Review of: "A New Family of Solids: The Infinite Kepler-Poinsot Polyhedra"

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This is a good survey of the history of regular polyhedra with planar faces. The illustrations are also very good to clarify the objects. It also highlights on a recently introduced structure.

I have two suggestions to make this paper more consistent with current literature.

The first suggestion is to make explicit citations for definitions. In the spirit of keeping the paper accessible for the readers of this journal it is good not to fill it with rigorous definitions and statements in the way mathematicians often do; nevertheless, anyone interested on definitions should be pointed in the right direction. In particular, it would be good to know what definition of regular polyhedron is to be understood in this paper (presumably that of reference [4] in Section 2.1?). I also suggest to make a citation to the paper by Coxeter where he exposes the three Petrie-Coxeter polyhedra, even if that paper talks about some other objects in 4 dimensions. The paper is called “Regular skew polyhedra in three and four dimensions, and their topological analogues”.

The second suggestion is to not call this object a regular polyhedron and instead to call it a regular compound; this suggestion is based on the following.

According to Section 1.2 of reference [4] (Coxeter's “Regular polytopes”), the polygons surrounding a vertex of a polyhedron should form a single circuit. That is intended to be a requirement to call the object ‘a polyhedron’. In the object constructed in the paper, when traversing along the hexagons surrounding a vertex we find two circuits (that intersect 4 times in midpoints of edges), not only one. Alternatively, the vertex-figure (as defined in Section 2.1 of reference [4]) is not a polygon, but a compound of two polygons (two skew quadrilaterals).

The object constructed in the paper is in fact formed by the hexagons of four copies of a Petrie-Coxeter polyhedron with type {6,4}. I give next two ways to visualize this.

When taking the arrangement of cubohemioctahedra and all its hexagons as explained in Page 5, take only one hexagon (out of the 4) of some cubohemioctahedron. Then take the 6 hexagons of neighboring cubohemioctahedra that share edges with the initial hexagon (every edge belongs indeed to only two hexagons of the cubohemioctahedra). Then take all hexagons that share edges with the hexagons considered so far, and repeat this process. The result is a Petrie-Coxeter polyhedron with type {6,4}. The other three polyhedra appear from the same process by taking as initial hexagon any other of the four of the initial cubohemioctahedron.

Alternatively, we can start with a Petrie-Coxeter polyhedron with type {6,4}. Duplicate it and translate it so that the center of one hexagon meets the center of another hexagon sharing an edge with the original one (many vertices of the initial polyhedron will be vertices of the second polyhedron as well). Then duplicate these two polyhedra and translate them so that the centers of hexagons meet the centers of other suitably chosen hexagons to obtain the four polyhedra {6,4} whose hexagons are those in Figure 5 of the paper.

In this compound every vertex belongs to precisely two of the four polyhedra with type {6,4}, and every hexagon belongs to only one of those four polyhedra.

To conclude, in my view this object resembles the compound of 5 cubes in the Plate III of reference [4] (Coxeter's “Regular polytopes”) in that several identical copies of a regular polyhedron are put together to form a compound, and every vertex belongs to two of such polyhedra. I find it consistent with literature to call it a compound, and not a polyhedron, based on that.