

Peer Review

Review of: "Parameterized Deceleration in $f(Q, C)$ Gravity: A Logarithmic Approach"

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The formulation of $f(Q, C)$ gravity in a cosmological setup is presented. The formalism of this alternative gravitational theory is developed and then applied to a cosmological scenario. In particular, a homogeneous and isotropic universe is considered, and the modified Friedmann-like equations are obtained. Later, a statistical analysis is performed, and some features of the model are extracted through the $r - s$ parameter and the $Om(z)$ diagnostic. Some conclusions are given at the end.

The paper is well-written in general, and the mathematical framework of $f(Q, C)$ gravity is very well developed. I would like to add several comments that I consider can help the authors to improve their work:

1.- As motivation, some challenges for the Λ CDM model are presented, such as the fine-tuning problem, the coincidence problem, and the age problem, all of them related to the cosmological constant. Recent results from the Dark Energy Spectroscopy Instrument (DESI) show that it is possible that the accelerated expansion of the universe is not due to a cosmological constant, which motivates the exploration of theoretical proposals such as that of the present work. As complementary motivation, some discussion about DESI results presented in Section 3 of <https://arxiv.org/abs/2411.12022> could be added in relation to BAO-CMB-SN Ia data indicating a departure from the Λ CDM model.

2.- The modified Friedmann-like equations are obtained in Eqs.(21) and (22), with the new dark energy component given by Eqs.(23) and (24) in terms of $f(Q, C)$ and its derivatives. If these expressions constitute a generalization of the standard FLRW equation of GR, probably there would be a limit where GR is recovered from $f(Q, C)$ gravity. Maybe this could be done at the level of equations of

motion, but also from the action in Eq.(11). The suggestion to show such a limit is due to the fact that any alternative theory of gravity should explain not only the accelerated expansion of the universe but also that it is valid within the regimes where it is very well known that GR works.

3.- Something to review is the constant of integration in Eq.(29). It seems that Eq.(30) does not recover H_0 for $z = 0$.

4.- I am a little bit confused about the deceleration parameter $q(z)$. The work is based on a specific proposal for the deceleration parameter $q(z)$ given by Eq.(27). However, later in Eq.(36), a new function $q(z)$ is shown. Then, the question arises: which one is the function describing the deceleration parameter, Eq.(27) or Eq.(36)? It would be useful to see how such expressions are consistent with each other. An appendix may be useful to show the procedure where the Hubble function of Eq.(30) leads to the deceleration parameter $q(z)$.

5.- One of the main goals of the work is to perform a statistical analysis, and being the main topic the cosmological consequences of $f(Q, C)$ gravity, the parameters γ_1 , γ_2 and n should be included in the statistical analysis as well. This is because whereas there is a phenomenological model for $q(z)$, the fundamental parameters of the theory are given by the constants in the $f(Q, C)$ parameterization. Thus, observational data should be used to constrain them, and not numerical values imposed by hand.

6.- Figures 6 and 7 show energy density ρ and pressure p respectively. By eye, it seems that approximately from $z \simeq 1.5$ to $z = 0$ the curves describe the same values for each of these physical quantities, i.e., the red and blue lines are overlapped. However, Figure 8 shows the equation of state ω and both curves in this figure are clearly separated. In fact, taking the value of ρ and p at $z = 0$, it is not possible to recover the value of ω at such a redshift. An option to show the evolution of ρ and p at low redshifts is by plotting the relative differences between each set of data for both energy density and pressure in order to appreciate in an evident way that there are different values for such a range of redshifts for these quantities, if it is the case.

7.- Equally important, a fundamental ingredient of Bayesian analysis is the prior, that is, the probability distribution function assigned to the parameters of the model based on previous knowledge (theoretical/observational) about them. It is important to specify the priors used, as well as why such a choice was made.

Declarations

Potential competing interests: No potential competing interests to declare.