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Cosmology in five dimensions. Is our universe the surface of a four-dimensional hypersphere? Application to dark matter and dark energy.

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Summary:

Two of the main problems of cosmology today are dark matter and dark energy. While dark energy was introduced to try to explain the accelerating expansion of the universe, dark matter was postulated as a solution to the high rotation speeds of stars around galaxies, among other observed experimental results.

In this paper, we will assume that our universe is contained on the surface of a 4D hypersphere, we will propose its metric so that its radius *R* be dependent on cosmological time *t*. In this way, a 4D metric similar to the FLRW will be obtained in which the role of its scale factor will be played by the radius *R(t)* of the hypersphere, that is, its size.

With the new metric obtained, when applying the equations of Einstein's General Relativity Theory (GRT) new Friedman equations will be obtained but changing *a(t)* for *R(t).* In this way, and assuming the cases of a universe dominated by matter and energy, it will be obtained that both universes will be in decelerated expansion, a deceleration that is due to the gravitational force of the initial mass and energy of the universe.

The novelty of this paper and of the use of the new metric is the proposal that this gravitational force is transmitted to our 3D universe so that it is possible, in the case of a universe dominated by matter, to explain the rotation curves of galaxies and the Tully-Fisher relationship without having to resort to dark matter.

Finally, in this paper, we will propose that the expansion rate of the universe will generate a relativistic effect of time dilation that, without resorting to dark energy, will be able to explain the experimental results that have so far been interpreted as an accelerated expansion when in fact they correspond to a decelerated expansion of the universe.

Keywords: FLRW metric, Friedman's equations, dark matter, dark energy, Tully-Fisher relationship, accelerated expansion of the universe.

1. Introduction

Current cosmology is based on the Friedman-Lemaître-Robertson-Walker metric, or *FLRW metric*. This four-dimensional metric (three spatial dimensions and one temporal dimension) is considered the most general metric possible that describes a space-time that conforms to the cosmological principle. In addition, thanks to its scale factor, *a(t),* it allows us to explain the cosmological redshift by means of an expansion of the universe parameterized by this scale factor. The most common form of such an FLRW metric is:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a(t)^2\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2\right) \tag{1}$$

The time coordinate *t* is known as *cosmological time* and is the time measured by an observer whose peculiar motion is negligible, i.e., whose only motion is due to the expansion or contraction of homogeneous and isotropic space-time. These observers who share the same cosmological time

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are sometimes called *fundamental observers*. In an expanding universe like ours, fundamental observers would all move with the Hubble flow.

The spatial coordinates (in this case r, θ, ϕ) assigned by a fundamental observer are known as comoving coordinates and remain constant over time for any point. k, on the other hand, is the *curvature parameter* that can be taken by one of the three discrete values 0, +1, or -1 that correspond, respectively, to flat, positively curved, or negatively curved three-dimensional hypersurfaces.

On the other hand, the FLRW metric, when applying Einstein's field equation of General Relativity, provides the well-known Friedman equations from which, and only by setting the curvature parameter k and the initial densities of matter and energy of the universe, they allow to obtain the equations that describe the dynamics of the Cosmos. In this way, it is possible to obtain all imaginable models of universes (open, closed, in accelerated or decelerated expansion, static, etc.). Therefore, it is considered that Friedman's equations constitute the starting point of cosmology since it can be considered that cosmology begins from the obtaining of these equations.

To deduce them, it is sufficient to apply Einstein's equation of General Relativity

$$G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4}T_{\alpha\beta} \tag{2}$$

Although, to simplify the terms related to the metric, we will use the form:

$$G^{\alpha}_{\beta} \equiv R^{\alpha}_{\beta} - \frac{1}{2}Rg^{\alpha}_{\beta} + \Lambda g^{\alpha}_{\beta} = \frac{8\pi G}{c^4}T^{\alpha}_{\beta} \tag{3}$$

Using the FLRW metric and assuming that the energy-momentum tensor of the universe is that of a perfect fluid given by:

$$T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta + pg_{\alpha\beta} \tag{4}$$

where, if we consider that in the absence of peculiar motions, the velocity quadrivector has components *(1, 0, 0, 0),* then we can write the energy-momentum tensor with a contravariant index and a covariant index of the form:

$$T^{\alpha}_{\beta} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \tag{5}$$

The above form of the energy-momentum tensor allows us to simplify calculations by also using the contravariant and covariant versions of Einstein's tensor, G.

In this way, the two Friedman equations are obtained, whose form are:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{6}$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \tag{7}$$

2. Proposal for a new metric of the universe

In our case, although the FLRW metric is considered to be the most generic possible, we will start from the metric of the 4-Sphere, whose form is:

$$\mathrm{d}s^2 = \mathrm{dR}^2 + R^2 d\chi^2 + R^2 \sin^2\chi\, \mathrm{d}\theta^2 + R^2 \sin^2\chi \sin^2\theta\, \mathrm{d}\phi^2 \tag{8}$$

If we now make the variable change:

$$r = \sin\chi \tag{9}$$

we will have:

$$\mathrm{d}r = \cos\chi\, \mathrm{d}\chi \tag{10}$$

and if we square the previous expression:

$$\mathrm{d}r^2 = \cos^2\chi\, \mathrm{d}\chi^2 = (1 - \sin^2\chi)\mathrm{d}\chi^2 \tag{11}$$

from where we can clear:

$$\mathrm{d}\chi^2 = \frac{\mathrm{d}r^2}{(1 - \sin^2\chi)} = \frac{\mathrm{d}r^2}{(1 - r^2)} \tag{12}$$

Then the original metric of the 4-Sphere would be of the form (note that *k* = 1):

$$\mathrm{d}s^2 = \mathrm{d}R^2 + R^2\left(\frac{\mathrm{d}r^2}{1 - r^2} + r^2\mathrm{d}\theta^2 + r^2 sin^2\theta \mathrm{d}\phi^2\right) \tag{13}$$

If we add the temporal component to the previous metric, using the trace (1,-1,-1,-1,-1), we would finally get:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}R^2 - R^2\left(\frac{dr^2}{1 - r^2} + r^2\mathrm{d}\theta^2 + r^2 sin^2\theta \mathrm{d}\phi^2\right) \tag{14}$$

On the other hand, if we consider that the coordinate *R* is a function of cosmological time *t*, *R=R(ct),* then:

$$\mathrm{d}R(ct) = \frac{\mathrm{d}R(ct)}{\mathrm{d}ct}\mathrm{d}ct = \dot{R}(ct)\mathrm{d}ct = c \cdot \dot{R}(ct)\mathrm{d}t \tag{15}$$

(in this document, the notation with one or two points will be used to indicate first and second derivatives with respect to *ct*).

Then the metric would be as follows:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - c^2\dot{R}^2(ct)\mathrm{d}t^2 - R^2(ct)\left(\frac{\mathrm{d}r^2}{1 - r^2} + r^2\mathrm{d}\theta^2 + r^2 sin^2\theta \mathrm{d}\phi^2\right) \tag{16}$$

And if we group all the *ct* terms:

$$\mathrm{d}s^2 = \left(1 - \dot{R}^2(ct)\right)c^2\mathrm{d}t^2 - R^2(ct)\left(\frac{\mathrm{d}r^2}{1 - r^2} + r^2\mathrm{d}\theta^2 + r^2 sin^2\theta \mathrm{d}\phi^2\right) \tag{17}$$

If we compare the above expression with that of the FLRW metric (1) we see that the spatial part can be considered equivalent if we make *a(t) = R(t)*. However, the temporal part of the metric is different since a dependency has been introduced on the speed of the coordinate *R.* This dependence will introduce a time dilation that, as we will see later, will provide the solution, without the need to resort to dark energy, to the Hubble diagrams that are currently interpreted as the universe being in accelerated expansion.

With the above metric, the energy-momentum tensor will have the form:

$$T^{\alpha}_{\beta} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \quad (18)$$

If we now check if the new metric given by (17) meets Einstein's equations (3), we will obtain that for this to happen, it must be fulfilled that:

$$G^0_0 = \frac{3}{R^2(ct)\left(1 - \dot{R}^2(ct)\right)} = \frac{8\pi G}{c^4} T^0_0 = \frac{8\pi G}{c^2} \rho \quad (19)$$

and for the other diagonal components of the Einstein tensor we obtain that it must be fulfilled that:

$$G^1_1 = G^2_2 = G^3_3 = \frac{2R(ct)\ddot{R}(ct) - \dot{R}(ct)^2 + 1}{R(ct)^2\left(1 - \dot{R}(ct)^2\right)^2} = \frac{8\pi G}{c^4} T^1_1 = \frac{8\pi G}{c^4} T^2_2 = \frac{8\pi G}{c^4} T^3_3 = -\frac{8\pi G}{c^4} p \quad (20)$$

If now in (19) we clear the denominator, we get that:

$$\frac{1}{R(ct)^2\left(1 - \dot{R}(ct)^2\right)} = \frac{8\pi G}{3c^2} \rho \quad (21)$$

which we can replace in (20) to obtain that:

$$\frac{2R(ct)\ddot{R}(ct) - \dot{R}(ct)^2 + 1}{1 - \dot{R}(ct)^2} \cdot \frac{8\pi G}{3c^2} \rho = -\frac{8\pi G}{c^4} p \quad (22)$$

that is:

$$p = -\frac{2R(ct)\ddot{R}(ct) - \dot{R}(ct)^2 + 1}{1 - \dot{R}(ct)^2} \cdot \frac{1}{3} \rho c^2 \quad (23)$$

Therefore, we can consider that the equations (21) and (23) would be the new equations that would replace the Friedman equations obtained with the new metric (17).

3. Radiation-dominated universe

In this model of the universe the *cosmological constant*, *Λ,* is zero and the dominant energy density is that of radiation and relativistic matter. It is believed that a radiation-dominated universe can describe the early universe reasonably well.

If we assume that the energy density ρ_E has a dependence on *t* through *R(ct):*

$$\rho_E(ct)c^2 = \frac{E(ct)}{V(ct)} = \frac{E(ct)}{\frac{4\pi}{3}R^3(ct)} \tag{24}$$

now we must take into account that, when the radius *R(ct) increases,* there will be an increase in the wavelength, *λ*, of the radiation that will decrease its energy *E(ct)* in inverse proportion to *R(ct).* The problem is that we need to scale this reduction so that the dimensions of the previous expression are correctly maintained. To do this, we can, assuming that in this type of universe it will be limited by a maximum radius, *Rs*, make that:

$$E(ct) = \frac{R_S}{R(ct)}E_U \tag{25}$$

with E_U being the initial energy of the universe.

So, from (19), we have:

$$\frac{3}{R(ct)^2\left(1-\dot{R}^2(ct)\right)} = \frac{8\pi G}{c^4}\rho_E(ct) = \frac{8\pi G}{c^4}\cdot\frac{E(ct)}{\frac{4\pi}{3}R^3(ct)} = \frac{2GE_U}{c^4}\cdot\frac{R_S}{R^4(ct)} \tag{26}$$

and by reorganizing and simplifying we obtain:

$$\frac{1}{\left(1-\dot{R}^2(ct)\right)} = \frac{2GE_U}{c^4}\cdot\frac{R_S}{R^2(ct)} \tag{27}$$

then we have to clear:

$$R^2(ct) = \frac{2GE_U}{c^4}R_S\left(1-\dot{R}^2(ct)\right) \tag{28}$$

If we now define R_S as:

$$R_S = \frac{2GE_U}{c^4} \tag{29}$$

we will finally obtain that:

$$R(ct) = R_S\sqrt{1-\dot{R}^2(ct)} \tag{30}$$

And from expression (30) we can clear the velocity at the coordinate *R* to obtain:

$$\dot{R}(ct) = \sqrt{1-\frac{R^2(ct)}{R_S^2}} \tag{31}$$

from which it can be deduced that *R(t)* can never be higher than R_S as we had supposed.

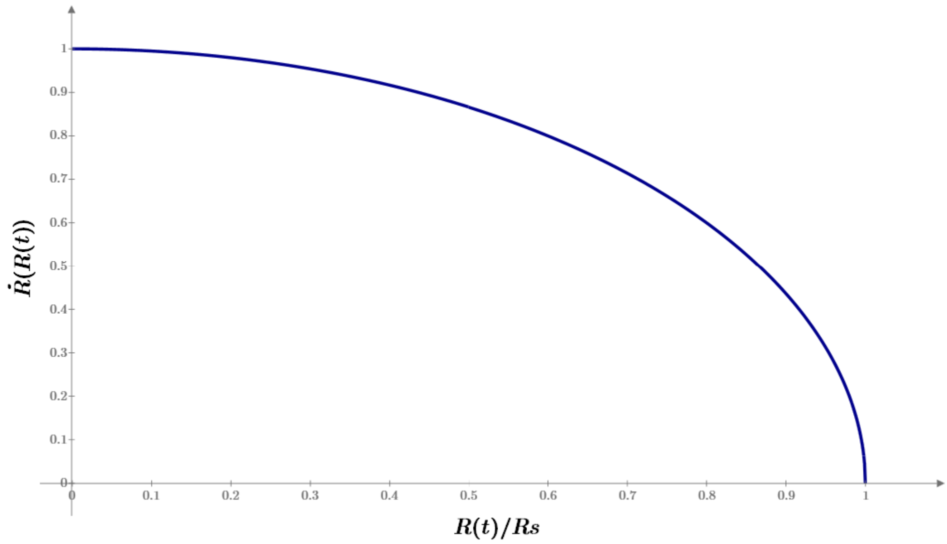

Figure 1: The rate of expansion of the universe as a function of its radius R.

If we now derive the expression (31) again we will obtain:

$$\ddot{R}(ct) = \frac{-2R(ct)\dot{R}(ct)}{R_S^2 \cdot 2\sqrt{1 - \frac{R^2(ct)}{R_S^2}}} = \frac{-R(ct)}{R_S^2} \tag{32}$$

that is, this universe dominated by energy is expanding but has a negative acceleration that is slowing it down, acceleration that is due to the gravitational force caused by the E_U energy of the universe.

If we now enter the values obtained in (31) and (32) in the equation (23) we will obtain that:

$$p = -\frac{\frac{-2R^2(ct)}{R_S^2} - \left(1 - \frac{R^2(ct)}{R_S^2}\right) + 1}{1 - \dot{R}^2(ct)} \cdot \frac{1}{3}\rho c^2 = \frac{R^2(ct)}{R_S^2\left(1 - \dot{R}^2(ct)\right)} \cdot \frac{1}{3}\rho c^2 \tag{33}$$

And if we take into account (30) then we finally obtain that:

$$p = \frac{1}{3}\rho c^2 \tag{34}$$

just as we expected in the case of radiant energy.

Finally, we can integrate the equation (31) performing the change of variable:

$$\frac{R(ct)}{R_S} = \sin x \tag{35}$$

to obtain that, assuming that the initial radius of the universe is zero:

$$R(ct) = R_S \sin\left(\frac{ct}{R_S}\right) \tag{36}$$

And deriving the previous expression with respect to *ct* we will have to:

$$\dot{R}(ct) = cos\left(\frac{ct}{R_S}\right) \tag{37}$$

And returning to derive a second time we obtain that the acceleration of *R* will be:

$$\ddot{R}(ct) = \frac{-1}{R_S} \sin\left(\frac{ct}{R_S}\right) \tag{38}$$

From the above expressions it can be deduced that the universe began with a size *R(0)* = 0 and an expansion speed equal to the speed of light, *c*, a speed that decreases until it stops in time:

$$ct = \frac{\pi}{2} \cdot R_S \tag{39}$$

at which point it will start a contraction process until it returns to its initial point of zero length.

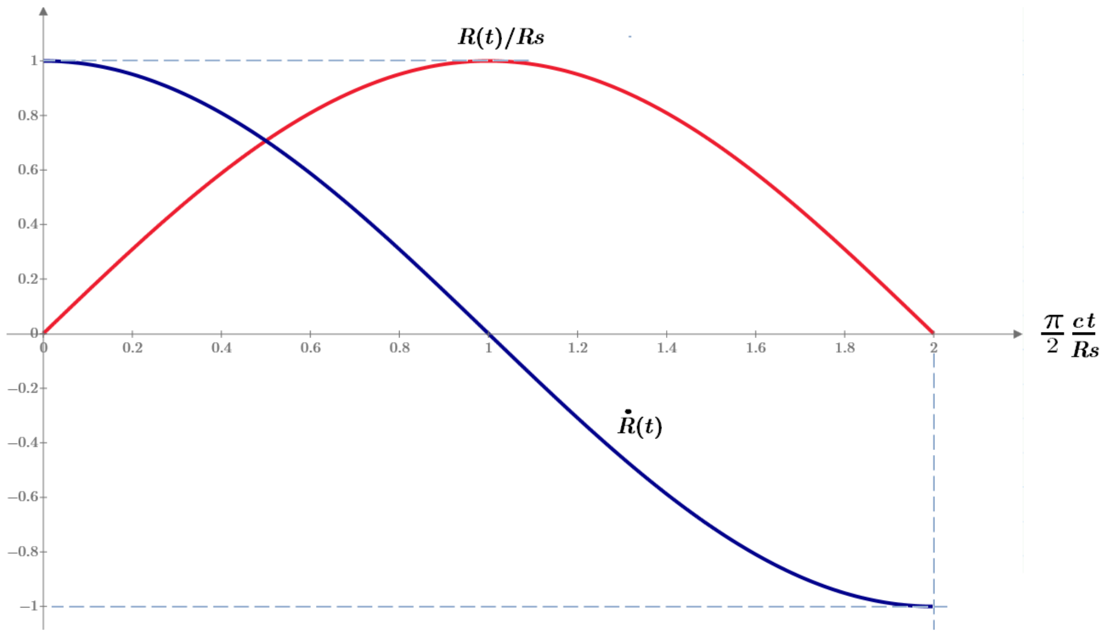

Figure 2: Expansion rate (blue) and size of the universe (red) as a function of cosmological time t.

If we now substitute the expression (31) in the metric (17) we will obtain that:

$$ds^2 = \frac{R^2(ct)}{R_S^2} c^2 dt^2 - R^2(ct)\left(\frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right) \tag{40}$$

But if we replace them again (37) in the previous expression we would obtain that:

$$ds^2 = \sin^2\left(\frac{ct}{R_S}\right) c^2 dt^2 - R_S^2 \sin^2\left(\frac{ct}{R_S}\right)\left(\frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right) \tag{41}$$

And if we define time τ as:

$$d\tau = \sin\left(\frac{ct}{R_S}\right) dt \tag{42}$$

we would have:

$$ds^2 = c^2 d\tau^2 - R^2(\tau)\left(\frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right) \tag{43}$$

which is again the expression of the FLRW metric.

Now, we can integrate both sides of the expression (42) to obtain that:

$$\tau = -\frac{R_S}{c}\cos\left(\frac{ct}{R_S}\right) + C \tag{44}$$

and assuming that $\tau = 0$ when $t = 0$, then we can calculate the value of C to have

$$\tau = \frac{R_S}{c}\left(1 - \cos\left(\frac{ct}{R_S}\right)\right) \tag{45}$$

from where we can clear t as a function of τ as:

$$t = \frac{R_S}{c} \cdot \arccos\left(1 - \frac{c\tau}{R_S}\right) \tag{46}$$

We can now calculate the values of velocity and acceleration based on τ substituting the previous expression in the value of (37):

$$\dot{R}(ct) = \frac{\mathrm{d}R}{\mathrm{d}ct} = \frac{\mathrm{d}R}{\mathrm{d}c\tau} \cdot \frac{\mathrm{d}c\tau}{\mathrm{d}ct} = \dot{R}(c\tau)\frac{\mathrm{d}\tau}{\mathrm{d}t} = \dot{R}(c\tau)\frac{R(ct)}{R_S} = \sqrt{1 - \frac{R^2(ct)}{R_S^2}} \tag{47}$$

from where we can clear:

$$\dot{R}(c\tau) = \sqrt{\frac{R_S^2}{R^2(c\tau)} - 1} \tag{48}$$

And if we derive again with respect to $c\tau$ we will obtain that:

$$\ddot{R}(c\tau) = -\frac{R_S^2}{R^3(c\tau)} \tag{49}$$

and if we substitute the value of R_S given by (29) we get:

$$\ddot{R}(\tau) = -\frac{\left(\frac{2GE_U}{c^4}\right)^2}{R^3(\tau)} \tag{50}$$

Finally, if we take into account (48) then we can clear:

$$\mathrm{c}\mathrm{d}\tau = \frac{\mathrm{d}R(c\tau)}{\sqrt{\frac{R_S^2}{R^2(c\tau)} - 1}} \tag{51}$$

that we can integrate to obtain that:

$$R(\tau) = \sqrt{2c\tau R_S - c^2\tau^2} \tag{52}$$

4. Universe dominated by non-relativistic matter

Since the radiation-dominated universe is expanding, and since the density of radiation decreases faster than the density of matter (it decreases with R^4 versus R^3 the second), there will come a time when the density of matter will exceed the density of radiation and it will be this non-relativistic or dust matter that dominates the universe. Therefore, the previous equations deduced in the paragraph 3 will be no longer valid.

If we assume, again, that the cosmological constant is zero and that the total mass of the universe is M_U, then the mass density $\rho_M(ct)$ will have a dependence on t through $R(ct)$ of the form:

$$\rho_M(t) = \frac{M_U}{V(t)} = \frac{M_U}{\frac{4\pi}{3}R(ct)^3} \tag{53}$$

Then substituting in (21) we will have:

$$\frac{3}{R^2(ct)\left(1 - \dot{R}^2(ct)\right)} = \frac{8\pi G}{c^2} \cdot \frac{M_U}{\frac{4\pi}{3}R^3(ct)} = \frac{2GM_U}{c^2} \cdot \frac{3}{R^3(ct)} \tag{54}$$

If we define R_S as:

$$R_S = \frac{2GM_U}{c^2} \tag{55}$$

(note the equality with the Schwarzschild radius) then we can simplify (54) as:

$$\frac{1}{1 - \dot{R}(ct)^2} = \frac{R_S}{R(ct)} \tag{56}$$

Reorganizing we finally obtain that:

$$R(ct) = R_S\left(1 - \dot{R}^2(ct)\right) \tag{57}$$

and from expression (57) we can clear the speed of R to obtain:

$$\dot{\mathrm{R}}(\mathrm{ct}) = \sqrt{1 - \frac{\mathrm{R(ct)}}{\mathrm{R_S}}} \tag{58}$$

from which it can be deduced that $R(ct)$ can never be greater than R_S.

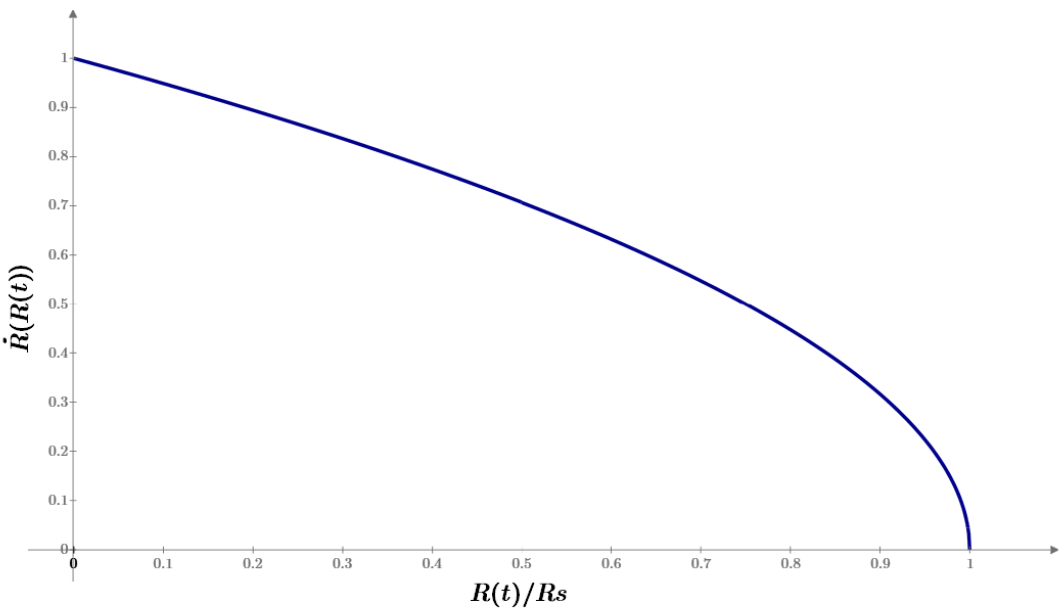

Figure 3: Dependence of the expansion rate of the universe on its radius R(t).

If we now derive the expression (58) again we get:

$$\ddot{R}(ct) = \frac{-\dot{R}(ct)}{R_S \cdot 2\sqrt{1 - \frac{R(ct)}{R_S}}} = \frac{-1}{2R_S} \tag{59}$$

That is, this universe is expanding but has a negative acceleration that is slowing it down, acceleration that is due to the gravitational force caused by the mass M_U of the universe.

If we now enter the values obtained in (58) and (59) in the equation (23) we will obtain that:

$$\mathrm{p} = -\frac{\frac{2R(ct)}{-2R_S} - \left(1 - \frac{R(ct)}{R_S}\right) + 1}{1 - \dot{R}^2(ct)} \cdot \frac{1}{3}\rho c^2 = -\frac{0}{1 - \dot{R}^2(ct)} \cdot \frac{1}{3}\rho c^2 = 0 \tag{60}$$

from which we have obtained that for a universe with mass M_U it must be fulfilled that:

$$p = 0 \tag{61}$$

as was necessary for a universe of matter.

Finally, we can integrate the equations (58) and (59) to obtain that:

$$R(ct) = ct - \frac{(ct)^2}{4R_S} \tag{62}$$

from which, in turn, we can clear the value of *t*:

$$t(R) = \frac{2R_S}{c}\left(1 - \sqrt{1 - \frac{R}{R_S}}\right) \tag{63}$$

On the other hand, deriving (62) we obtain the expression of the expansion rate as a function of cosmological time *t*:

$$\dot{R}(ct) = 1 - \frac{ct}{2R_S} \tag{64}$$

From the above expressions it can be deduced that the universe began with a size $R(0) = 0$ and an expansion velocity equal to the speed of light, *c,* a speed that slows down until it stops in time

$$t = \frac{2R_S}{c} \tag{65}$$

and will contract back to its starting point of null length in

$$R\left(\frac{4R_S}{c}\right) = 0 \tag{66}$$

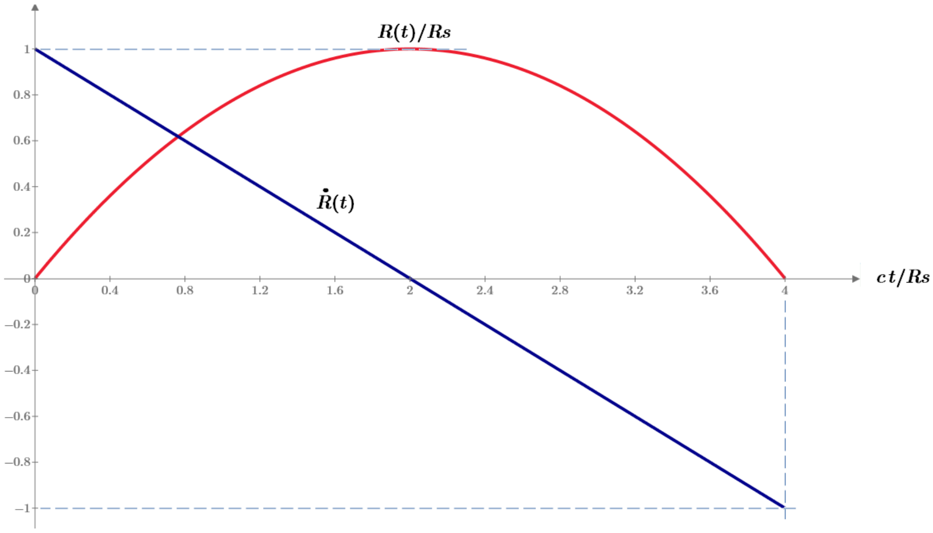

Figure 4: Expansion rate (blue) and size of the universe (red) as a function of cosmological time t.

Finally, if we replace (58) in the expression (17) of the metric, we would obtain that:

$$\mathrm{d}s^2 = \frac{R(ct)}{R_S} c^2 \mathrm{d}t^2 - R^2(ct)\left(\frac{\mathrm{d}r^2}{1-r^2} + r^2 \mathrm{d}\theta^2 + r^2 sin^2\theta \mathrm{d}\phi^2\right) \tag{67}$$

And if we define time τ as:

$$\mathrm{d}\tau = \sqrt{\frac{R(t)}{R_S}}\,\mathrm{d}t \tag{68}$$

we would have:

$$\mathrm{d}s^2 = c^2 \mathrm{d}\tau^2 - R^2(c\tau)\left(\frac{\mathrm{d}r^2}{1-r^2} + r^2 \mathrm{d}\theta^2 + r^2 sin^2\theta \mathrm{d}\phi^2\right) \tag{69}$$

which is again the expression of the FLRW metric.

If we use the value of *R(t)* given by (62) we can enter it in (68) to integrate this function:

$$\int \mathrm{d}\tau = \int \sqrt{\frac{R(ct)}{R_S}}\,\mathrm{d}t = \int \sqrt{\frac{ct - \frac{(ct)^2}{4R_S}}{R_S}}\,\mathrm{d}t = \frac{1}{2R_S}\int \sqrt{4R_S \cdot ct - (ct)^2}\,\mathrm{d}t \tag{70}$$

and obtain:

$$\tau = \frac{R_S}{c}\arcsin\left(\frac{ct - 2R_S}{2R_S}\right) + \frac{ct - 2R_S}{4R_S \cdot c}\sqrt{4R_S ct - c^2t^2} + C \tag{71}$$

And if we assume that $\tau = 0$ when $t = 0$ then we can calculate the value of C and have:

$$\tau = \frac{R_S}{c}\arcsin\left(\frac{ct-2R_S}{2R_S}\right) + \frac{ct-2R_S}{4R_S\cdot c}\sqrt{4R_Sct-c^2t^2} - \frac{3\pi}{2}\cdot\frac{R_S}{c} \tag{72}$$

Now we can calculate the values of the velocity and acceleration in R as a function of τ. Since:

$$\dot{R}(ct) = \frac{dR}{dct} = \frac{dR}{dc\tau}\frac{dc\tau}{dct} = \dot{R}(c\tau)\frac{d\tau}{dt} = \dot{R}(c\tau)\sqrt{\frac{R(ct)}{R_S}} = \sqrt{\frac{R_S - R(ct)}{R_S}} \tag{73}$$

then we can clear:

$$\frac{dR(c\tau)}{dc\tau} = \dot{R}(c\tau) = \sqrt{\frac{R_S - R(ct)}{R(ct)}} = \sqrt{\frac{R_S - R(c\tau)}{R(c\tau)}} \tag{74}$$

And if we derive again with respect to $c\tau$ we obtain that:

$$\ddot{R}(c\tau) = \frac{1}{2\sqrt{\frac{R_S - R(c\tau)}{R(c\tau)}}}\cdot\frac{-\dot{R}(c\tau)\big(R(c\tau) + R_S - R(c\tau)\big)}{R^2(c\tau)} = \frac{-R_S}{2R^2(c\tau)} = \frac{-R_S}{2R^2(c\tau)} \tag{75}$$

If we now substitute the value of R_S given by (55) we get:

$$\ddot{R}(\tau) = \frac{-c^2R_S}{2R^2(\tau)} = \frac{2GM_U}{c^2}\cdot\frac{-c^2}{2R^2(\tau)} = -\frac{GM_U}{R^2(\tau)} \tag{76}$$

which is nothing more than the **acceleration of gravity that would be obtained using Newton**.

Finally, if we take into account (74) then we can clear:

$$cd\tau = \sqrt{\frac{R(c\tau)}{R_S - R(c\tau)}}\cdot dR(c\tau) \tag{77}$$

that we can integrate to obtain:

$$\tau(R) = \frac{R_S}{c}\arctan\left(\sqrt{\frac{R}{R_S - R}}\right) - \frac{(R_S - R)}{c}\sqrt{\frac{R}{R_S - R}} \tag{78}$$

4.1. Proper distance and Hubble's law

If we start from the metric (69) with the proper time τ in which we assume that angular coordinates, θ and ϕ, don't change, then:

$$ds^2 = c^2d\tau^2 - R(\tau)^2d\chi^2 \tag{79}$$

If we define

$$d\sigma = R(\tau)d\chi \tag{80}$$

then for an observer with time τ who tries to determine the spatial distance $\sigma(\tau)$ along the coordinate χ (radial coordinate) we will have

$$\sigma(\tau) = \int_0^\chi R(\tau)d\chi = R(\tau)\chi \tag{81}$$

Now we can define the proper radial velocity, v_p, as:

$$v_p = \frac{d\sigma(\tau)}{d\tau} = \frac{dR(\tau)}{d\tau}\chi = \dot{R}(\tau)\chi \tag{82}$$

And if from (81) we clear χ:

$$\chi = \frac{\sigma(\tau)}{R(\tau)} \tag{83}$$

then we can substitute in (82) and obtain:

$$v_p = \frac{\dot{R}(\tau)}{R(\tau)}\chi = \mathrm{H}(\tau)\cdot\chi \tag{84}$$

where we arrive to the *law of velocity-distance*, with the *Hubble parameter*, *H(τ).*

Now we can use (74) to get that the Hubble parameter would be:

$$H(\tau) = c\cdot\sqrt{\frac{R_S - R(\tau)}{R(\tau)}}\cdot\frac{1}{R(\tau)} = c\cdot\sqrt{\frac{R_S - R(\tau)}{R^3(\tau)}} \tag{85}$$

which we can represent graphically in the area in which *H(τ)* takes the range of values among which it is considered to be at present:

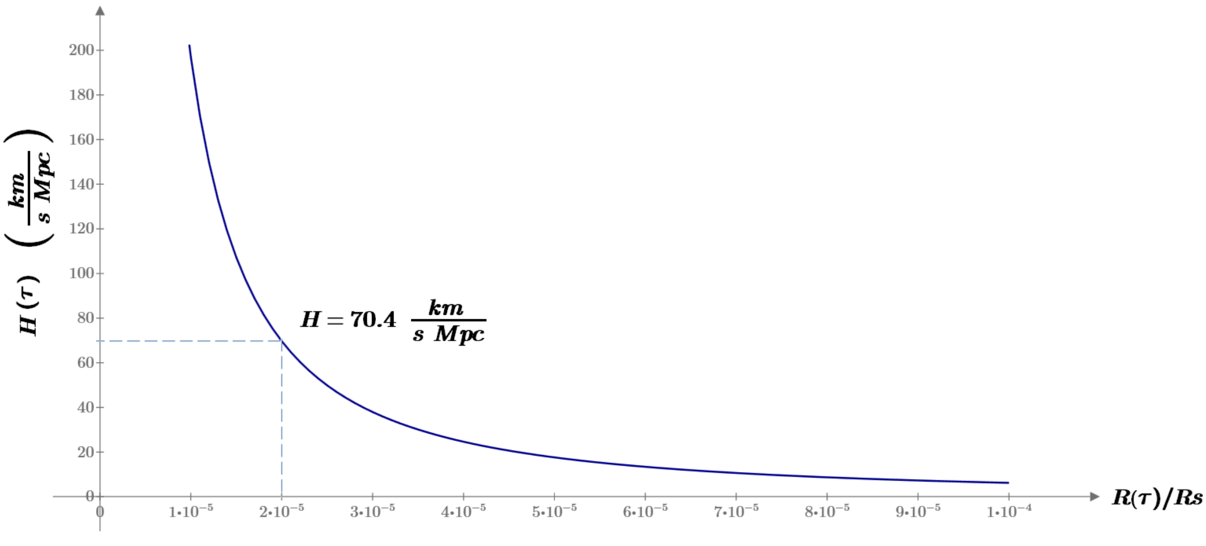

Figure 5: The value of the Hubble constant as a function of the size of the universe.

From the above expression we can operate a little and obtain:

$$R^3(\tau)\cdot\frac{H^2(\tau)}{c^2} + R(\tau) - R_S = 0 \tag{86}$$

which allows us to calculate the value of *R(τ)* as a function of *H(τ)* and R_S by solving the above equation of the third degree.

Let's put some numbers to the above amounts. Let's suppose that

$$\mathrm{M_U} = 1.51\cdot 10^{53}\ \mathrm{kg} \tag{87}$$

then we get that

$$R_S = 2.2411 \cdot 10^{26}\ \text{m} = 23.70 \cdot 10^{9}\ \text{light-years} \tag{88}$$

If we take the value

$$\mathrm{H}(\tau) = 70.4\ \frac{\text{km}}{\text{s}\cdot\text{Mpc}} \tag{89}$$

we can solve (86) to get that the current value of *R(τ)* is

$$R(\tau) = R(t) = 1.212 \cdot 10^{26}\ \text{m} = 12.81 \cdot 10^{9}\ \text{light-years} \tag{90}$$

Now we can use (63) to obtain the current coordinate time *t* as:

$$t = 15.27 \cdot 10^{9}\ \text{years} \tag{91}$$

and (78) to have the current proper time *τ* as:

$$\tau = 7.767 \cdot 10^{9}\ \text{years} \tag{92}$$

4.2. Dark matter and galactic rotation velocity curves

In the expression (76) we have obtained that, for a universe dominated by matter, there is a proper acceleration in the radial direction R of expansion that coincides with the acceleration of gravity generated by the mass M_U of the universe, an acceleration that from now on we will call ***cosmological acceleration*** $g_C(\tau)$:

$$g_C(\tau) \equiv \ddot{R}(\tau) = -\frac{GM_U}{R^2(\tau)} \tag{76}$$

The fact that cosmological acceleration occurs in R, which is one more spatial dimension, which would not be the case with the FLRW metric and its scale factor *a(t),* allows us to suppose or propose that part of this acceleration could be transmitted or affect other dimensions. Let's see.

For simplicity and by analogy, let's think that our universe is included on the surface of a 2D sphere immersed in 3D space. We can then assume that on this surface there will be deformations of the surface in areas where there are massive bodies, such as stars or galaxies. We can, to further simplify the reasoning, assume that such deformations of space-time in our universe conform to Schwarzschild's metric, so that if we restrict them again to a 2D universe, such local deformations on the surface of our universe would be shaped like Flamm's paraboloid.

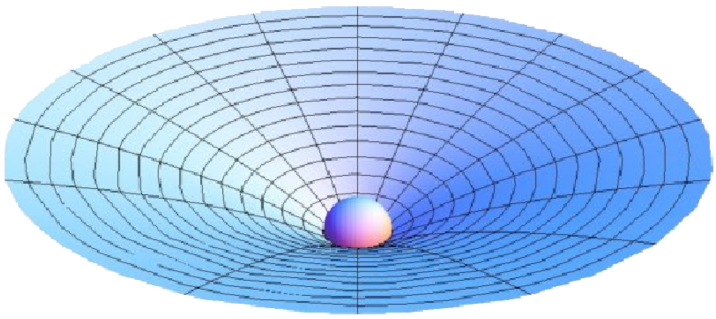

Figure 6: Flamm's paraboloid in a 2D universe.

If we assume the above as correct, then we can think that part of the cosmological acceleration $g_C(\tau)$ can affect the radial direction of r. To do this, it is enough to multiply $g_C(\tau)$ by $\sin(\theta)$ being θ the angle formed by the surface of the paraboloid with the surface of the hypersphere, as shown in the figure below.

On the other hand, we know that the slope at each point r of the Flamm paraboloid has the expression:

$$\tan(\theta) = \frac{dR(\tau)}{dr} = \frac{1}{\sqrt{\frac{r}{R_s} - 1}} = \sqrt{\frac{R_s}{r - R_s}} \tag{93}$$

where R_S is now the Schwarzschild radius generated by the mass M located at $r = 0$:

$$R_s = \frac{2GM}{c^2} \tag{94}$$

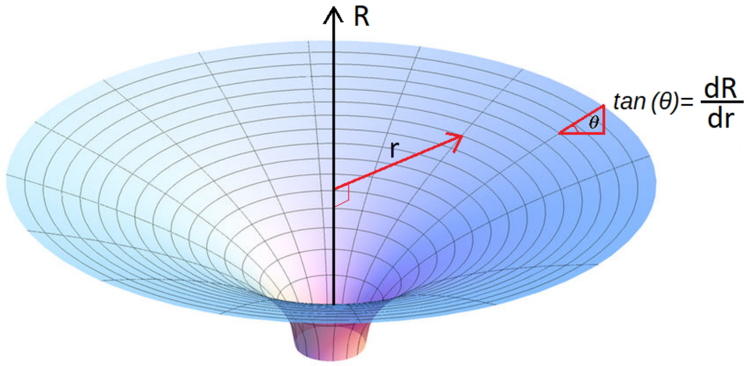

Figure 7: Diagram with the tan(θ) in a Flamm paraboloid.

For r much bigger than R_S (let's think that the mass M necessary to obtain a R_S of 1 *Kpc* would be of $M > 1 \cdot 10^{16}$ solar masses) the next approximation can be used:

$$\sin(\theta) \simeq \tan(\theta) \tag{95}$$

Therefore, we can now assume that the acceleration in the direction of r, $g_r(\tau)$, will be:

$$g_r(\tau) = \sin(\theta)\, g_C(\tau) \simeq \frac{\mathrm{d}R(\tau)}{\mathrm{d}r} g_C(\tau) = \sqrt{\frac{R_s}{r - R_s}} \cdot \frac{-GM_U}{R^2(\tau)} \tag{96}$$

an expression that for r much greater than R_S can be approximated to:

$$g_r(\tau) \simeq \sqrt{\frac{R_s}{r}} \cdot \frac{-GM_u}{R^2(\tau)} \tag{97}$$

If we now apply the equality of centripetal acceleration for an object of mass m rotating in a circular orbit at a distance r from the mass M, we will have to:

$$m\frac{v^2(r)}{r} = m \cdot g_N(r) + m \cdot g_r(\tau) = \frac{GMm}{r^2} + m\sqrt{\frac{R_s}{r}} \cdot \frac{GM_U}{R^2(\tau)} \tag{98}$$

where $g_N(r)$ is the acceleration of gravity according to Newton.

If now, for very large r, we ignore the part due to Newton's acceleration of gravity since it has a dependence on $1/r^2$ compared to the dependence $1/r^{1/2}$ of the second summation, we will obtain what we can define as *cosmological velocity*, $v_C(r)$:

$$v_C^2(r) = \sqrt{\frac{R_s}{r}} \cdot \frac{GM_U}{R^2(\tau)} r = \frac{GM_U}{R^2(\tau)}\sqrt{R_s r} = g_C(\tau)\sqrt{R_s r} \tag{99}$$

And if we take the square root to obtain $v_C(r)$:

$$v_C(r) = \sqrt{g_C(\tau)} \cdot \sqrt[4]{R_s r} \tag{100}$$

If, finally, we substitute the value of R_S we get:

$$v_C(r) = \sqrt{g_C(\tau)} \cdot \sqrt[4]{r\frac{2GM}{c^2}} = \sqrt{\frac{GM_U}{R^2(\tau)}} \cdot \sqrt[4]{r\frac{2GM}{c^2}} \tag{101}$$

from where we can also clear the *M* mass of the galaxy:

$$M = v_C^4 \cdot \frac{c^2R^4(\tau)}{2G^3M_U^2r} \tag{102}$$

that is, *M* is proportional to:

$$M \propto \frac{v^4}{r} \propto v^4 \tag{103}$$

what is the Tully-Fisher relationship [4].

Before to check if the expression (101) adjusts to the rotation speeds of galaxies, we only have to make one more assumption to improve this adjustment. To do this, we must take into account that in the equation (101) it has been assumed that the whole mass *M* of the galaxy is concentrated at its center, which makes the profile of *v(r)* is too pronounced for small values of *r*. We can correct this if we assume an exponential distribution of masses in the galaxy with the form:

$$M(r) = M_o\left(1 - e^{-\left(\frac{r}{r_o}\right)^2}\right) \tag{104}$$

This is a distribution that holds that 63% of the total mass of the galaxy, *Mo*, is at a r_o distance and that 90% of the mass is at 1.5 r_o. In the following figure you can see a graphical representation of this distribution for r_o = 4 and M_o = 1.

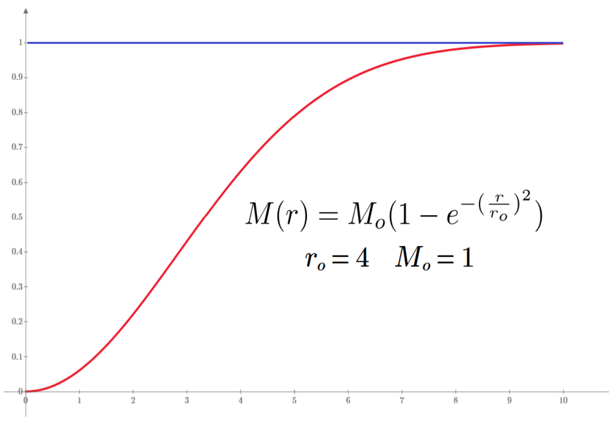

Figure 8: Value of the exponential mass distribution.

Now we can introduce *M(r)* in (101) to finally have:

$$v_C(r) = \sqrt{\frac{GM_U}{R^2(\tau)}} \cdot \sqrt[4]{\mathrm{r}\frac{2GM_o\left(1 - e^{-\left(\frac{r}{r_o}\right)^2}\right)}{c^2}} \tag{105}$$

Obviously, the exponential distribution function used is a proposal, so it could be replaced by any other. Similarly, the calculations made in other papers usually include the gravitational effects caused by radiation and galactic gas, but in this study we will assume that these effects are already included in the mass distribution of (104).

Finally, we must calculate the rotational speed according to Newton generated by the distribution (104) to obtain (for this we can use Gauss's theorem) that:

$$v_N(r) = \sqrt{\frac{GM_o\left(1 - e^{-\left(\frac{r}{r_o}\right)^2}\right)}{r}} \tag{106}$$

Now, to calculate the total velocity, $v_T(r)$, of a star inside a galaxy as a function of its distance r, we will have to be:

$$v_T(r) = \sqrt{v_N(r)^2 + v_C(r)^2} = \sqrt{\frac{GM_o\left(1 - e^{-\left(\frac{r}{r_o}\right)^2}\right)}{r} + \frac{GM_U}{R^2(\tau)}\sqrt{r\frac{2GM_o\left(1 - e^{-\left(\frac{r}{r_o}\right)^2}\right)}{c^2}}^{2}} \tag{107}$$

where $v_N(r)$ is the velocity according to Newton and $v_C(r)$ is the new term due to cosmological velocity.

Now, once the expression is obtained (107) we can already check whether this expression allows us to obtain the rotation speeds of the observed galaxies. To do this, data have been taken from eight galaxies with different sizes, masses and rotation speeds, and an attempt has been made to adjust the different values used in (107) until the rotational speed profiles measured experimentally can be obtained.

Of the values that need to be adjusted for use in (107) there are those referring to the entire universe, that is, its total mass, M_U, and its current radius, $R(\tau)$ which, in turn, will be determined by the value of the Hubbel parameter through the expression (86). These values, obviously, once established, must be the same for all galaxies, having to be fixed in a particular way the value of the mass of the galaxy, M_o, and the value of the radius r_o.

A first attempt with values $M_U = 1.53 \cdot 10^{53}$ kg and $R(\tau) = 1.212 \cdot 10^{26}$ m obtained in (90), provide values of $v_C(r)$ despicable. After several attempts at manual adjustment, it was found that the M_U value that achieved the best adjustments for all the selected galaxies was

$$M_U = 1.0 \cdot 10^{60}\ \text{kg} \tag{108}$$

which would imply that the maximum size of the universe would be

$$R_S = 1.4842 \cdot 10^{33}\ \text{m} = 1.70 \cdot 10^{17}\ \text{light-years} \tag{109}$$

that allows, resolving (86), obtain that the current size of the universe is

$$R(\tau) = 2.949 \cdot 10^{28}\ \text{m} = 3.119 \cdot 10^{12}\ \text{light-years} \tag{110}$$

Now we can use (63) to obtain coordinate or cosmological current time t as:

$$t = 3.119 \cdot 10^{12}\ \text{years} \tag{111}$$

and (78) to have the current proper time τ as:

$$\tau = 9.267 \cdot 10^{9}\ \text{years} \tag{112}$$

Finally, and to conclude this section, the following figures show the rotation velocity curves for eight galaxies. The black dots, together with their error bars, are the values experimentally measured and obtained from [1]. The blue dashed lines correspond to Newton's velocity values, $v_N(r)$, and the strokes and dots in black are the values of cosmological velocity $v_C(r)$. The red line is the total value,

$v_T(r)$, sum of the above according to the equation (107). On the other hand, the values of the masses of galaxies, M_o, are given at 10^9 $M_\odot$ (solar masses) and radial distances in kpc.

The conclusion that emerges from the following figures and from the adjustments obtained is that the expression (107) approximates quite well the rotation curves obtained experimentally. Therefore, we can conclude that the cosmological acceleration given by (76) provides an additional velocity term, the cosmological velocity, **which makes it possible to explain the rotation velocity curves of galaxies without the need to resort to the introduction of dark matter**.

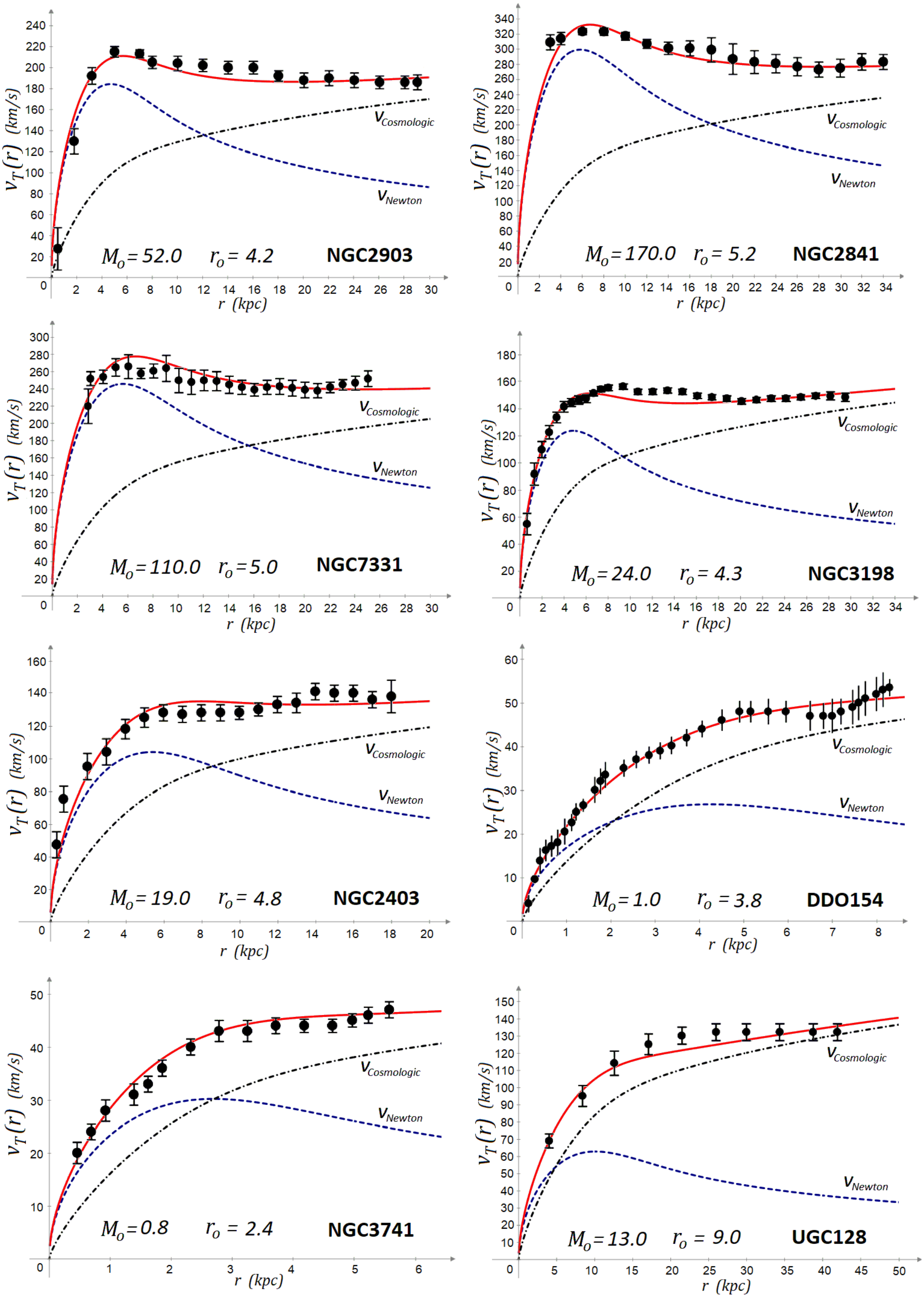

Figure 9: Rotation curves for different galaxies including cosmological velocity.

4.3. Dark energy and the accelerated expansion of the universe.

In the previous section, it has been proposed that cosmological velocity (101) due to the negative acceleration of the universe given by (76), the cosmological acceleration, can explain the rotation velocity curves of galaxies without having to resort to dark matter. Therefore, now it is necessary to give a satisfactory explanation compatible with the above to the results and measurements that seem to indicate that the universe, contrary to what is obtained in this text, is not expanding accelerately but is doing so decelerately, that is, more and more slowly.

The figure below shows the Hubble diagram obtained by the Supernova Cosmology Project in 2003 [2] showing the effective value of M_B based on redshift *z,* where we can assume that it is defined as:

$$m_B^{effective} = 5 \cdot log\big(D_L(z, \Omega_M, \Omega_{Lambda})\big) + 25 \tag{113}$$

with D_L being the distance of light, which can be considered to have the expression:

$$D_L(z) = rR_o(1+z) \tag{114}$$

where *r* is the comoving radial coordinate.

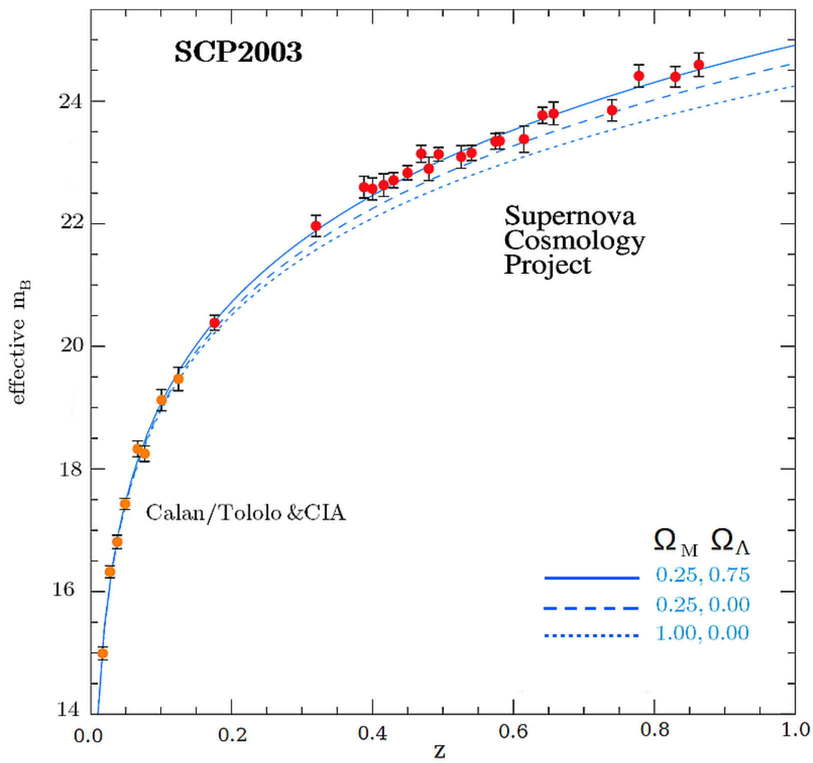

Figure 10: Hubble diagram obtained by the Supernova Cosmology Project in 2003.

The value of D_L indicated by (114) has been obtained by assuming that:

$$\frac{\nu}{\nu_o} = \frac{dt_o}{dt} = \frac{\lambda_o}{\lambda} = \frac{R_o}{R} = (1+z) \tag{115}$$

where *ν* and *λ* are the frequency and wavelength of the observed radiation, and where the subscript 'o' indicates the current value. The above equalities can be obtained in different ways through the FLRW metric.

At this point, it is time to recover the expression of the metric obtained in (67):

$$\mathrm{d}s^2 = \frac{R(ct)}{R_S} c^2 \mathrm{d}t^2 - R^2(ct)\left(\frac{\mathrm{d}r^2}{1-r^2} + r^2\mathrm{d}\theta^2 + r^2 sin^2\theta \mathrm{d}\phi^2\right) \tag{67}$$

and the definition of *dτ* given by (68):

$$\mathrm{d}\tau = \sqrt{\frac{R(t)}{R_S}} \cdot \mathrm{dt} \tag{68}$$

According to the above definition, it is obtained that the $d\tau_1$ *and* $d\tau_2$ for two observers located at a different R_1 and R_2 will comply with the relationship:

$$\mathrm{d}\tau_1 \cdot \sqrt{\frac{R_S}{R_1}} = \mathrm{d}t = \mathrm{d}\tau_2 \cdot \sqrt{\frac{R_S}{R_2}} \tag{116}$$

that is:

$$\mathrm{d}\tau_1 = \sqrt{\frac{R_1}{R_2}} \cdot \mathrm{d}\tau_2 \tag{117}$$

an expression that relates the proper times of both observers through the value of the radio of the universe *R*, and according to which, the proper clock of an observer located in R_1 will run at a different speed than the proper clock of another observer located in R_2.

The above expression could also have been obtained if we assume that the clocks of each observer are affected by the time dilation of Special Relativity, but assuming that the velocity of each observer is its *absolute velocity* which would be given by (58). That is, we must make use of equation (30) obtained using the generic transformations (or Tangherlini transformations) obtained in [3]:

$$\frac{d\tau_1}{d\tau_2} = \frac{\gamma_2}{\gamma_1} = \sqrt{\frac{c^2 - v_1^2}{c^2 - v_2^2}} = \sqrt{\frac{c^2 - \dot{R}_1^2}{c^2 - \dot{R}_2^2}} \tag{118}$$

in which we assume that the absolute velocity *v(t)* that generates temporal dilation is the velocity of the radial component *R* given by (58):

$$v(t) = \dot{R}(t) = c\sqrt{1 - \frac{R(t)}{R_S}} \tag{58}$$

so if we substitute the previous value in (118) we get the relationship (117) again.

On the other hand, if in the expression (117) we take into account the relationship (115) we obtain that:

$$\frac{d\tau_1}{d\tau_2} = \sqrt{\frac{R_1}{R_2}} = \sqrt{1+z} \tag{119}$$

Finally, if we consider that the anterior temporal dilation is increased or scaled by the metric FLRW in the proportion R_1/R_2 through the relationship(115), then we get that:

$$\frac{d\tau_1}{d\tau_2} = \sqrt{\frac{R_1}{R_2} \cdot \frac{R_1}{R_2}} = \sqrt{1+z} \cdot (1+z) = (1+z)^{3/2} \qquad (120)$$

Now, using the above ratio, it can be deduced that the distance of light given by (114) will be modified by taking into account the temporal dilation, obtaining that:

$$D_{Ld}(z) = rR_o(1+z)^{3/2} = D_L(z)\sqrt{1+z} \qquad (121)$$

being $D_L(z)$ the distance of light obtained in (114) without taking into account the time dilation.

Therefore, if we now use the previous expression in (113) we will obtain that the effective value of $m_{Bd}^{effective}(z)$ if temporal dilation is taken into account, it will be:

$$m_{Bd}^{effective}(z) = 5 \cdot log\big(D_{Ld}(z)\big) + 25 = 5 \cdot log\big(D_L(z)\sqrt{1+z}\big) + 25 \qquad (122)$$

and if we continue to operate, we get that:

$$m_{Bd}^{effective}(z) = 5 \cdot log\big(D_L(z)\big) + \frac{5}{2} \cdot log(1+z) + 25 \qquad (123)$$

Therefore, the effective value of $m_{Bd}^{effective}(z)$ will be:

$$m_{Bd}^{effective}(z) = m_B^{effective}(z) + \frac{5}{2} \cdot log(1+z) \qquad (124)$$

If we now represent graphically (red line) the previous expression taking as $m_B^{effective}(z)$ the theoretical values for a universe composed only of cold matter (i.e. Ω_M = 1.0 and Ω_Λ = 0.0) and superimpose it on the Hubble diagram obtained by the Supernova Cosmology Project, we will see that the values of $m_{Bd}^{effective}(z)$ obtained with the expression (124) coincides perfectly with those obtained for a universe with dark matter and dark energy with densities Ω_M = 0.25 and Ω_Λ = 0.75 respectively.

That is, when considering the temporal dilation given by (119) and scale it across the radius of the universe *R* through the metric FLRW to achieve (120), we have obtained (124) which, applied to a universe dominated solely by matter, manages to explain the Hubble diagram obtained by the Supernova Cosmology Project without the need to resort to or introduce the concept of dark energy. Therefore, we can say that **the universe is in decelerated expansion.**

To conclude this section, it should be said that according to the expression (58) and the values of $R(t)$ and R_S obtained in the previous section, the current expansion rate of the universe is

$$\dot{R}(ct) = \mathrm{c}\sqrt{1 - \frac{R(t)}{R_S}} = \mathrm{c}\sqrt{1 - \frac{9.267 \cdot 10^9}{1.1842 \cdot 10^{33}}} = 0.99999 \cdot c \qquad (125)$$

and, therefore, according to (68) we have that our time dilation factor is:

$$\mathrm{d}\tau = \sqrt{\frac{R(t)}{R_S}}\mathrm{d}t = 0.004457 \cdot \mathrm{d}t \qquad (126)$$

that is, our proper clock goes 224.4 times slower than the coordinate or cosmological time clock, which would correspond to the clock of a stationary observer located in R_S.

On the other hand, it should be noted that for a clock located at a distance such that its z = 1, its time will be slower than ours by one factor:

$$\frac{d\tau_1}{d\tau_2} = \sqrt{\frac{R_1}{R_2}} = \sqrt{1+z} = \sqrt{2} \tag{127}$$

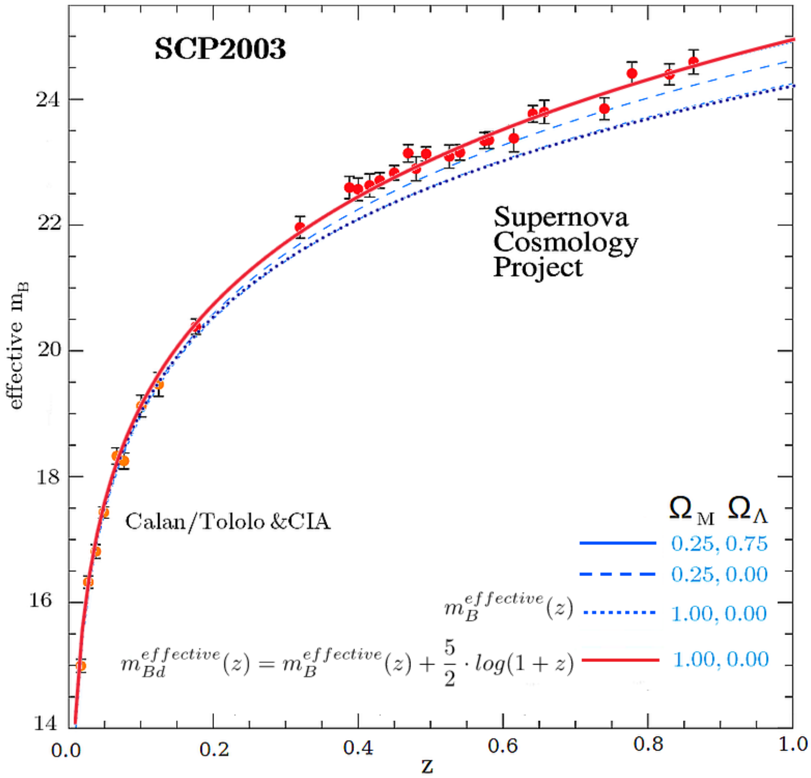

Figure 11: In red, the $m_{Bd}^{effective}(z)$ values superimposed on the Hubble diagram of the Supernova Cosmology Project.

5. Universe dominated by dark energy

At this point in the paper, it would be normal to end this study by analyzing a universe dominated solely by dark energy. But since we have shown in the section 4.3 that it is not necessary to introduce it to explain the Hubble diagrams of supernovae, we believe that we can affirm the non-existence of such dark energy and, therefore, we will ignore this study and end here the analysis of the different models of universes.

6. Conclusions

This paper has been based on the assumption that the universe is contained on the surface of a 4D hypersphere. With this assumption, a metric has been proposed so that its radius, *R(t)*, is dependent on cosmological time. In this way, a 4D metric similar to FLRW has been obtained in which the role of its scale factor is played by the radius of the hypersphere, that is, its size. This metric, given by (17), retains the same shape in its spatial part, but introduces a temporal dilation due to the expansion velocity of the coordinate *R(t)*.

Einstein's equations have been applied to the new proposed metric in such a way that new equivalent Friedman's equations, (20) and (21), have been obtained. In this way, and assuming the case of a universe dominated by energy and matter, it is obtained that in both cases, universe is in decelerated expansion due to the gravitational force of the energy and mass of the universe.

Although the fact that the universe is in decelerated expansion due to gravity is also a result that can be obtained directly from the FLRW metric and Friedman's equations, the novelty of this paper and the use of the new metric is that this gravitational force is transmitted to our universe in such a way that it is possible to explain the rotation curves of galaxies without the need to resort to dark matter. In addition, the expressions obtained also manage to explain the Tully-Fisher relationship ($M = kv^4$), a fact that the introduction of dark matter could not do.

Finally, the new proposed metric also manages to explain the Hubble diagrams obtained by the Supernova Cosmology Project without having to introduce the concept of dark energy as responsible for the acceleration of the expansion of the universe, since what current cosmology interprets as an acceleration, is nothing more than an effect of relativistic time dilation generated by the expansion velocity in *R.* Therefore, not only is the expansion of the universe not accelerating, but it is being stopped due to the gravitational "pull" produced by the mass of the universe.

In short, if the statements and deductions in this document are correct, we can say that our universe is contained on the surface of a 4D hypersphere. Its size would be expanding at speeds close to the speed of light, an expansion that is being stopped by the gravity of the mass of the universe and that will end up stopping when it reaches the size R_S given by (55) at which point it will contract again until it becomes the singularity from which it started. In addition, we can say that it is the effects caused by the decelerating expansion of the universe that allow us to explain the effects that we now attribute to dark matter and dark energy.

Therefore, we can assume that we live inside a five-dimensional black hole and that, currently, we are approaching its Schwarzschild radius at 99.999% of the speed of light.

7. Discussions

After the statements made in this paper, it would be time to apply the effects of cosmological gravity to clusters of galaxies to see if their dynamics can also be explained without the need to resort to dark matter. In the same way, it would be necessary to study what other implications in other ages and phases of the universe would have the replacement of the effects attributed to dark energy and dark matter by time dilation and cosmological gravity.

Finally, it should be discussed whether, if what is stated here is true, it can be considered a total success for Einstein's General Relativity and, therefore, all those rival gravitational theories that seek to replace it by trying to explain the effects of dark matter and dark energy but without the need for their use could be discarded.

Furthermore, it should also be discussed whether the fact that the relativistic time dilation produced by the expansion of the universe in its dimension *R*, time dilation that would depend on its absolute velocity (58) through the Lorentz factor according to the expression (118), would not invalidate the Special Relativity Theory and the Lorentz transformations in favor of the generic transformations deduced in [3].

8. References

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