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Subjective Probability Theory for Decision Making

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Abstract

In this paper, we present de Finetti-Ramsey's subjective probability theory and indicate an interpretation that is suitable for contemporary decision-making theory. In a final section, we take up the case of some "Paradoxes in Expected Utility Theory" and try to reconcile them with the help of subjective probabilities.

1. Introduction: Frank Ramsey ([8], [9])and Bruno de Finetti ([1], [2], [3]) independently defined the probability of an event assessed by an individual as the individual's degree of belief that the event will occur and measured as the price (using slight discretion in the present context) measured in utils the individual is willing to pay (without being vulnerable to "sure loss"), for a lottery ticket that would yield 1 util to the individual if the event did occur and nothing otherwise, the price of the ticket itself being non-refundable. It is simply because these assessments may differ from one individual to another that leads to lottery tickets being bought and sold.

Note that, for the definition of probability due to Ramsey and de Finetti to be meaningful it is required that: (a) "willingness to pay" and "what is at stake" be measured in the same units; and (b) one unit of what the subject or the DM is willing to pay gives the same satisfaction as one unit of what is at stake. Thus, if utility is measured in money units, one unit of money that one is willing to pay needs to give the same satisfaction as one unit of money that is at stake.

An immediate consequence of the notion of subjective probability due to Ramsey and de Finetti is that while there may be many ways to measure individual satisfaction of which (individual specific) utils may be just one, the uniquely defining characteristic of utils, i.e. the units in which prices and stakes in a lottery are measured, satisfies the property that if an individual is willing to pay 'x' utils for a lottery ticket how-so-ever it is specified, the same individual would be willing to pay ‘nx’ utils for ‘n’ such lottery tickets. We may refer to this defining characteristic as the “**linearity criterion**”. Whether such a measure of satisfaction, which may vary from one person to another – since here we are not concerned with an interpersonal comparison of satisfaction – exists or not is a separate issue. In situations where such a measure of satisfaction does not exist, the Ramsey-de Finetti, and consequently our theory of decision-making under uncertainty that follows, would fail to be applicable. In order for our theory to be applicable to an individual, it is imperative that the above “linearity criterion” holds for the utils of the concerned individual.

Thus, probability is a personal assessment based on a “thought experiment” prone to subjectivity and unless what is at stake is a unit of money, the process of probability assessment is possibly not a conscious decision of the individual. It may require to be “elicited” through the revealed behaviour of the concerned individual.

This subjective probability which forms the cornerstone of our discussion here is formally presented beginning with the next section. (See [7] for an interesting discussion of the concept).

In the final section of this paper, we take up the case of “Paradoxes in Expected Utility Theory” whose starting point is the critique of “Expected Monetary Value” posed by the St. Petersburg paradox. Our position is that such paradoxes arise because of unwarranted “mathematical liberties” that are taken while describing the behaviour of individuals facing uncertain monetary prospects. We do not question the robust validity of expected utility theory as opposed to a theory that argues for expected monetary value and no further. We only insist that the reasons for claiming the superiority of one theory over another should be right and not incorrect. We try to reconcile the “apparent inconsistencies” by using subjective probabilities.

2. The framework of analysis: Let S be a non-empty set, subsets of which are **events**.

An event is a state of nature that is of concern to us, the status of its occurrence (which may be either “true” or “false”) being unknown in the current state.

An event E occurs if the status of its occurrence is true. The non-occurrence of E is also a state of nature and is denoted by E^c. In such a situation we say that E^c occurs.

We consider an individual who chooses to measure his/her personal satisfaction (to the extent possible and the science of human psychology permitting so) in units of a (perhaps abstract) quantifiable entity called “utility” whose unit of measurement is called “utils”. In particular, utility could be money in which case utils would be monetary units used by the individual.

A **finite “utilitarian” lottery ticket** is a pair $(\{E_1,...,E_n\}, \pi)$, where $\{E_1,...,E_n\}$ is a finite partition for some positive integer ‘n’, each member of which is an event and a pay-off function $\pi:\{E_1,...,E_n\}\rightarrow\mathbb{R}$, where for each $i\in\{1,...,n\}$, $\pi(E_i)$ is the pay-off in utility (measured in utils) to a buyer of the lottery in the current state, if the event E_i occurs.

It is easy to see that if n = 2, then $(E_1)^c = E_2$ and $(E_2)^c = E_1$.

Hereafter, in what follows, we will refer to a finite utilitarian lottery ticket, as a "utilitarian lottery ticket". If utility is measured in monetary units then we will refer to it simply, as a "lottery ticket". A "lottery ticket" is also known as an "uncertain prospect".

3. The calculus of subjective probability: A non-empty subset $\mathfrak{B}$ of 2^S (i.e. power set of S) is said to be a **Boolean algebra** (of subsets of S) if and only if it satisfies the following properties:

(a) $S\in \mathfrak{B}$ and $\phi\in\mathfrak{B}$;

(b) If $E\in\mathfrak{B}$, then $E^c\in \mathfrak{B}$;

(c) If $E,F\in\mathfrak{B}$, then $E\cup F\in\mathfrak{B}$.

A member of $\mathfrak{B}$ is said to be an **event**.

Since for all $E,F\in\mathfrak{B}$, $E\cap F = (E^c\cup F^c)^c$, it must be the case that $E\cap F\in \mathfrak{B}$.

In particular, if for some positive integer n, $(E_1,...,E_n)$ is a partition of S, then the set $\mathfrak{B}(E_1,...,E_n)$ comprising of all unions in the partition $(E_1,...,E_n)$ and the null (empty) set, is said to be the **Boolean algebra generated by** $(E_1,...,E_n)$.

A **simple bet** on an event E is (i) if $E^c\neq\phi$, a utilitarian lottery $(\{E, E^c\}, \pi)$ with $\pi(E) = 1$ and $\pi(E^c) = 0$; and (ii) if $E^c = \phi$, in which case $E = S$, the utilitarian lottery $(\{S\}, \pi)$ with $\pi(S) = 1$.

Let P(E) a real number denote the **price** of a simple bet on an event E.

If an agent **buys** a simple bet on an event E at a price P(E) then the agent gains 1-P(E) if the state of nature E occurs and gains –P(E) (i.e. loses P(E)) if the event E does not occur.

In order for the paragraph above to make sense it is required that: (a) "willingness to pay" and "what is at stake" be measured in the same units; and (b) one unit of what the subject or the DM is willing to pay gives the same satisfaction as one unit of what is at stake. Thus, if utility is measured in money units, one unit of money that one is willing to pay needs to give the same satisfaction as one unit of money that is at stake.

A function $P:\mathfrak{B}\rightarrow\mathbb{R}$ which associates with each simple bet based on an event E the maximum price P(E) at which a unit of it is voluntarily bought is called a **price function**.

Since for all $E\in \mathfrak{B}$, the simple bet based on E has to be voluntarily bought, it must be the case that $P(E) \leq 1$ for all $E\in \mathfrak{B}$.

Given a price function P a simple bet on an event E is said to yield an **unconditional gain compared to a finite (possibly empty) set of simple bets** if the simple bet yields a net gain which is strictly greater than the aggregate net gains from the finite set of simple bets, regardless of whether the event E occurs or not.

It is precisely here (in the definition of "the net gain at E from the simple portfolio) that the "linearity criterion" discussed in the introduction is applicable.

Given a price function P a simple bet on an event E is said to yield an **unconditional loss compared to a finite (possibly empty) set of simple bets** if, the simple bet yields a net gain which is less than the aggregate net gains from the finite set of simple bets, regardless of whether the event E occurs or not.

Note: The aggregate net gain from the empty set of simple bets is 'zero'.

A price function P is said to be a **de Finetti price function** if for no simple bet based on an event is there an unconditional gain compared to a finite set of simple bets or an unconditional loss compared to a finite set of simple bets.

Since the aggregate net gain from the empty set of simple bets is 'zero', a de Finetti price function cannot assume negative values. Combined with the property that a price function is a voluntary payment we get that if P is a de Finetti price function then for all $E \in B$, $0 \leq P(E) \leq 1$.

A function $\psi: \mathfrak{B} \rightarrow \mathbb{R}$ is said to be a **(finitely additive) probability measure** if the following conditions are satisfied:

(i) $\psi(E) \in [0,1]$ for all $E \in \mathfrak{B}$;

(ii) $\psi(S) = 1$, $\psi(\phi) = 0$;

(iii) For all $E,F \in \mathfrak{B}$ with $E \cap F = \phi$, it is the case that $\psi(E \cup F) = \psi(E) + \psi(F)$.

From (i) and (iii) it follows that for $E,F \in \mathfrak{B}$, $E \subset F$ implies $\psi(F) = \psi(E) + \psi(F \backslash E)$.

It follows from (iii) of the definition of a probability measure that for all $E,F \in \mathfrak{B}$ it is the case that $\psi(E \cup F) = \psi(E) + \psi(F) - \psi(E \cap F)$.

The reasoning is as follows: $E \cup F = E \cup (F \backslash E)$ where $E \cap (F \backslash E) = \phi$ and $F = (E \cap F) \cup (F \backslash E)$ where $(E \cap F) \cap (F \backslash E) = \phi$. Thus, $\psi(E \cup F) = \psi(E) + \psi(F \backslash E)$ and $\psi(F) = \psi(E \cap F) + \psi(F \backslash E)$. Substituting for $\psi(F \backslash E)$ from the second equation to the first we get, $\psi(E \cup F) = \psi(E) + \psi(F) - \psi(E \cap F)$.

Theorem 1: Every de Finetti price function is a finitely additive probability measure.

Proof: Let P be a de Finetti price function. Thus, as already noted $0 \leq P(E) \leq 1$ for all $E \in \mathfrak{B}$.

Suppose towards a contradiction it is the case that $P(S) < 1$. Then by buying 1 unit of a simple bet based on S, the net gain is $1 - P(S) > 0$, contradicting P is a de Finetti price function. Hence $P(S) = 1$.

Now suppose that $P(\phi) > 0$. Then by buying 1 unit of a simple bet on ϕ, the net gain at any non-empty event F is $-P(\phi) < 0$, since $F \cap \phi = \phi$, contradicting P is a de Finetti price function. Hence $P(\phi) = 0$.

(iii) Let $E,F \in \mathfrak{B}$ with $E \cap F = \phi$.

First, suppose $P(E \cup F) > P(E) + P(F)$.

Thus, 1- $P(E\cup F) < 1 - P(E) - P(F)$.

Then by buying 1 unit of a simple bet based on $E\cup F$, 1 unit of a simple bet based on E and 1 unit of a simple bet on F, the net gain from 1 unit of a simple bet based on $E\cup F$ is (a) $[1 – P(E\cup F)] < [1-P(E)-P(F)]$ if E occurs but F does not, (b) $[1 – P(E\cup F)] < [1-P(F)-P(E)]$ if G occurs but E does not, and (c) $– P(E\cup F) < -P(E)-P(F)]$ if neither $E\cup F$ does not occur.

Thus, the net gains from simple bet $E\cup F$ is strictly less than the aggregate net gains from a simple bet based on E and a simple bet based on F. Thus, $P(E\cup F) \leq P(E) + P(F)$.

Now suppose, $P(E\cup F) < P(E) + P(F)$.

Then by buying 1 unit of a simple bet based on $E\cup F$, buying 1 unit of a simple bet based on E and 1 unit of a simple bet based on F, the net gain from a simple bet based on $E\cup F$ is $1- P(E\cup F) > 1-P(E)-P(F)$ if $E\cup F$ occurs and $- P(E\cup F) > - P(E)-P(F)$ if $E\cup F$ does not occur. $[-P(E)- P(F)+P(E\cup F)] < 0$ regardless of whether $G\cap(E\cup F) = \phi$ or $G\cap(E\cup F) \neq\phi$, since $E\cap F = \phi$.

Thus, the net gains from simple bet $E\cup F$ is strictly greater than the aggregate net gains from a simple bet based on E and a simple bet based on F. Thus, $P(E\cup F) \geq P(E) + P(F)$.

Combining the two inequalities we get $P(E\cup F) = P(E) + P(F)$. Q.E.D.

4. Expected utility value as the willingness to pay or accept: A function $X:S\rightarrow\mathbb{R}$ is said to be a **random variable**.

For $s\in S$, $X(s)$ is the utility (measured in utils) that the individual gets if s is chosen and is said to be the **realization of the random variable** X **at** s.

The random variable X is said to be bounded if there exists a positive real number M such that for all $s\in S$, $|X(s)| \leq M$.

Given a de Finetti price function P, the **Expected Utility Value (EUV) or Expectation or Reservation Price** of a random variable X for an individual (provided it exists) denoted E(X) is the number of utils at which random variable X is voluntarily bought or sold, i.e. the maximum number of utils the DM is willing to pay and simultaneously the minimum number of utils the DM is willing to accept in lieu of random variable X.

EUV or Expectation or Reservation Price are meaningful in our context only if the random variables and probabilities are measured in the same units. The maximum deviation we can allow for is to assume that the units in which the random variables are measured are an affine transformation of the units in which probabilities are measured, the slope of the transformation being strictly positive.

A random variable X is said to be **finitely generated** if there exists a finite partition $\{E_1,\ldots,E_n\}$ if for all $j\in\{1,\ldots,n\}$ and $s, s^{'}\in E_k$: $X(s) = X(s^{'})$ (= x_k say).

It is very easy to establish the following result.

Proposition 1: If X is a finitely generated random variable, then $E(X) = \sum_{j=1}^{n} x_j P(E_j)$.

Proof: In this case X is the simple portfolio $((E_1, E_2,...,E_n), z)$ with $z_j = x_j$ for $j\in\{1,...,n\}$.

Since $P(E_j)$ is the price at which 1 unit of the simple bet based on E_j is voluntarily bought or sold, $x_jP(E_j)$ is the amount at which x_j units of E_j are voluntarily bought or sold.

Hence the simple portfolio $((E_1, E_2,...,E_n), z)$ which in this case is the random variable X, is voluntarily bought and sold for the amount $\sum_{j=1}^{n} x_j P(E_j)$.

Thus, $E(X) = \sum_{j=1}^{n} x_j P(E_j)$. Q.E.D.

A random variable X is said to be **countably generated** if there exists a countably infinite collection $\{E_1,\ldots,E_k,\ldots\}$ of mutually exclusive (i.e. disjoint) and exhaustive (i.e. $\bigcup_{k=1}^{\infty} E_k = S$) non-empty events such for all $k\in\mathbb{N}$ and $s, s'\in H_k$: $X(s) = X(z')$ (= x_k say).

The proof of the following is not much difficult either.

Proposition 2: If X is a bounded and countably generated random variable, then $E(X) = \sum_{k=1}^{\infty} x_k P(E_k)$.

Proof: Suppose there exists a positive real number M such that for all $s\in S$, $|X(s)| \leq M$.For $K\in\mathbb{N}$, let X^{+K} be the finitely generated random variable on $\{E_1,\ldots,E_K, S\backslash(\bigcup_{k=1}^{K} E_k)\}$ such that $X^{+K}(s) = X(s)$ for all $s\in\bigcup_{k=1}^{K} E_k$ and $X(s) = M$ for all $s\in S\backslash\bigcup_{k=1}^{K} E_k$. For $K\in\mathbb{N}$, let X^{-K} be the finitely generated random variable on $\{E_1,\ldots,E_K, S\backslash(\bigcup_{k=1}^{K} E_k)\}$ such that $X^{-K}(s) = X(s)$ for all $s\in\bigcup_{k=1}^{K} E_k$ and $X(s) = - M$ for all $s\in S\backslash\bigcup_{k=1}^{K} E_k$.

Thus, $E(X^{-K}) \leq E(X) \leq E(X^K)$ for all $K\in\mathbb{N}$.

Thus, $\sum_{k=1}^{K} x_k P(E_k) - M\sum_{k=K+1}^{\infty} P(E_k) \leq E(X) \leq \sum_{k=1}^{K} x_k P(E_k) + M\sum_{k=K+1}^{\infty} P(E_k)$ for all $K\in\mathbb{N}$.

Since $\lim_{K\to\infty} \sum_{k=K+1}^{\infty} P(E_k) = 0$, we get the desired result. Q.E.D.

Given a de Finetti price function P and events G and E, since G is a disjoint union of $G\cap E$ and $G\cap E^c$, we know that $P(G) = P(G\cap E) + P(G\cap E^c)$.

Thus, if $P(G) > 0$, then $1 = \frac{P(E\cap G)}{P(G)} + \frac{P(E^c\cap G)}{P(G)}$

Given a de Finetti price function P and an event G with $P(G) > 0$, a function $P(\cdot|G):\mathfrak{B}\to\mathbb{R}$ is said to be a **price function conditional on** G **and consistent with** P if, for each event E, $P(E|G)$ is the "expectation" or "maximum willingness to pay" (reservation price) for the random variable $X(E|G)$ where $X(E|G)(s) = 1$ if $s\in E\cap G$, $X(E|G) = 0$ if $s\in E^c\cap G$ and $X(E|G) = P(E|G)$ if $s\in G^c$.

In other words, $P(E|G)$ is the reservation price of the random variable that yields 1 util if $E\cap G$ occurs, nothing if $E^c\cap G$ occurs and the reservation price of the random variable if G does not occur.

Since, E(X(E|G)) = P(E∩G) + P(E|G)(1-P(G)), E(X(E|G)) = P(E|G) implies P(E|G) = $\frac{P(E\cap G)}{P(G)}$.

The following theorem can be established in a manner analogous to the proof of Theorem 1.

Theorem 2: Given a de Finetti price function P, an event G with P(G) > 0, and a de Finetti price function conditional on G and consistent with P,P(·|G), the function P(·|G) satisfies the following properties:

(i) P(E|G)∈[0,1] for all E∈$\mathfrak{B}$;

(ii) P(S|G) = 1 and P(ϕ|G) = 0.

(iii) P(E∪H|G) = P(E|G)+P(H|G) for all E,H∈ $\mathfrak{B}$ with E∩H = ϕ.

Given a de Finetti price function P, an event G with P(G) > 0, and a de Finetti price function conditional on G and consistent with P, P(·|G), the **Expected Utility Value (EUV) conditional on** G **or Expectation conditional on** G of a random variable X for an individual (provided it exists) denoted E(X|G) is the number of utils at which random variable X is voluntarily bought or sold conditional on the occurrence of event G.

The proofs of the following two propositions are analogous to the proofs of propositions 1 and 2.

Proposition 3: If X is a finitely generated random variable, then E(X|G) = $\sum_{j=1}^{n} x_j P(E_j|G)$.

Proposition 4: If X is a bounded and countably generated random variable, then E(X|G) = $\sum_{k=1}^{\infty} x_k P(E_k|G)$.

5. Explanation of Paradoxical Behaviour

Thus far we have not invoked the "utility of money" while discussing matters concerning risk or aversion towards it, since in our framework money itself could be used to measure the utility of/for/by some individuals, in much the same way as "years of being alive" could be used of/for/by others. At the same time, there is the extremely realistic example related to portfolio diversification discussed in Section 1.1 of the book by Eeckhoudt, Gollier and Schlesinger [4] which compels us to conclude that people may not always use the expected value of random variable denoting gains and losses of money to measure the reservation price of (what we may refer to as) the "uncertain prospect" associated with the random returns (variable). Thus, the utils in which an agent measures probability may usually be different from money, in which case given the possibility of random gains and losses of money, we may need to find the utility of money and the relationship between the two may not always be a linear function. In fact, the only interpretation of "expected monetary value" that is unconditionally justifiable in our framework, is that of the expected utility of a "hypothetical individual" whose utility function for money is affine and has a positive slope. In [6] we provide a simple proof, using a small number of very reasonable assumptions, of the existence of a (Bernoulli) utility function of money such that the reservation prices of uncertain gains and losses are measured by their expected utility.

Having said that, in this section, we will provide scenarios which have either been misunderstood or misrepresented, to justify expected utility – instead of expected monetary values – to measure reservation prices of random monetary returns. However, the very realistic example (considerably more realistic than the St. Petersburg Paradox) about a hypothetical individual by the name of Sempronius in section 1.1 of [4], shows that although two pieces are sufficient for explaining "loss aversion" more than two pieces are required for explaining the preference for a more diversified portfolio of assets over a less diversified one.

A major concern is about the frequent observation that a potential buyer of a fair bet may refuse to buy it, in the sense that the potential buyer may attach a negative price to such a lottery ticket. Why?

The reason for this appears to us to be an asymmetry in the roles of the buyers and sellers. True to his word, the seller of a fair bet foresees the real possibility of an equal number of gains and losses on the utilitarian lottery tickets that he sells, whereas this is not the case with the buyer who may buy just one such ticket. A simple example may help to illustrate what we are trying to suggest. Consider a seller of lottery tickets, each of which yields its buyer a "gain" of $1/- if the toss of an unbiased coin shows up heads and the buyer incurs a "loss" of $1/- if the toss of the same coin shows up tails. As far as the seller is concerned the coin is an unbiased one and not loaded in favour of any outcome. Hence it would not be unreasonable for him to assume that he would gain $1 almost as many times as he would lose $1 and thus his net gain from selling such lottery tickets is zero. But how about a buyer of the lottery ticket who gets the opportunity to lose or gain from a toss of the unbiased coin exactly once? What the outcome of that particular toss is going to be is neither implied nor does it have any implications for the kind of observations that one would expect from multiple repetitions of the toss. The perspective of the buyer is completely different from that of the seller, since the question that confronts the buyer is: will the outcome of this toss be among the approximately 50% times "heads" show up or will the outcome of this toss be among the approximately 50% times "tails" show up? The price the buyer would be willing to pay for the lottery ticket would very likely depend on whether the buyer is an optimist or a pessimist, or what his mood is at the time of buying the ticket. While there may be definite results in physics which could determine the outcome of a coin toss on the basis of force, spin etc. of the toss, rarely are such considerations invoked when one has to decide on buying a lottery ticket or not – even if the potential buyer is a professional physicist. It is quite possible that the buyer may be feeling pessimistic at the moment, and seek a non-refundable compensation from the seller of the lottery ticket in case he did agree to participate in the lottery. Of course, it is important to note, that while the seller of the lottery ticket considers the two possible outcomes of the toss of the coin as heads and tails, the same is not true for the potential buyer. The potential buyer would very likely be viewing the two possible outcomes as lucky and unlucky or as a good day and a bad day or elements in the set {$1, $(-1)}. More formally, the potential buyer views the two outcomes as elements in the set {this particular toss is among the approximately 50% times "heads" show up, this particular toss is among the approximately 50% times "tails" show up}. It is precisely for this reason, that the subjective price or the de Finetti price of the event "less than or equal to zero" may be greater than $\frac{1}{2}$, the latter being the de Finetti price of the event to the seller.

A well-known but not well-recognized red-herring in decision-making theory is the so-called St. Petersburg paradox which led to Bernoulli utility functions and expected utility maximization. The experiment consists of repeated trials of an unbiased coin till the first head shows up. If the first head shows up on the nth toss, then the participant in this game gets $\$2^n$. How much should a person be willing to pay to play this game? While no reasonable person would be willing to pay more than a couple of dollars for it, apparently an expected monetary value maximiser should be willing to pay an unbounded sum of money to play the game, provided one believes the coin is fair and not loaded in favour of showing either heads or tails. However, the conclusion that a person would be willing to stake any amount of money that is conceivable, to participate in such a game, rests crucially on the assumption that simply because the person initially started off by believing that the coin under consideration is fair and unbiased continues to do so after no head has appeared till 'n' tosses, however large 'n' may be. Hence after observing a string of one million consecutive tails showing up – if that was humanly possible to endure – one would continue to abide by one's initial belief that heads and tails would show up almost equally often. That clearly requires a "Giant Leap of Faith" – and absolutely nothing less. In fact, that such a game is on offer would make one suspect, whether the coin is "fair", i.e. the two sides are indeed head and tail and the engineering behind the coin has not favoured any one side to show up more often than another. The approach using Kolmogorov (mathematical) probability on which the St. Peterburg paradox is based, is that there is no justification required for assuming an infinite sequence of independent and identically distributed (IID) random variables. It is perfectly consistent with mathematical probability to assume a sequence of randomizations to be IID even if such an assumption is inconsistent with science and/or empirical observations and it is the assumption of IID random outcomes of an unbiased coin in the St. Petersburg paradox which leads to the conclusion that the probability of the first head appearing after 'n' tails is $(\frac{1}{2})^{n+1}$for all positive integers 'n' and thus the expected monetary value of this so-called paradox proposed by Nicolas Bernoulli, who first stated it in a letter to Pierre Raymond de Montmort on September 9, 1713. However it was not he, but his cousin Daniel Bernoulli who is considered to be the pioneer of Expected Utility Theory with his arguments in the *Commentaries of the Imperial Academy of Science of Saint Petersburg* (1738) in favour of strictly increasing and bounded utility functions of money whose expectation rather than the expected monetary value he suggested as a criterion for choosing uncertain prospects. It is perfectly fine for us if the Bernoulli brothers or whoever else subscribes to the kind of probability theory that finds nothing unreasonable about the probability of the first head appearing after 'n' tails to be $(\frac{1}{2})^{n+1}$ for all positive integers 'n'. The approach of subjective probability theory we subscribe to, unlike the approach adopted above, would allow the conditional probability of a head on the $(n+1)^{th}$ toss given tails on the previous 'n' toss, to be equal to $\frac{1}{2}$ for n= 0,1,2, equal to $\frac{1}{4}$ for n = 3, $\frac{1}{8}$ for n = 4 and the same to be equal to 0 for n ≥ 5. While expected utility maximization with an increasing and bounded utility function and IID Bernoulli random variables would theoretically accommodate the "mathematical madness" of the type that confused Nicolas Bernoulli, the subjective probability of the kind that leads to our more sober (gu-)es(s)timates, would recommend updating and evaluating probabilities and then using expected monetary value to evaluate lotteries/uncertain monetary prospects. In fact, we would be inclined to view the St. Petersburg paradox as an example against thoughtless invocation in the science of an infinite sequence of IID random variables with a well-behaved probability distribution, rather than a reason for expected utility theory. By this, we do not wish to cast aspersions on or deny "the possibly immense significance" of expected utility theory. Not at all. We simply do not view the St. Petersburg paradox as a valid argument against using expected monetary value as a reasonable evaluator of uncertain prospects, wherever it is applicable.

For the uncertain prospect in the St. Petersburg paradox, the rational decision would be to offer no more than a dollar or two and that too for the sake of some entertainment, which is what most people would do.

On the other hand, a very realistic example (considerably more realistic than the St. Petersburg Paradox) about a hypothetical individual by the name of Sempronius in section 1.1 of [4], shows the need for using expected utility instead of expected monetary value to evaluate the worth of uncertain monetary prospects.

An apparent violation of expected utility that may challenge the confidence of mathematicians in "mathematical probabilities" but can be easily explained using subjective probabilities is available in [5], where we consider the example of a uniformly distributed random variable on the closed interval [0,1] with a "huge" monetary loss occurring if the random variable realises a rational number and a comparatively small monetary gain otherwise. How such a random variable could be operationalised is also discussed in [5]. The two alternatives available to an individual are to either opt for the random variable or not opt for it. Most individuals- including mathematicians- rejected the random variable considering it to be too risky, although the mathematical expectation associated with the random variable was a positive gain. The comparatively less mathematically trained individuals confessed that they were associating a probability of 1/2 to each of the two outcomes, thereby ending up with a huge expected loss. Clearly, such probabilities disagree with those prescribed by the uniform probability distribution on [0,1], according to which the probability of realizing a rational number is '0'. Even mathematicians preferred to use subjective probabilities that were different from the mathematically "objective" probabilities, in the context of the experiment discussed in [5].

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