Research Article

A Possible Secular Drift of Atomic Clocks

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Arguments in favor of the hypothesis that the tick rate of atomic clocks is drifting are examined. The main one is the existence of a preferred value for the relative drift of the period of millisecond pulsars, which is otherwise left unexplained. Other arguments, like the drift of the frequency of sapphire oscillators, the Earth-Moon distance and the Pioneer anomalies are less convincing since other factors are known to play a key role. Interestingly, the corresponding drift of atomic spectra could contribute significantly to the value of the Hubble constant, thus eliminating the age problem of ΛCDM, which has recently been exacerbated by the discovery of mature galaxies at redshifts as high as 14. Such a drift is likely due to an increase of the gravitational field felt by the atomic clocks.

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Introduction

Modern atomic clocks are so accurate that they would not have gained or lost a second if they had started running billion years ago [1][2]. However, compared with atomic clocks at rest in an inertial reference system, a moving atomic clock is ticking at a slower rate [3][4], its tick rate slowing also in the presence of massive bodies [5][6] as checked, for instance, by Joseph Hafele and Richard Keating in their trip around Earth aboard commercial airliners [3]. As a matter of fact, if such effects were not taken into account, the Global Positioning System would not work [7].

So, if the gravitational field felt by an atomic clock on Earth were changing as a function of time, its tick rate would drift. Moreover, if the gravitational field were changing in the same way all around the Earth, either as a consequence of a variation of the gravitational field of the Earth [8], of the mass of the Sun [9], of the gravitational constant [10][11][12] or of the mean gravitational field of the local Universe [13][14][15], the tick rate of all atomic clocks would drift in the same way, meaning that they would keep on looking highly accurate, when compared to each other.

As a corollary, the tick rate of clocks based on phenomenons not affected by the value of the local gravitational field would drift with respect to atomic clocks [16][17]. Interestingly, the tick rate of clocks based on different and, ideally, independant physical phenomenons would drift at the same rate, with respect to atomic clocks, allowing both to determine the common origin of their drift and to measure the variations of the local gravitational field as a function of time.

The main goal of the present study is to gather arguments in favor of the hypothesis that the tick rate of atomic clocks is drifting. Some of these arguments have already been used to back other non-mainstream hypotheses, like a universal expansion of space [18][19][20][21], a spacetime expansion [22], a varying gravitational constant [23][24] or a varying speed of light [25][26]. Note however that these other hypotheses have significantly different physical consequences.

Main hypothesis

Since atomic clocks rely on the fact that a given kind of atom can only be excited by higlhy specific frequencies, let us assume that $\nu_{mn}(t_u)$, the frequency required for a photon to be absorbed by an atom when it jumps from state m to state n, varies slowly as a function of time, so that:

$$\nu_{mn}(t_u) = C_{mn} t_u + \frac{1}{2}\dot{C}_{mn} t_u^2 + \dots$$

where t_u is the time given by a clock that is not sensitive to a change of the gravitational field, C_{mn}, the time derivative of $\nu_{mn}(t_u)$ and $\dot{C}_{mn}$ the time derivative of C_{mn}. Hereafter, for the sake of simplicity, only the first term of the above expansion is retained. In other words, it is assumed that $\nu_{mn}(t_u)$ varies so slowly over the timescales considered herein that it can be well approximated by [27]:

$$\nu_{mn}(t_u) = C_{mn} t_u$$

As a consequence:

$$\frac{\nu_{mn}(t_u)}{\nu_{mn}(t_0)} = \frac{t_u}{t_0} \qquad (1)$$

where t_0 is the reference time, for instance the time when a series of observations begins. Note that the above approximation implies that atoms were not able to emit any observable light at $t_u = 0$.

Δt_{ac}, the duration measured by an atomic clock, is, by definition, proportional to a given $\nu_{mn}(t_u)$, namely, to the atomic frequency used by the clock to control its ticking rate, at the moment a

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measurement is performed. So, as a consequence of eqn 1, durations measured by atomic clocks are expected to increase in such a way that:

$$\frac{\Delta\dot{t}_{ac}}{\Delta t_0} = \frac{1}{t_0} \tag{2}$$

where Δt_0 is the duration measured at time t_0.

On the other hand, f_{ac}, the frequency measured by an atomic clock is expected to decrease, so that:

$$\frac{\dot{f}_{ac}}{f_0} = -\frac{1}{t_0} \tag{3}$$

where f_0 is the frequency measured at time t_0.

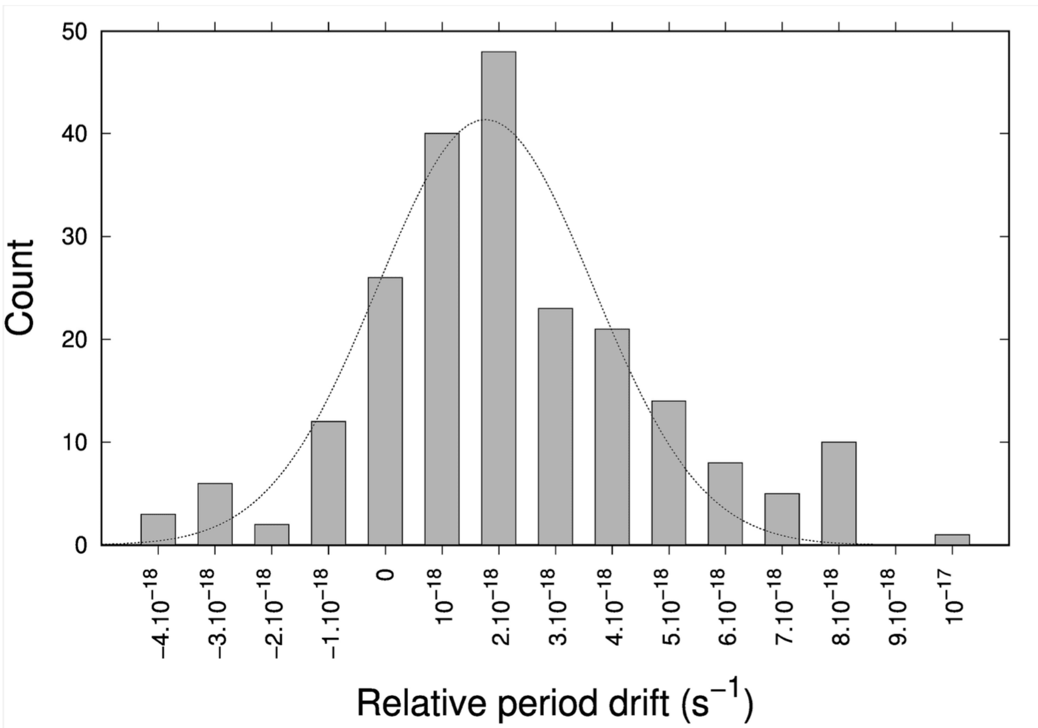

Figure 1. Relative drift of the period of millisecond pulsars. The 301 pulsars of the ATNF pulsar database with a period less than 15 ms, a known period drift, distance and transverse velocity were considered. 25% of them have a relative period drift outside the range shown above. Dotted line: gaussian fit.

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Results

Millisecond pulsars

As a consequence of the law of inertia, the rotation frequency of a freely rotating body is expected to be constant in time.

Rotating bodies with very stable periods have actually been found in space, millisecond pulsars [28] being so stable that they have been used to establish timescales that rival the best atomic-clocks in long-term stability [29]. Such timescales are nowadays provided by several pulsar timing arrays, like the Parkes [30], the European [31], the Indian [32] or the MeerKAT [33] ones.

In the ATNF pulsar database [34], there are 778 millisecond pulsars[1], namely, pulsars with a measured period, P_{obs}, of less than 15 ms. Among them, 402 have a known period drift, $\dot{P}_{obs}$, which ranges between -1.1×10^{-18}, for pulsar B2127+11D, and $1.3 \times 10^{-17} s^{-2}$, for pulsar J1748-2446aw.

The apparent drift of the period of a pulsar needs to be corrected with the Shklovsky term, the actual drift of its period, $\dot{P}$, being so that [35][36]:

$$\frac{\dot{P}}{P} \approx \frac{\dot{P_{obs}}}{P_{obs}} - \frac{v_\perp^2}{c_0 d_p}$$

where $v_\perp$ is the velocity of the pulsar perpendicular to the line of sight, d_p, its distance, and c_0, the speed of light.

In the ATNF pulsar database, $v_\perp$ and d_p are nowadays available for 301 millisecond pulsars with a known period drift. As shown in Figure 1, this allows to confirm that there is a preferred value for $\frac{\dot{P}}{P}$ [22][37], namely, as obtained with a gaussian fit:

$$\frac{\dot{P}}{P} = 1.77 \pm 0.19 \times 10^{-18} s^{-1} (\text{std error})$$

Upon the hypothesis that the drift of the period of millisecond pulsars is a consequence of magnetic dipole radiation [38], this would mean that millisecond pulsars have a preferred characteristic age [36][39], τ_c, of:

$$\tau_c \approx \frac{P}{2\dot{P}} = 9 \pm 1 \text{ Gyr}$$

Such a preferred value seems difficult to justify [37]. In the following, it is instead assumed that it is due to a drift of the tick rate of the atomic clocks used to measure the timing of millisecond pulsars. According

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to eqn 2, this would mean that:

$$t_0 \approx 18 \pm 2 \text{ Gyr}.$$

In this context, fluctuations around the preferred value of $\frac{\dot{P}}{P}$ (see Fig. 1), of $1.90 \pm 0.19 \times 10^{-18} s^{-1}$, could be due to an acceleration of the pulsar, a_p, along the line of sight, since it would add an additional drift to its measured period of [36][40]:

$$\frac{\dot{P}}{P} = \frac{a_p}{c_0}$$

with an order of magnitude for a_p of nearly $6 \times 10^{-10} m/s^2$. Note that this is the acceleration experienced by a body at 22 AU of a planet with the mass of Neptune. Note also that the period of revolution of planets at this distance of a solar-mass star is $\approx$ 100 yr, explaining why they may have escaped detection so far. Indeed, though planets have been discovered around pulsars [41][42], bodies in the ATNF database with a period of revolution around a pulsar of more than a year have a mass of at least 0.15 $M_\odot$.

Sapphire oscillators

If the tick rate of atomic clock is drifting, clocks based on other physical principles are expected to exhibit the same, apparent drift [16][24].

Compared to atomic clocks over many years, cryogenic sapphire oscillators (CSO) indeed exhibit linear drift rates [43][44], with an order of magnitude often found consistent with the drift observed for the period of millisecond pulsars [21] given that, according to the present study, frequencies measured using atomic clocks are expected to decrease as a function of time (eqn 3).

For instance, the fractional frequency shift of the CSO at CNES has been recorded from 2003 to 2005. During the single cryogenic cycling, there was an abrupt frequency jump of about 0.5 Hz, however the drift otherwise remained at a constant level of $-2.4 \times 10^{-13} d^{-1}$ [44], that is, of $-2.8 \times 10^{-18} s^{-1}$. In the case of the CSO at Paris observatory, a constant linear drift of $-1.5 \times 10^{-13} d^{-1}$, that is, of $-1.7 \times 10^{-18} s^{-1}$, was also observed over a period of more than three years, also in spite of a frequency shift during cycling [44].

On shorter timescales, smaller drifts have however been reported [21][45][46]. For instance, a mean fractional frequency drift rate of $-0.04 \times 10^{-18} s^{-1}$ was measured over a 190-day-long period [47]. But since it has been argued that CSO drifts could be due to an aging of the resonators, associated to the relaxation of the mechanical stress induced during their assembly [48][49], such phenomenons could as well help canceling out their actual frequency drift with respect to atomic clocks. Of course, like for atomic clocks, the tick rate of CSOs could prove sensitive to variations of the gravitational field. To my knowledge, in spite of their high stability, effects of the gravitational field on the frequency of CSOs remain however to be exhibited.

Earth–Moon distance anomaly

Since durations measured by atomic clocks are expected to increase (eqn 2), d_{ac}, the distance measured using the time taken by light to go from a source to an observer, is also expected to increase so that:

$$d_{ac} = d_0 + c_0 \Delta \dot{t}_{ac} (t_u - t_0)$$

where d_0 is the distance measured at time t_0, c_0, the speed of light, the apparent drift of this distance being, according to eqn 2:

$$\frac{d_{ac} - d_0}{t_u - t_0} = \frac{d_0}{t_0} \tag{4}$$

In the case of the Earth–Moon distance, with the value of t_0 determined above, an apparent drift of 2.0 $\pm$ 0.2 cm yr^{-1} is thus expected.

An apparent increase of the Earth–Moon distance has indeed been observed. It is however of 3.82 $\pm$ 0.07 cm yr^{-1} [50]. In the context of the present study, this means that the actual increase of the Earth–Moon distance is only of 1.8 $\pm$ 0.2 cm yr^{-1}. Interestingly, with the later value, the present rate of tidal dissipation in the Earth–Moon system does not look as anomalous as it does when atomic clocks are not assumed to drift [26][51].

As a consequence of such an increase, and of momentum conservation, the length of the day is also expected to increase, namely, of 1.1 $\pm$ 0.1 msec cy^{-1}, instead of 2.3 $\pm$ 0.1 msec cy^{-1}, when it is assumed that atomic clocks are not drifting [52]. Unfortunately, fluctuations of the length of the day of several milliseconds have been observed over the last centuries, likely to be due to mantle-core interactions [53][54]. As a matter of fact, the analysis of an extensive compilation of ancient eclipses yields an intermediary value of 1.78 $\pm$ 0.03 msec cy^{-1} over the last 2500 years [55].

On the other hand, as a consequence of an increase of the Earth–Moon distance, d_M, a decrease of η_m, the angular velocity of the Moon, is expected, since

$$\eta_m = \sqrt{\frac{GM_E}{d_M^3}}$$

where M_E is the mass of the Earth, G, the gravitational constant, and where it has been assumed that the orbit of the Moon is circular. So, if the actual increase of the Earth–Moon distance is a linear function of time, then $\dot{\eta}_m$, the angular acceleration of the Moon, is so that:

$$\dot{\eta}_m = -\frac{3}{2}\frac{\dot{d}_M}{d_M}\eta_m \tag{5}$$

Thus, in the context of the present study, an actual angular acceleration of the Moon of -11 $\pm$ 1 $''$/cy^2 is expected. However, as the consequence of the drift of atomic clocks, an additional, apparent acceleration of the Moon of -9 $\pm$ 1 $''$/cy^2 should also be observed (eqn 2), the total acceleration of the Moon being of $\dot{\eta}_m$ = -20 $\pm$ 2 $''$/cy^2.

On the other hand, if it is assumed that the drift of atomic clocks does not contribute to the measured increase of the Earth–Moon distance [50], according to eqn 5, the acceleration of the Moon is instead expected to be of -23.7 $\pm$ 0.1 $''$/cy^2, a more detailled analysis of lunar laser ranging data providing $\dot{\eta}_m$ = -25.858 $\pm$ 0.003 $''$/cy^2 [56].

As noticed previously [56][57], this later value is in perfect agreement with the acceleration measured through the analysis of timings of transits of Mercury across the Sun between 1677 and 1973, namely, -26 $\pm$ 2 $''$/cy^2 [58]. However, other values obtained with the help of atomic clocks but before lunar laser ranging data became available do not look that consistent [59]. For instance, an analysis of 8249 lunar occultation timings between 1955 and 1980 gave -21.4 $\pm$ 2.8 $''$/cy^2 [24][60], in better agreement with the value predicted within the frame of the present study.

Pioneer anomaly

It has been noticed that an acceleration of the atomic clocks of $2.9 \pm 0.4 \times 10^{-18} s^{-1}$ could explain the so-called Pioneer anomaly [13][14][61][62]. This is significantly above the values mentioned above. But since it has been shown that the Pioneer anomaly could be due to the recoil force associated with an anisotropic emission of thermal radiation off the Pioneer spacecrafts [63][64], the present study suggests that this recoil force is responsible for only 40% of the observed effect.

Cosmological consequences

Let us assume that the energy of photons traveling in free space is conserved. Thus, if the frequency emitted by a given kind of atom increases as a function of time, the wavelength measured by a remote observer is expected to be redshifted, so that, as a consequence of eqn 1:

$$\frac{z}{1+z} = \frac{\Delta t_\gamma}{t_0} \tag{6}$$

where Δt_γ is the photon time-of-flight between its source and the observer, z being the redshift, defined as usual with respect to the wavelength known by the observer for this atom.

It has been shown that eqn 6 can account for a variety of cosmological observations [27][65][66], if it is assumed that $t_0 = t_H$, t_H being the Hubble time. Indeed, it is a straightforward consequence of the $R_h = ct$ cosmological model [67], which has been claimed to be favored by various model selection criteria [68][69], when compared to ΛCDM, that is, to the standard one. However, recent measurements of the local Hubble constant yield values of t_H [21][70] that are significantly smaller than the t_0 value obtained above. For instance, by studying cepheids in the hosts of 42 supernovae Ia the SH0ES team obtained t_H = 13.7 $\pm$ 0.2 Gyr (H_0 = 73.0 $\pm$ 1.0 km s^{-1} Mpc^{-1} [71]).

So, according to the present study, the drift of atomic spectra could contribute significantly to the local value of the Hubble constant, by as much as 75%. Interestingly, a value for the Hubble constant around 30 km s^{-1} Mpc^{-1} would cure all of the ills of a spatially flat Universe composed predominantly of cold dark matter [72]. It would also eliminate the cosmic age problem of ΛCDM [73][74][75][76], which has recently been exacerbated by the discovery of mature galaxies at a redshift around 14 [77][78][79][80]. The tension with the value of the Hubble constant obtained by analyzing the fluctuations of the cosmic microwave background [70][81] would however become more severe.

Discussion

The tick rate of a static atomic clock can increase only if the gravitational field felt by the clock increases.

The local gravitational field could for instance increase if the gravitational constant were decreasing [10][11][12]. Based on a variety of physical arguments, and high-accuracy measurements, a significant enough variation of the gravitational constant has however been excluded [82][83][84]. Though other speculative hypotheses can be considered [8], an increase of the local gravitational field likely means that the average mass density of the local Universe is decreasing. Such a decrease is, for instance, expected as a consequence of the expansion of space [13]. It could also be due to a loss of gravitational mass [15].

Conclusion

A drift of the tick rate of atomic clocks can explain the fact that there is a preferred value for the relative drift of the period of millisecond pulsars [22][37] (Fig. 1), while the corresponding drift of atomic spectra may contribute significantly to the local value of the Hubble constant.

In order to further support this hypothesis, it is necessary to study with a high accuracy physical phenomena that are not expected to vary in time. For instance, phenomena linked to inertia, like the angular rotation of objects in almost frictionless media, like in the case of millisecond pulsars, or the length of rods expected to remain rigid, like in the far from ideal case of the Earth–Moon distance.

As an example of a forthcoming opportunity, the Laser Interferometer Space Antenna will measure picometer–level fluctuations in the distance between drag–free proof masses over baselines of approximately $5\ 10^6$ km [85][86]. According to eqn. 4, an apparent drift of the length of its arms, of 9 ± 1 Å s^{-1}, should be observable.

Statements and Declarations

Acknowledgements

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Footnotes

[1] In version 2.6.3, as retrieved on August 2025, 7^{th}, on https://www.atnf.csiro.au/research/pulsar/psrcat

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