

## Peer Review

# Review of: "The Pauli Spin Algebra, Related Clifford Algebra Representation, and the Dirac Hamiltonian Operator Derivation"

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## General assessment

The manuscript proposes an algebraic derivation of the Dirac Hamiltonian starting from the Pauli spin algebra, Clifford-algebra representations, and second-quantized Fock-space structures. The general idea is interesting, and an algebraic connection between Pauli spin symmetry, Clifford structures, and the Dirac Hamiltonian is certainly worth discussing. However, the present manuscript requires substantial clarification and correction. It contains several fundamental mathematical and physical problems that directly affect its central claim. I therefore cannot recommend publication in its present form.

## 1. Clifford algebra and its Pauli-matrix representation

The discussion around the algebra generated by the three  $2 \times 2$  Pauli matrices with the eight-dimensional complex Clifford algebra  $Cl_3(\mathbb{C})$ . Abstractly,

$$Cl_3(\mathbb{C}) \simeq M_2(\mathbb{C}) \oplus M_2(\mathbb{C}),$$

and hence it has complex dimension eight. However, the usual Pauli matrices provide only one irreducible representation, whose image is  $M_2(\mathbb{C})$ , of dimension four.

In this representation,  $\sigma_1 \sigma_2 \sigma_3 = iI$ , so that the pseudoscalar used in the manuscript satisfies

$$\xi = -i\sigma_1 \sigma_2 \sigma_3 = I.$$

Consequently,

$$P_+ = 1 + \xi/2 = I, \quad P_- = 1 - \xi/2 = 0,$$

don't produce two non-trivial components as required in Eqs. (2.5)-(2.11). The manuscript must therefore carefully distinguish the abstract Clifford algebra from a particular irreducible matrix representation.

## 2. Construction of the Dirac Hamiltonian

The dynamical assumption

$$dS_p/dt = 0,$$

is too weak to derive  $H = \alpha \cdot p + \beta m$

since other Hamiltonians satisfy the same commutation condition; for example, any rotational scalar

$$H = f(p^2)$$

commutes with  $S \cdot p$ . To obtain the Dirac Hamiltonian, you need to impose additional assumptions: linearity on momentum, a specific representation space, relativistic covariance... These assumptions should be stated explicitly.

### 3. Lorentz invariance and helicity

The discussion of Lorentz invariance should be substantially revised. For Lorentzian signature,

$$\text{Spin}(1, 3) \simeq \text{SL}(2, \mathbb{C}),$$

whereas  $\text{SU}(2) \times \text{SU}(2)$  is associated with  $\text{Spin}(4)$  in Euclidean signature. The manuscript should clarify the stated relation.

Moreover, the reduction to the Klein–Gordon operator does not establish Lorentz covariance of the first-order equation by itself. The appropriate spinorial transformation law

$$S(\Lambda)^{-1} \gamma_\mu S(\Lambda) = \Lambda_\mu$$

$$\gamma^\mu \gamma^\nu$$

should be demonstrated explicitly.

On the other hand, for a massive spin-1/2 particle, helicity is not invariant under arbitrary Lorentz boosts. Its conservation for a free Dirac particle is a dynamical property of the specified Hamiltonian. The manuscript should distinguish between the quantization of a spin component and the conservation of helicity.

### 4. Novelty claim

The introductory claim that a rigorous derivation connecting spin symmetry and the Dirac Hamiltonian has remained essentially unexplored since Dirac is too strong. There is an extensive literature on Clifford algebras, Lorentz-group representations, spinors, and algebraic constructions of the Dirac equation.

The novelty needs to be identified, and the manuscript should distinguish clearly between a new derivation, a new representation-theoretic reformulation, and a reorganization of known structures.

### 5. Additional corrections

Remark 4.5 suggests that the electron, being a lepton, does not fit naturally into the Higgs symmetry-breaking mechanism. But, in the Standard Model, the electron mass arises through its Yukawa coupling

to the Higgs field.

The English language also requires extensive revision throughout the manuscript.

#### Final recommendation

In summary, the manuscript contains potentially interesting algebraic ideas, but the problems listed above affect the core of the claimed derivation. Resolving them would require a substantial reconstruction of the paper rather than a conventional revision.

I therefore recommend rejection in the present form. A thoroughly rewritten manuscript with a moderate and precise claim might merit future consideration.

**Attachments:** available at <https://doi.org/10.32388/RC41CU>

#### Declarations

**Potential competing interests:** No potential competing interests to declare.