same time, the conditions of Corollary 1 are satisfied, and this can be taken as a new theoretical foundation of quantum decision theory, a discipline where most of the literature is based upon empirical investigations, see for instance Busemeyer and Bruza [28]. This foundation is further discussed in [7].

Conclusions

The ordinary formal basis for quantum mechanics is very difficult to understand. It has led to very many interpretations, and for people outside the physical community, it seems to be nothing but pure formalism.

By contrast, the foundation discussed in this article is given by 6 postulates, in my opininion, fairly easy to understand, also for outsiders. Still, perhaps, interpretation can be discussed, but a general epistemic interpretation seems to be natural. The basis is just theoretical variables, accessible or inaccessible, and connected to an observer or to a group of communicating observers in some physical context. Since basic quantum theory may be derived from these postulates, everything in the literature which is derived from this formalism can in principle also be deduced from the postulates. An exception is that I restrict the superposition somewhat, as discussed above, and also in [1].

The mathematics behind these derivations is not given here; it is all contained in the recent articles [1] and [2].

So-called paradoxes like Schrödinger's cat can be understood using this theory. The results of the EPR type experiments and the Bell experiments can be explained using the theory.

It is also important that this theory has consequences outside basic quantum theory. A foundation of quantum decision theory may be found. Links to statistical inference theory are beginning to appear. Finally, links to special and general relativity and to quantum field theory can be discussed. A thorough discussion will be given in the forthcoming book [4]; a beginning can be found in [8].

I am open to any criticism of this theory, but I also have a hope that further research, using my theory as a basis, can be carried out.

References

1. *Helland, I.S, (2024a). An alternative foundation of quantum theory. Foundations of Physics 54, 3.*
2. *Helland, I.S. (2024b). On probabilities in quantum mechanics. submitted to Foundations of Physics*

3. *Helland, I.S, (2021). Epistemic Processes. A Basis for Statistics and Quantum Theory. 2. edition. Springer, Berlin.*
4. *Helland, I.S. and Parthasarathy, H. (2024). Theoretical Variables, Quantum Theory, Relativistic Quantum Field Theory, and Quantum Gravity. Book in preparation.*
5. *Helland, I.S, (2022a). On reconstructing parts of quantum theory from two related maximal conceptual variables. International Journal of Theoretical Physics 61, 69.*
6. *Helland, I.S. (2022b). The Bell experiment and the limitation of actors. Foundations of Physics 52, 55.*
7. *Helland, I.S. (2023a). On the foundation of quantum decision theory. arXiv:2310.12762 [quant-ph]*
8. *Helland, I.S. (2023b). Possible connections between relativity theory and a version of quantum theory based upon theoretical variables. arXiv: 2305.15435 [physics.hist-ph]*
9. *Helland, I.S. (2023c). Quantum mechanics as a theory that is consistent with the existence of God. Dialogo Conferences and Journal 10 (1), 127-134.*
10. *Helland, I.S. (2023d). A simple quantum model linked to decisions. Foundations of Physics 53, 12.*
11. *Berger, J. O., & Wolpert, R. L. (1988). The likelihood principle. Hayward, CA: Institute of Mathematical Statistics.*
12. *Zwirn, H. (2016). The measurement problem: Decoherence and convivial solipsism. Foundations of Physics 46, 635-667.*
13. *Zwirn, H. (2023). Everett's interpretation and Convivial Solipsism. Quantum Rep. 5 (1), 267-281.*
14. *Plotnitsky, A. (2013). Niels Bohr and Complementarity. An Introduction. Springer, New York.*
15. *Hall, B.C. (2013). Quantum Theory for Mathematicians. Graduate Texts in Mathematics, 267, Springer, Berlin.*
16. *Busch, P. (2003). Quantum states and generalized observables: A simple proof of Gleason's Theorem. Physical Review Letters, 91(12), 120403.*
17. *Caves, C. M., Fuchs, C. A., & Schack, R. (2002). Quantum probabilities as Bayesian probabilities. Physical Review, A65, 022305.*
18. *Wright, V.J. and Weigert, S. (2019). A Gleason-type theorem for qubits based on mixtures of projective measurements. journal of Physics A: Mathematical and Theoretical 52 (5), 055301.*
19. *Wright, V.J. and Weigert, S. (2021). General probabilistic theories with a Gleason-type theorem. Quantum 5, 588.*
20. *Auffeves, A. and Granger, P. (2019). Deriving Born's rule from an inference to the best explanation. arXiv:1910.13738 [quant-ph].*
21. *Maccone, L. (2024). Schrödinger cats and quantum complementarity. Foundations of Physics 54, 17.*
22. *Skotiniosis, M., Dür, W. and Sekatski, P. (2017). Macroscopic superpositions require tremendous measurement devices. Quantum 1, 34.*
23. *Helland, I.S. (2023e). On the Bell experiment and quantum foundation. arXiv: 2305.05299 [quant-ph]. J. Mod. Appl. Phys. 6 (2), 1-5.*
24. *Susskind, L. and Friedman, A. (2014) Quantum Mechanics. The Theoretical Minimum. Basic Books, New York.*
25. *Einstein, A., Podolsky, B. and Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? Physical Review 47, 777-780.*
26. *Helland, I.S., Sæbø, S. and Tjelmeland, H. (2012). Near optimal prediction from relevant components. Scand. J. Statist. 39, 695-713.*

27. *Helland, I.S. (2024c). Towards optimal linear predictions. Submitted.*
28. *Busemeyer, J.R. and Bruza, P.D. (2012). Quantum Models of Cognition and Decision. Cambridge University Press, Cambridge.*