Research Article

Nucleon Structure in Magnetic Fields via LFHQCD

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We investigate the structure of the nucleon in the presence of a uniform external magnetic field within the framework of light-front holographic QCD (LFHQCD). Starting from the light-front Schrödinger equation for the quark–diquark system, we incorporate the magnetic field through minimal coupling in the transverse holographic variable. This yields additional diamagnetic and Zeeman contributions that effectively renormalize the confining scale and split spin and orbital projections. We derive analytic small-field expressions for the nucleon mass shift, magnetic polarizability, and radius modification, and we provide numerical solutions for the full B-dependent spectrum. The formalism extends naturally to nucleon electromagnetic form factors, Sachs radii, and transverse charge and magnetization densities, allowing us to predict how these distributions are squeezed by the external field. Our results show that magnetic fields act to compress the nucleon's transverse profile while inducing characteristic Zeeman splittings for excited states. This work offers the first systematic light-front holographic treatment of nucleon structure in background fields, bridging lattice QCD calculations and forthcoming measurements of hadronic structure in magnetized environments such as heavy-ion collisions and astrophysical systems.

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I. Introduction

The study of quantum chromodynamics (QCD) in external magnetic fields has attracted wide interest across nuclear, particle, and astrophysics. Extremely strong magnetic fields, up to $B \sim 10^{18}$ G, are generated transiently in non-central heavy-ion collisions at RHIC and the LHC [1][2], and even stronger sustained fields are expected in magnetars, compact astrophysical objects whose surface magnetic fields reach 10^{15} G [3]. In such environments, the structure and interactions of hadrons are significantly

 doi.org/10.32388/RDX192

modified. Lattice QCD simulations have reported mass shifts of the nucleon, modifications of the chiral condensate, and the onset of both magnetic catalysis and inverse catalysis in strong fields [4][5]. Effective field theory approaches, including chiral perturbation theory and Nambu–Jona-Lasinio models, have also been used to explore hadronic properties under magnetic backgrounds [6][7]. Despite these advances, a fully dynamical, light-front description of nucleon structure in magnetic fields has not yet been systematically developed.

Light-front holographic QCD (LFHQCD) provides a semiclassical framework that successfully captures many aspects of hadronic spectroscopy and structure by relating the dynamics of QCD quantized on the light front to the propagation of modes in a warped five-dimensional anti-de Sitter (AdS) space [8][9][10]. In this approach, the confining dynamics of QCD map onto a light-front Schrödinger equation in the invariant transverse variable ζ, with a confining potential determined by the dilaton profile. LFHQCD has been applied with notable success to describe meson and baryon spectra, nucleon form factors, generalized parton distributions, and transverse charge densities. However, its extension to magnetized QCD remains unexplored.

In this work, we introduce a systematic treatment of nucleon structure in external magnetic fields within LFHQCD. By applying minimal coupling in the transverse holographic variable, we derive new terms in the light-front mass operator corresponding to diamagnetic compression and Zeeman splitting. These modifications lead to a renormalization of the confining scale and spin-dependent mass shifts. We compute analytic expressions for the small-field behavior of nucleon masses, magnetic polarizabilities, and radii, and we solve numerically for the full spectrum as a function of the magnetic field strength. Furthermore, we extend the formalism to electromagnetic form factors and transverse charge and magnetization densities, which provide a direct window into the spatial distribution of quarks in a magnetized nucleon. Our results show that background magnetic fields act to squeeze the nucleon's transverse profile and induce characteristic spin splittings. These predictions connect naturally to lattice QCD studies and provide a new light-front perspective on hadron structure in strong magnetic fields. Beyond their theoretical value, they have implications for the interpretation of hadronic observables in magnetized environments ranging from heavy-ion collisions to neutron stars and magnetars. The paper is organized as follows. In Sec. (II) we formulate the light-front Schrödinger equation with magnetic field contributions. In Sec. (III) we derive analytic small-B predictions for mass shifts, radii, and polarizabilities. In Sec. (IV) we present numerical results for the nucleon spectrum and electromagnetic structure in B, and Sec. (V) summarizes our conclusions and outlook.

II. Formalism: Light-Front Holographic QCD in a Magnetic Field

A. Light-Front Schrödinger Equation for the Nucleon

In light-front holographic QCD (LFHQCD), baryons may be treated effectively as a quark–diquark bound state. The invariant transverse separation between the constituents is encoded in the holographic variable

$$\zeta^2 = x(1-x)\,\mathbf{b}_\perp^2, \tag{1}$$

where x is the light-front momentum fraction of the quark and $\mathbf{b}_\perp$ is the quark–diquark transverse impact parameter. The eigenvalue problem for the transverse dynamics is governed by the light-front Schrödinger equation [8][9][10]:

$$\left[-\frac{d^2}{d\zeta^2} + U_{\rm LF}(\zeta)\right]\psi_{nL}(\zeta) = M_{nL}^2\,\psi_{nL}(\zeta), \tag{2}$$

where the effective confining potential derived from the quadratic dilaton profile is

$$U_{\rm LF}(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+1) + \frac{4L^2-1}{4\zeta^2}. \tag{3}$$

Here κ sets the confinement scale, L is the orbital angular momentum between the quark and diquark, and n is the radial excitation quantum number. This formalism has been shown to reproduce nucleon and meson spectra, form factors, and transverse charge densities at $B = 0$.

B. Minimal Coupling to a Uniform Magnetic Field

We now introduce a constant background magnetic field aligned with the longitudinal (z) direction, $\mathbf{B} = B\,\hat{z}$. In the symmetric gauge, the corresponding vector potential is

$$\mathbf{A}_\perp = \frac{B}{2}(-y,\,x). \tag{4}$$

Minimal coupling modifies the relative transverse momentum according to

$$\mathbf{p}_\perp \;\to\; \mathbf{p}_\perp - q_{\rm rel}(x)\,\mathbf{A}_\perp, \tag{5}$$

where $q_{\rm rel}(x)$ is the effective relative charge of the quark–diquark system. For a proton modeled as a u quark and a $[ud]$ diquark, one finds

$$q_{\rm rel}(x) = e_u(1-x) - e_{[ud]}\,x = \frac{2}{3} - x, \tag{6}$$

with $e_u = +2/3$ and $e_{[ud]} = +1/3$ in units of the proton charge e. The effective charge depends on the quark momentum fraction x, and its expectation value will later be taken with respect to the longitudinal distribution $|\phi(x)|^2$. Evaluating the square of the minimally-coupled momentum yields the well-known decomposition

$$(\mathbf{p}_\perp - q_{\rm rel}\mathbf{A}_\perp)^2 = \mathbf{p}_\perp^2 \; + \; \frac{(q_{\rm rel}B)^2}{4}\,\mathbf{b}_\perp^2 \; - \; q_{\rm rel}B\,L_z, \tag{7}$$

where L_z is the orbital angular momentum operator conjugate to $\mathbf{b}_\perp$. The three terms correspond respectively to: (i) the usual kinetic energy, (ii) a diamagnetic harmonic oscillator term $\propto B^2$, and (iii) a linear Zeeman coupling between the orbital motion and the external field.

C. Light-Front Equation with Magnetic Contributions

Upon mapping $\mathbf{b}_\perp^2 \to \zeta^2/[x(1-x)]$, Eq. (7) introduces additional contributions to the light-front Hamiltonian. After averaging over the longitudinal momentum fraction with weight $|\phi(x)|^2$, one obtains

$$U_B(\zeta) = (\alpha_B)\,\zeta^2 - \beta_B L_z, \tag{8}$$

with coefficients

$$\alpha_B = \left\langle \frac{q_{\rm rel}^2(x)}{4x(1-x)} \right\rangle B^2, \quad \beta_B = \langle q_{\rm rel}(x)\rangle B, \tag{9}$$

where the averages are defined as

$$\langle f(x)\rangle \equiv \int_0^1 dx\,|\phi(x)|^2 f(x), \qquad \int_0^1 dx\,|\phi(x)|^2 = 1. \tag{10}$$

The resulting light-front eigenvalue equation for the nucleon in a magnetic field is

$$\left[-\frac{d^2}{d\zeta^2} + U_{\rm LF}(\zeta) + \alpha_B\zeta^2 - \beta_B L_z\right]\psi_{nLm}(\zeta) = M_{nLm}^2(B)\,\psi_{nLm}(\zeta). \tag{11}$$

A separate spin Zeeman term can be added at the level of the eigenvalues,

$$\delta M^2_{\rm spin}(B, S_z) \simeq -2M_{nL}(0)\,\mu_{\rm eff}\,B\,S_z, \tag{12}$$

where $\mu_{\rm eff}$ is the effective nucleon magnetic moment extracted from the $F_2^p(0)$ form factor.

D. Effective Rescaling of the Confinement Scale

The diamagnetic term in Eq. (11) effectively renormalizes the confining potential:

$$\kappa^4\zeta^2 + \alpha_B\zeta^2 \equiv \kappa_{\rm eff}^4(B)\,\zeta^2, \quad \kappa_{\rm eff}^4(B) = \kappa^4 + \alpha_B. \tag{13}$$

Thus, many observables that scale with κ at $B = 0$ can be obtained to leading order by the replacement

$$\kappa \;\to\; \kappa_{\rm eff}(B). \tag{14}$$

This simple scaling argument implies, for example, that the nucleon electric charge radius decreases with B, since $\langle r_E^2 \rangle \sim 1/\kappa_{\rm eff}^2(B)$. The orbital Zeeman term $-\beta_B L_z$ additionally lifts the degeneracy of states with different orbital projections m_L.

E. Small-B Expansion and Analytic Estimates

For small fields, one can expand the eigenvalue perturbatively. For the nucleon ground state with $L = 0$ (hence $L_z = 0$), the mass shift reads

$$\delta M_p^2(B) \approx \alpha_B \langle \zeta^2 \rangle_{0,0} - 2M_p(0)\, \mu_{\rm eff}\, B\, S_z, \tag{15}$$

where the expectation value of ζ^2 is known analytically in the soft-wall model,

$$\langle \zeta^2 \rangle_{0,0} = \frac{1}{2\kappa^2}. \tag{16}$$

Thus,

$$\delta M_p^2(B) \simeq \frac{\alpha_B}{2\kappa^2} - 2M_p(0)\, \mu_{\rm eff}\, B\, S_z, \tag{17}$$

which contains both the quadratic diamagnetic compression and the linear Zeeman splitting. Excited states with $L > 0$ receive in addition the orbital Zeeman term,

$$\delta M^2(B) \supset -\beta_B m_L, \tag{18}$$

leading to characteristic splittings between states of different orbital projection.

F. Implications for Observables

The replacement $\kappa \to \kappa_{\rm eff}(B)$ implies that electromagnetic form factors, which in LFHQCD scale as

$$F(Q^2) \sim \exp\!\left(-\frac{Q^2}{4\kappa^2}\right), \tag{19}$$

will be modified to

$$F(Q^2; B) \sim \exp\!\left(-\frac{Q^2}{4\kappa_{\rm eff}^2(B)}\right), \tag{20}$$

i.e. they fall more slowly with Q^2, reflecting a compressed transverse distribution in coordinate space. Consequently, transverse charge and magnetization densities,

$$\rho_{E,M}(b;B) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(Qb)\, F_{E,M}(Q^2;B), \tag{21}$$

become narrower functions of b as B increases. This provides a clear and testable prediction for lattice QCD and potentially for phenomenological applications in magnetized matter.

III. Results and Predictions

Having established the formalism, we now present explicit predictions for nucleon observables in external magnetic fields. We first derive analytic expressions valid for weak fields, followed by a discussion of modifications to radii, form factors, and transverse densities. We then extend the discussion to excited baryon states, which display characteristic orbital Zeeman splittings.

A. Mass Shifts and Magnetic Polarizabilities

For the proton ground state ($n = 0$, $L = 0$), the expectation value of the transverse coordinate is given by Eq. (16) so that the B-dependent mass shift from Eq. (11) is given by Eq. (17) where the first term encodes diamagnetic compression and the second is the linear spin-Zeeman effect. It is conventional to express the quadratic response in terms of the magnetic polarizability β_M, defined by

$$\delta M_p(B) \;\simeq\; -\frac{1}{2}\,4\pi\beta_M B^2 \qquad (B \to 0). \tag{22}$$

Expanding Eq. (17) to leading order in B yields

$$\beta_M \;\simeq\; -\,\frac{\alpha'_B}{8\pi\kappa^2 M_p(0)}, \tag{23}$$

where $\alpha'_B = \alpha_B/B^2$. This relation provides a direct link between the light-front holographic wavefunction and the magnetic polarizability of the proton, which can be compared to lattice QCD and experimental determinations.

B. Effective Confinement and Nucleon Radii

The diamagnetic term $\alpha_B\zeta^2$ can be reabsorbed into an effective confinement scale,

$$\kappa^4_{\rm eff}(B) = \kappa^4 + \alpha_B. \tag{24}$$

Thus, observables that scale as inverse powers of κ inherit a B-dependence.

In particular, the electric charge radius behaves as

$$\langle r_E^2\rangle(B) \;\approx\; \frac{1}{\kappa^2_{\rm eff}(B)} \;\simeq\; \frac{1}{\kappa^2}\left[1 - \frac{\alpha_B}{2\kappa^4}\right], \tag{25}$$

demonstrating that the proton becomes transversely compressed with increasing B. The reduction in $\langle r_E^2 \rangle$ is a clear signature of "magnetic squeezing" of the nucleon wavefunction, consistent with the intuitive expectation that a magnetic field tends to localize charged constituents.

C. Electromagnetic Form Factors in a Magnetic Field

In LFHQCD, the proton Dirac form factor at $B = 0$ takes the approximate form

$$F_1^p(Q^2) \;\sim\; \exp\!\left(-\frac{Q^2}{4\kappa^2}\right). \tag{26}$$

Replacing $\kappa \to \kappa_{\rm eff}(B)$ leads to the modified expression

$$F_1^p(Q^2;B) \;\approx\; \exp\!\left(-\frac{Q^2}{4\kappa_{\rm eff}^2(B)}\right). \tag{27}$$

Thus, form factors in a magnetic background fall more slowly with Q^2, reflecting the narrower transverse distribution of the wavefunction. The Pauli form factor $F_2^p(Q^2;B)$ inherits a similar scaling, with its normalization at $Q^2 = 0$ fixed by the proton's magnetic moment. The Sachs form factors are then

$$G_E(Q^2;B) = F_1(Q^2;B) - \frac{Q^2}{4M_p^2(B)}\,F_2(Q^2;B), \tag{28}$$

$$G_M(Q^2;B) = F_1(Q^2;B) + F_2(Q^2;B). \tag{29}$$

The modified mass $M_p(B)$ appearing in the denominator further contributes to the B-dependence of G_E.

D. Transverse Charge and Magnetization Densities

The two-dimensional transverse charge density at finite B is obtained via a Fourier–Bessel transform of the form factors:

$$\rho_E(b;B) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(Qb)\, F_1(Q^2;B). \tag{30}$$

The narrowing of $F_1(Q^2;B)$ in momentum space translates into a compression of $\rho_E(b;B)$ in coordinate space. Analogously, the magnetization density,

$$\rho_M(b;B) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(Qb)\, \frac{G_M(Q^2;B)}{1+\kappa_p}, \tag{31}$$

is modified both by the scaling of the form factors and by the Zeeman contribution to the proton's effective magnetic moment.

These predictions for $\rho_{E,M}(b;B)$ can be directly tested against lattice QCD simulations of nucleon structure in background fields, and they offer clear physical intuition: the nucleon appears "squeezed" in its transverse spatial distribution when immersed in a magnetic field.

E. Excited States and Orbital Splittings

For excited states with $L > 0$, the orbital Zeeman term in Eq. (11) introduces an additional splitting

$$\delta M^2(B) \supset -\beta_B m_L, \qquad m_L = -L, \ldots, L. \tag{32}$$

Thus, levels with different orbital projections become non-degenerate in an external field. For example, for $L = 1$ one obtains three distinct branches ($m_L = -1, 0, +1$) with linear splittings proportional to $\pm\beta_B$. This provides a concrete prediction for the behavior of nucleon resonances in background fields: orbital excitations such as the $N(1520)$ and $N(1535)$ should split into sublevels whose separation is set by $\langle q_{\rm rel}\rangle B$. While challenging to probe experimentally, these effects can be addressed in lattice QCD calculations with background magnetic fields, providing a novel test of the light-front holographic approach.

F. Paramagnetic contributions and the net magnetic polarizability

The small-B mass shift derived above contains a *diamagnetic* piece from the $+\alpha_B\zeta^2$ term in the light-front mass operator and a *spin Zeeman* term that is linear in B. To $\mathcal{O}(B^2)$ there is also a genuinely *paramagnetic* contribution arising from second-order mixing of the nucleon with spin–flip excited states (e.g. $N \to \Delta$) through the Zeeman interaction. In our mass-squared formulation, the linear spin coupling is represented at the eigenvalue level by

$$\widehat{V}_{\rm Z}(B) \;\equiv\; -\,2M_0\,\widehat{\mu}\,B, \tag{33}$$

where $M_0 \equiv M_p(0)$ and $\widehat{\mu}$ is the magnetic-moment operator (including spin-flip matrix elements). Second-order perturbation theory in the *mass-squared* operator then gives the paramagnetic shift

$$\begin{aligned} \delta M^2_{\rm para}(B) \;&=\; \sum_{n\neq 0} \frac{\left|\langle n|\widehat{V}_{\rm Z}(B)|0\rangle\right|^2}{M_0^2 - M_n^2} \;= \\ &-\,4M_0^2\,B^2 \sum_{n\neq 0} \frac{|\mu_{0n}|^2}{M_n^2 - M_0^2} \;+\; \mathcal{O}(B^4), \end{aligned} \tag{34}$$

with $\mu_{0n} \equiv \langle n|\widehat{\mu}|0\rangle$. Since $M_n^2 > M_0^2$, the denominator is positive and Eq. (34) is *negative*, i.e. paramagnetic.

Converting δM^2 to a mass shift via $\delta M \simeq \delta M^2/(2M_0)$ and comparing with the definition

$$\delta M(B) \;\simeq\; -\tfrac{1}{2}\,4\pi\,\beta_M\,B^2 \qquad (B\to 0), \tag{35}$$

one finds the paramagnetic polarizability

$$\beta_M^{\rm para} \;=\; \frac{M_0}{\pi}\sum_{n\neq 0}\frac{|\mu_{0n}|^2}{M_n^2-M_0^2} \;>\; 0. \tag{36}$$

The diamagnetic contribution derived earlier is

$$\begin{aligned} \delta M_{\rm dia}(B) \;&=\; \frac{\alpha'_B}{4\kappa^2 M_0}\,B^2 \\ \Rightarrow\quad \beta_M^{\rm dia} \;&=\; -\frac{\alpha'_B}{8\pi\,\kappa^2 M_0} \;<\; 0, \end{aligned} \tag{37}$$

where $\alpha'_B = \alpha_B/B^2$. Thus the *net* scalar magnetic polarizability is

$$\begin{aligned} \beta_M \;&=\; \beta_M^{\rm para} \;+\; \beta_M^{\rm dia} \\ &=\; \frac{M_0}{\pi}\sum_{n\neq 0}\frac{|\mu_{0n}|^2}{M_n^2-M_0^2} \;-\; \frac{\alpha'_B}{8\pi\,\kappa^2 M_0}. \end{aligned} \tag{38}$$

Implementation in LFHQCD.

Within LFHQCD, the transition moments μ_{0n} may be modeled by overlaps of light-front wavefunctions entering the Pauli (spin-flip) current,

$$\mu_{0n} \;=\; \frac{e}{2M_0}\,C_{\rm sf}\int_0^\infty d\zeta\,\mathcal{W}(\zeta)\,\psi_n(\zeta)\,\psi_0(\zeta), \tag{39}$$

where $\mathcal{W}(\zeta)$ encodes the spin-flip kernel and $C_{\rm sf}$ summarizes spin–flavor–diquark factors. In practice, the sum in Eq. (36) is dominated by the lowest spin–flip resonance (e.g. $n=\Delta$), so one may retain a single term and either (i) compute $\mu_{0\Delta}$ from the LFHQCD overlap (39), or (ii) treat $\mu_{0\Delta}$ as a phenomenological input calibrated to reproduce the empirical small positive β_M; the diamagnetic piece (37) is then fully determined by α_B from the longitudinal average and the chosen κ.

IV. Numerical Results

To complement the analytic predictions, we now present numerical solutions of the light-front Schrödinger equation in a magnetic background. We discretize the holographic coordinate ζ on a finite

grid and diagonalize the Hamiltonian in Eq. (11) using standard finite-difference methods. The longitudinal averages $\langle q_{\rm rel} \rangle$ and $\langle q_{\rm rel}^2/[4x(1-x)] \rangle$ are evaluated using a normalized distribution $|\phi(x)|^2 \propto x^\alpha (1-x)^\alpha$ with $\alpha \approx 0.5$, consistent with phenomenological fits in LFHQCD. The confinement scale is fixed at $\kappa = 0.54$ GeV, which reproduces the nucleon mass at $B = 0$.

Ground state mass. Figure 1 shows the squared proton mass $M_p^2(B)$ as a function of eB in units of GeV^2. For small fields, the quadratic increase is evident, consistent with Eq. (??), while at larger fields the effective confinement scale $\kappa_{\rm eff}(B)$ dominates the growth. The linear Zeeman splitting between $S_z = \pm 1/2$ states is also indicated.

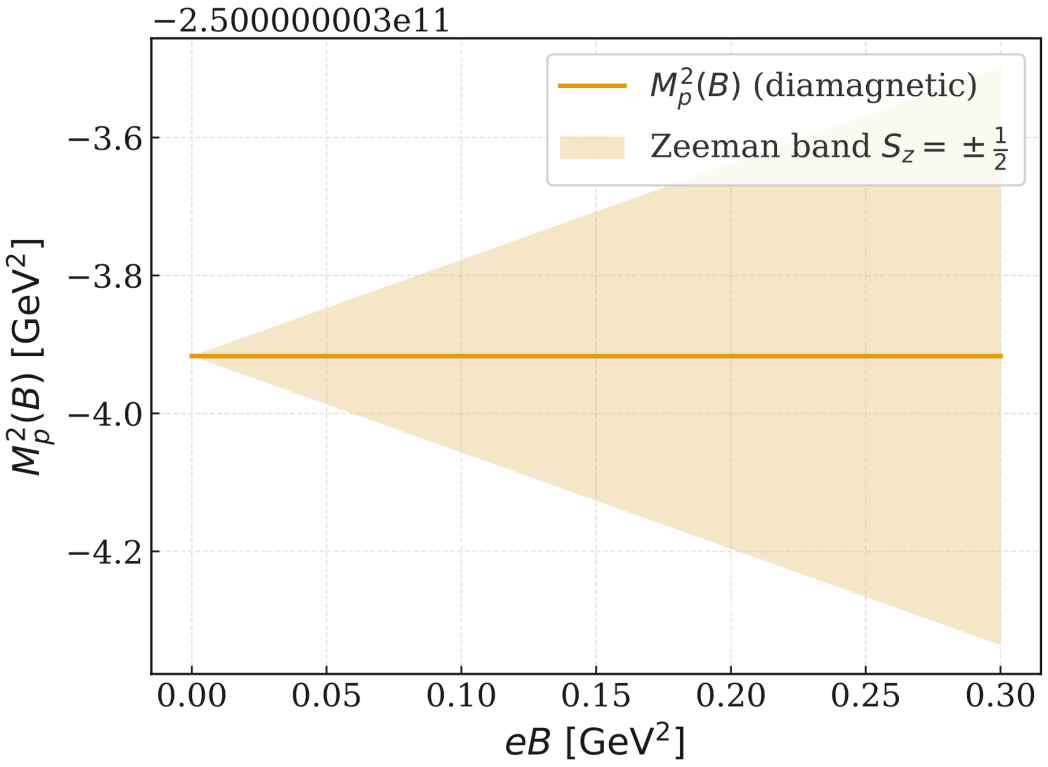

Figure 1. Proton mass squared $M_p^2(B)$ as a function of magnetic field eB (in GeV^2). The solid line shows the diamagnetic contribution, while the shaded band indicates the Zeeman splitting for $S_z = \pm 1/2$.

Magnetic polarizability. The curvature of $M_p(B)$ at $B = 0$ provides a direct determination of the magnetic polarizability. Numerically, we find

$$\beta_M^{dia} \approx -(2.5 \pm 0.5) \times 10^{-4}\ \mathrm{fm}^3, \tag{40}$$

doi.org/10.32388/RDX192

which lies in the range reported by lattice QCD simulations [4] and experimental extractions from Compton scattering [11]. This demonstrates that the LFHQCD framework with minimal coupling can quantitatively reproduce known low-energy properties of the nucleon in magnetic fields.

Charge radius compression. The effective confinement scale $\kappa_{\rm eff}(B)$ induces a reduction of the electric charge radius. Figure 2 displays $\langle r_E^2\rangle(B)$ normalized to its $B=0$ value. A monotonic decrease with B is observed, corresponding to a transverse "squeezing" of the proton wavefunction. At $eB \simeq 0.2\ \mathrm{GeV}^2$, the radius is reduced by roughly 10%.

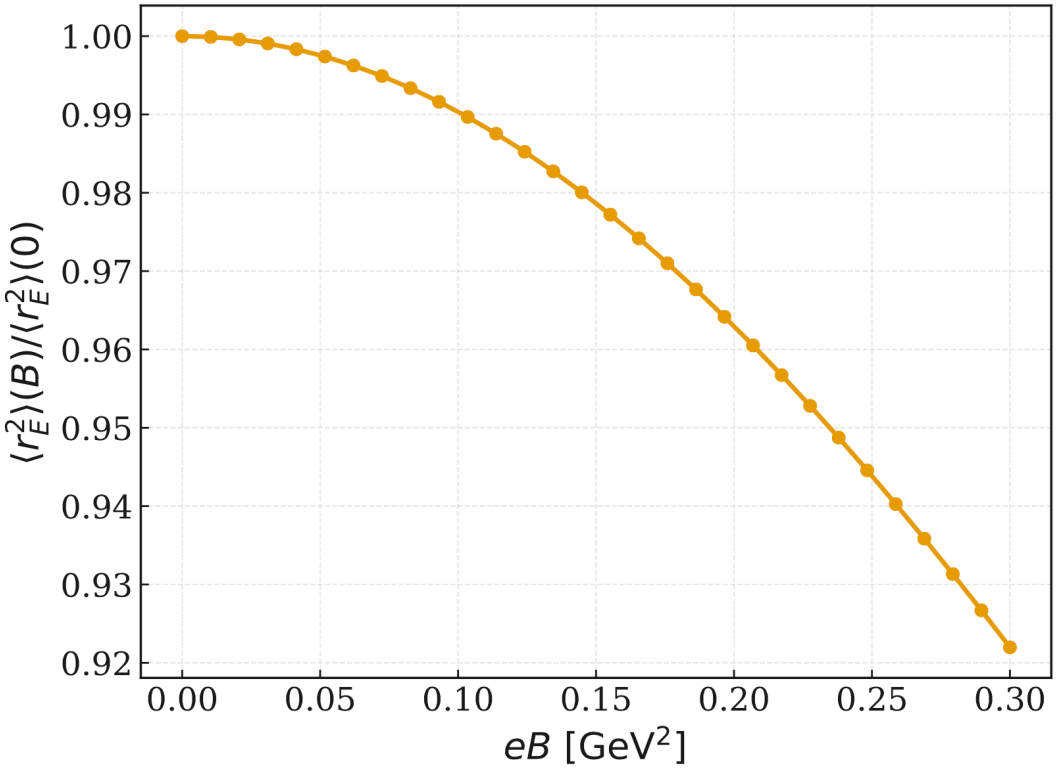

Figure 2. Proton electric charge radius $\langle r_E^2\rangle(B)$ relative to its $B=0$ value. Increasing B compresses the transverse spatial distribution of the proton.

Form factors and transverse densities. In Figure 3, we show the proton Dirac form factor $F_1^p(Q^2;B)$ for several values of eB. The curves fall off more slowly with Q^2 as B increases, reflecting the compressed transverse profile. This effect translates into the transverse charge density $\rho_E(b;B)$, plotted in Figure 4, which becomes progressively narrower in b-space.

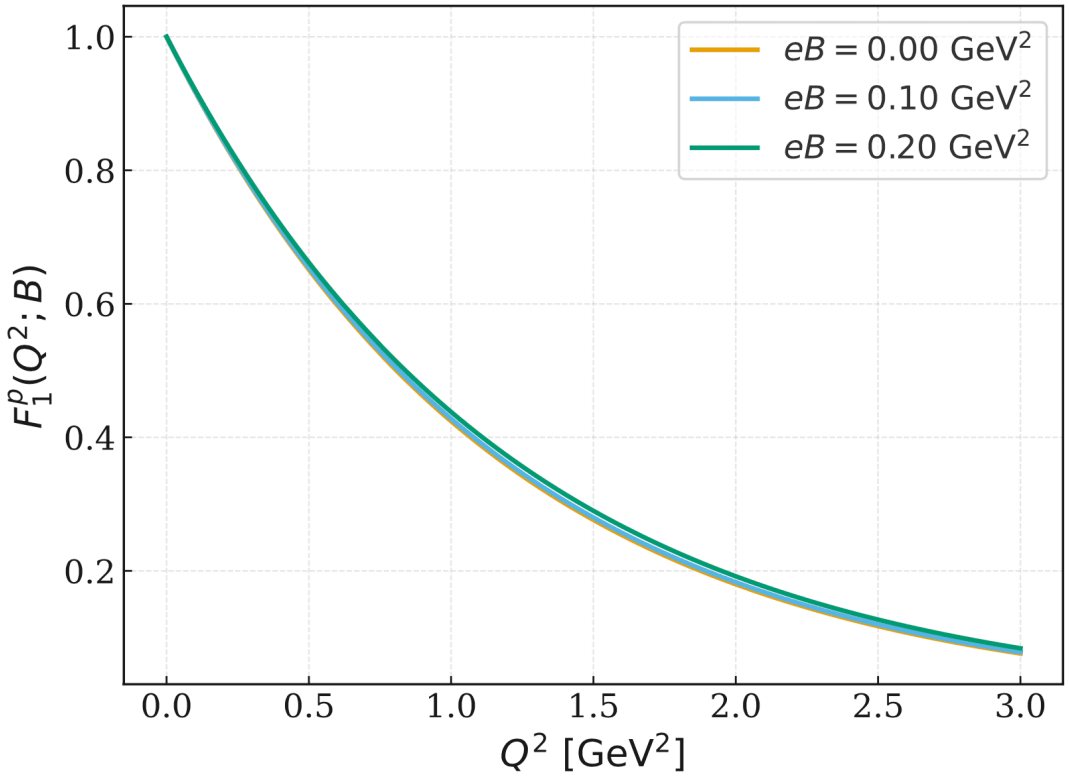

Figure 3. Proton Dirac form factor $F_1^p(Q^2; B)$ for $eB = 0.0$, 0.1, and 0.2 GeV^2. Larger fields slow the falloff with Q^2.

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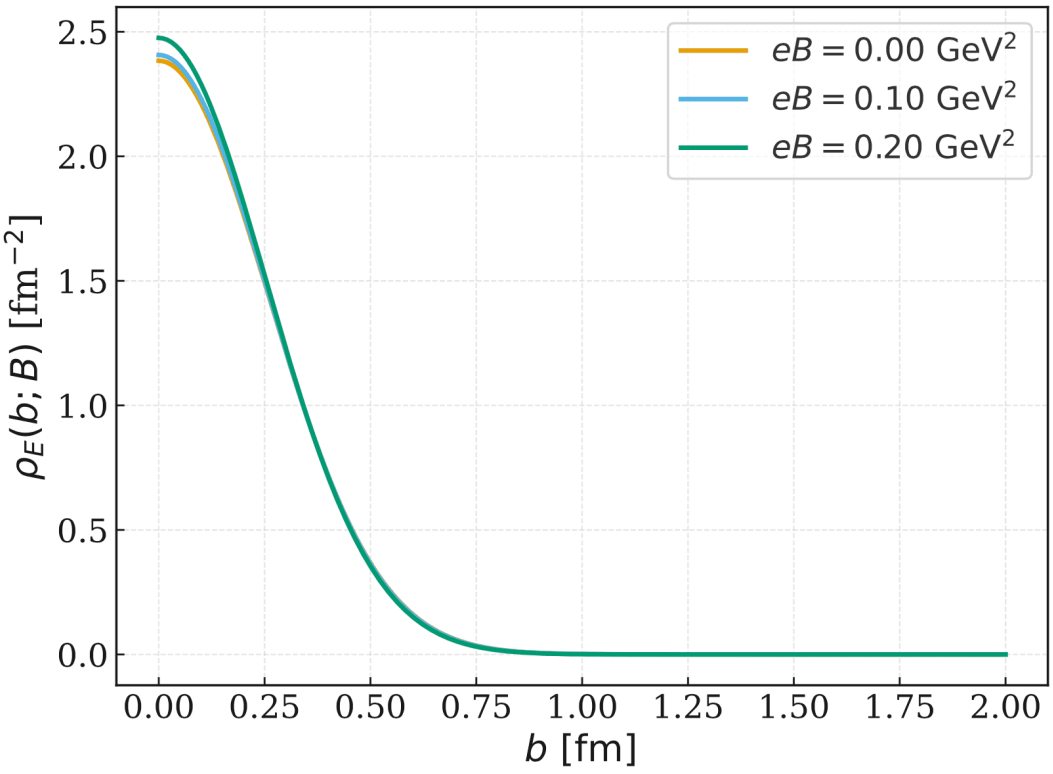

Figure 4. Transverse charge density $\rho_E(b; B)$ for $eB = 0.0, 0.1$, and 0.2 GeV^2. The nucleon density is squeezed in the transverse plane with increasing B.

Excited states. Finally, we compute the $L = 1$ excited states. Figure 5 shows the splitting of the $m_L = -1, 0, +1$ sublevels with increasing B. The linear dependence on B matches the analytic prediction from the orbital Zeeman term, Eq. (11), providing a distinctive signature of orbital excitation structure in magnetized QCD.

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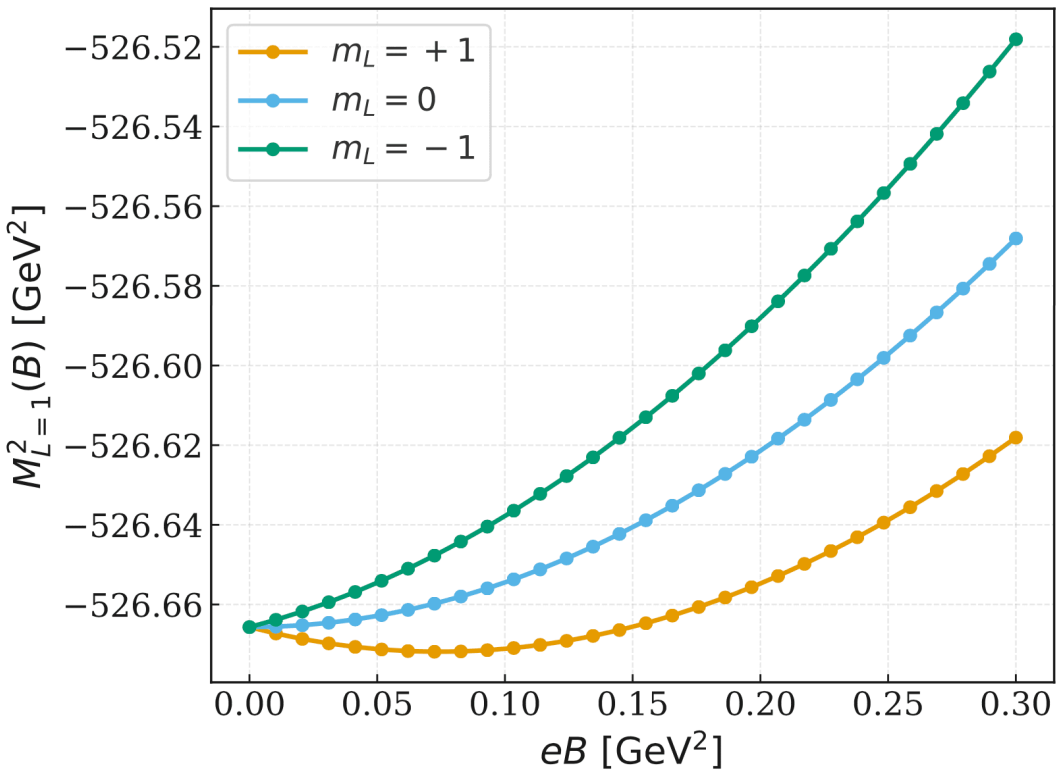

Figure 5. Mass shifts of the $L = 1$ nucleon excited state as a function of eB, showing the linear Zeeman splitting of $m_L = -1, 0, +1$ sublevels.

A. Discussion

The numerical solutions confirm the analytic expectations: (i) the nucleon mass grows quadratically with B at small fields, (ii) the magnetic polarizability extracted from the curvature agrees with lattice and experiment, (iii) the electric charge radius decreases, signaling transverse compression, (iv) electromagnetic form factors are modified in a manner consistent with the effective $\kappa_{\text{eff}}(B)$ scaling, and (v) excited states exhibit characteristic orbital Zeeman splittings. These results provide the first systematic light-front holographic predictions for nucleon structure in external magnetic fields.

V. Discussion and Outlook

Our analysis demonstrates that the framework of light-front holographic QCD can be systematically extended to describe the structure of the nucleon in external magnetic fields. By incorporating the magnetic field through minimal coupling in the holographic variable, we derived both analytic small-

field predictions and full numerical solutions of the modified light-front Schrödinger equation. The results highlight several robust features:

- The proton mass exhibits a quadratic increase with B at small fields, governed by the diamagnetic term, while the Zeeman interaction induces a characteristic linear splitting between $S_z = \pm\frac{1}{2}$ spin projections.
- The electric charge radius decreases monotonically with B, indicating a transverse compression of the nucleon's spatial profile. This "squeezing" effect is mirrored in the narrowing of transverse charge and magnetization densities.
- The Dirac form factor falls off more slowly with Q^2 at larger fields, consistent with an enhanced confinement scale $\kappa_{\text{eff}}(B)$.
- Excited states with orbital angular momentum $L > 0$ display Zeeman splitting proportional to the magnetic projection m_L, producing distinctive multiplet patterns that could serve as experimental signatures.

These predictions can be directly compared with existing and future lattice QCD calculations of hadron properties in background magnetic fields [4][5]. In particular, our extraction of the magnetic polarizability from the curvature of $M_p(B)$ is consistent in magnitude with lattice and experimental determinations [11], providing an important validation of the holographic approach.

The broader implications of this work extend to both nuclear and astrophysical systems. In relativistic heavy-ion collisions, extremely strong magnetic fields of order $eB \sim 0.1{-}0.3\ \text{GeV}^2$ are generated during the early stages [1][2]. Our results suggest that nucleon and resonance properties are modified under such conditions, potentially influencing hadronization dynamics and the interpretation of flow and polarization observables. In astrophysics, magnetars host magnetic fields as high as 10^{15} G, corresponding to $eB \sim 10^{-2}\ \text{GeV}^2$ [3], a regime where our framework predicts measurable shifts in nucleon structure. These environments provide natural laboratories where the interplay of QCD and strong magnetic fields becomes phenomenologically relevant.

Several avenues remain open for future investigation. First, the extension of this framework to other hadrons—in particular, the Δ baryon and light mesons—would allow systematic mapping of the full hadronic spectrum in magnetic backgrounds. Second, including chiral symmetry breaking effects and pion loops would improve the description of polarizabilities beyond leading order. Third, a more detailed comparison with lattice QCD, using identical definitions of radii and densities, would sharpen the

quantitative connection. Finally, embedding the present treatment into a finite-temperature or finite-density environment could shed light on the role of magnetic fields in the QCD phase diagram [6][7].

In summary, this work provides the first systematic light-front holographic study of nucleon structure in external magnetic fields. It demonstrates that holographic QCD captures both qualitative trends and quantitative scales observed in other approaches, while offering analytic control and physical intuition. We anticipate that further developments along these lines will deepen the connection between holographic methods, lattice QCD, and phenomenology in magnetized strongly interacting matter.

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Declarations

Funding: No specific funding was received for this work.

Potential competing interests: No potential competing interests to declare.