Qeios

Foundations of Quantum Mechanics Revealed by the Conservation Laws

Richard Oldani
Illinois Institute of Technology
https://orcid.org/0000-0001-6884-5614
email: oldani@juno.com

Abstract

It is often claimed that Einstein is wrong about quantum mechanics. However, when comparisons are made with respect to theoretical foundations rather than experimental results Einstein's theories are found to be superior. Because he did not complete his own quantum theory its far-reaching, intuitively motivated derivations were never properly appreciated despite the fact that his is the only theory based on a relativistically correct foundation. The development of his arguments reveals that although non-relativistic theory correctly predicts what it is possible to observe (the emissions) with remarkable accuracy it neglects the other half of natural phenomena, that which cannot be observed (the absorptions), thereby violating the conservation of energy. The deficiencies in non-relativistic theory are corrected by applying Hamilton's principle to derive relativistic equations of motion. This allows quantum mechanical formalism in abstract space to be interpreted as the juxtaposition of particle field geometries in real space. Direct application of the calculus of variations to an electron cycle reveals that the wave function is incomplete because it requires twice the allowable action minimum.

Keywords: Conservation laws; non-relativistic quantum mechanics; relativistic quantum mechanics; Hamilton's principle; energy; momentum; causality

1. Introduction

The preferred way to do science historically is by means of the scientific method. A theory is proposed and predictions made; experiments are performed to see how well it fits expectations; and finally improvements are made and corrections introduced. This is how all the founding theories of quantum mechanics; matrix mechanics, wave mechanics, and the path integral formulation; were initiated and carried out. Einstein paid close attention to the reality of experiment, but he was equally concerned with more subtle aspects such as conflicts that arise due to internal consistency. One of the ways he used to test the compatibility of theories was with thought experiments as when he imagined running beside a light wave or doing experiments while in free fall. The thought experiments would often suggest the

https://doi.org/10.32388/ROFQKG.7

possibility of real experiments. In his 1917 paper, "On the quantum theory of radiation" he imagined the effect photon momentum has on molecules during the absorption and emission of black body radiation allowing him to anticipate the invention of lasers [1]. The same reasoning led him to conclude that momentum conservation is critical for explaining Planck's radiation law of black body radiation. Following his lead we argue in the following pages that the energy and momentum conservation laws are of critical importance in deriving the foundations of quantum mechanics.

The three formulations of quantum mechanics that are successful in predicting the behavior of quantum phenomena use equations of motion that are specific to particular experiments. Because all of the equations describe aspects of atomic structure more than one of the formulations may be applied to any given experiment. Thus we are not always certain which formulation is the correct one. However, there is an alternative method we can use that applies equally to all the equations. This method allows one to collect together all the equations of motion and express them as the stationary property of a certain action function, the time integral of the Lagrangian [2,3]. The action S is defined formally as the time integral of the kinetic energy T minus the potential energy V; or equivalently, in terms of the Lagrangian.

$$S=\int_{t_1}^{t_2} (\mathrm{T}-\mathrm{V})\,dt=\int_{t_1}^{t_2} L\,dt \qquad 1)$$

The use of an action to describe a physical system has the advantage that it allows for conservation laws, since due to Noether's theorem for every continuous symmetry of an action it is possible to derive a local conservation law. In the first half of our paper we show that the conservation laws Einstein uses in his quantum theory of radiation evolve from the continuous symmetry of an action. In the second half of the paper we use the symmetries to identify the actions that correspond to an atomic system. Equations of motion are obtained by applying a variational principle to each action.

2. Perspectives of quantum mechanical foundations

2.1 A statistical theory due to Einstein

In Einstein's only attempt at a theory of quantum mechanics he derives Planck's law by statistical methods [1]. There he describes the dynamic equilibrium that exists between the thermal energy absorbed by molecules and its subsequent quantum mechanical emission as black body radiation. Heat energy is absorbed according to classically defined Maxwell Boltzmann statistics, transformed at the molecular level, and emitted at a later point in time according to the Planck radiation law. He continues by extrapolating the analysis to the level of individual molecules.

> "We now turn to the investigation of the motion which our molecules execute under the influence of radiation. In doing this we use a method which is well known from the theory of Brownian movement. . . . . Let a molecule of given kind be in uniform

> motion with velocity **V** along the x-axis of the coordinate system K. We inquire about the momentum transferred on the average from the radiation to the molecule per unit time. To calculate this we must consider the radiation from a coordinate system K′ that is at rest with respect to the given molecule. For we have formulated our hypotheses about emission and absorption only for molecules at rest."

The hypotheses he speaks of are the A and B coefficients. For an atom with energy states |1> and |2> the coefficient B_{12} refers to induced energy and momentum "transferred on average from the radiation to the molecule" in the coordinate system K. To study the motion of the individual molecules as they *absorb* energy he considers the viscosity and temperature of the gas, frequency of the radiation, and radiation density. A molecule subjected to these forces and confined to motion along the x-axis of K is accelerated by photon *absorption* in the direction of propagation, and in a direction opposite to that of propagation by photon *emission* due to recoil momentum. The sudden reversals and random nature of the impulses cause the molecular motions to be discontinuous, as observed in Brownian motion.

Einstein uses the conservation of momentum to justify the close causal relationship between momentum and energy exchange in black body radiation.

> "If a radiation beam with a well-defined direction does work on a Planck resonator [quantum oscillator], the corresponding energy is taken from the beam. According to the law of conservation of momentum, this energy transfer corresponds also to a momentum transfer from the beam to the resonator."

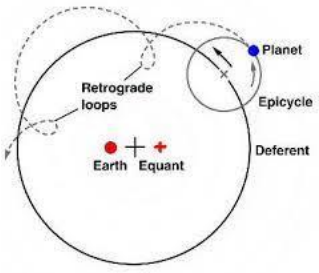
Heat energy E_K is absorbed from the environment by the oscillator which then causes it to emit energy $E_{K'}$ in the form of a photon, where $E_K \geq E_{K'}$. Emission occurs in one of two ways, by spontaneous emission according to the coefficient A_{21} or by induced emission according to B_{21}. Two equations of motion are required to describe the complete radiation process of a non-conservative system, a classical equation describing molecular impulses in K to satisfy Maxwell-Boltzmann statistics, and a quantum mechanical equation in K' to satisfy photon emission according to Planck statistics.

Momentum transfer "from the beam to the resonator" corresponds to *the continuous symmetry of an action*. And subsequently during photon emission "the molecule suffers a recoil of magnitude hν/c" corresponding to *the continuous symmetry of another action*. Each continuous symmetry corresponds to the space translation invariance of an event, the exchange of momentum. Whereas non-relativistic quantum mechanics treats photon emission as a single event Einstein describes emission as two events in succession. To emphasize the importance of momentum he adds.

> "In general one restricts oneself to a discussion of the energy exchange, without taking the momentum exchange into account. One feels easily justified in this, because the smallness of the impulses transmitted by the radiation field implies that these can almost always be neglected in practice, when compared with other effects causing the motion. For a theoretical discussion, however, such small effects should be considered

> on a completely equal footing with more conspicuous effects of a radiative energy transfer, since energy and momentum are linked in the closest possible way."

Energy is measurable as a magnitude and easily incorporated into the equations of motion. Momentum, on the other hand, is impossible to measure at the microscopic level so we tend to underestimate its contribution to the equations of motion. The advantage of an analysis by momentum is that it provides information about a system's time evolution. In the Ptolemaic planetary system, for example, equations of motion for retrograde loops are in clear violation of the conservation of momentum (see figure). The planets appear to reverse motion with respect to the background stars as the earth orbits the sun. If astronomers had understood this they would have rejected theories that suggest massive objects can reverse motion in empty space.



Ptolemaic system

Einstein places emphasis on determining the momentum of energy exchange rather than with the eigenvalues of energy states as in non-relativistic models because he is seeking a statistical relation describing the continuous time evolution of states rather than evaluating their values in absolute time. Excitation to a higher energy state is due to a transformation of heat energy from K to K', while emission from K' is spontaneous and "corresponds to that of a radioactive reaction". The time lapse between absorption and emission is not instantaneous, but it "should be negligible compared with the times which the molecule spends in states". Thus a time dependent separation exists between absorption and emission during spontaneous emission, and the equations of motion on either side of the discontinuity must differ because of it. Einstein describes the time evolution of energy states in *real time*; that is, relativistically, by closely examining momentum exchange, and we find in the next section that it contrasts sharply with the non-relativistic description in absolute time.

2.2 Heisenberg's non-relativistic matrices

Matrix mechanics evolved from long-standing attempts to describe dispersion phenomena, the continuous change in the angle of refraction of different frequencies of light by a prism or other medium. Although light disperses continuously across the entire spectrum, at certain specific frequencies characteristic of the medium, it is completely absorbed forming lines. Heisenberg discovered complex sets of mathematical rules describing the relationships of the observed frequencies and intensities of spectral lines. This allowed him to formulate a theory of quantum mechanics reconciling the continuity of radiation fields

with the discrete energy states of a hydrogen atom by expressing electron transitions in the form of a matrix [4].

$$\sum_k (p_{nk} q_{km} - q_{nk} p_{km}) = \begin{matrix} i\hbar \text{ for } n=m \\ 0 \text{ for } n \neq m \end{matrix} \quad \text{or equivalently:} \quad \mathbf{pq} - \mathbf{qp} = i\hbar \mathbf{I} \qquad 2)$$

The **p** and **q** are matrices representing the conjugate variables of a quantum system based on the assumption that the Hamiltonian can be expressed as the sum of the momentum and position as in classical theory. Although we are not given any information about why or how they occur we may safely assume that the **p** and **q** in the above equation do not refer to the continuous, classically described momentum and position in K used by Einstein, rather they refer to the discrete properties of photons designated by two indices and emitted or absorbed in K'. When $n=m$, **H** is a diagonal matrix and refers to possible energy states of a quantum system. Because the theory is based on the determination of observables, such as emission frequencies and transition probabilities, it is formulated in K' in coordinates relative to a molecule.

Off-diagonal elements, $n \neq m$, are assigned a value of zero because absorptions and emissions differ only with respect to phase and are assumed to average out, but they also include kinetic energy in K due to exchanges of momentum. They originate with these same absorptions and emissions but are ignored because they are unobservable. Although the collisions cannot be observed individually they are evident statistically in measurements of gas temperature. Heisenberg did not take these microscopic interactions into account because he believed that quantum mechanics should be "founded exclusively upon relationships between quantities which in principle are observable". However, unless there are thermal inputs there are no observables at all. If the off-diagonal elements of a molecule are zero it indicates that the ambient temperature is absolute zero and that all matrix elements both diagonal and non-diagonal are zero. Because Heisenberg was only concerned with observables in K' he ignored the causal influence that thermal energy has on quantum mechanical emission and consequently he was unable to establish a formal link between the classical and quantum theories.

Describing the energy of a material system by means of emission processes alone is a violation of energy conservation. Absorption must precede emission whether the material system is quantum mechanical or classical. The meaning of energy is not embodied only in the form of emissions, or only in absorptions, for due to the conservation laws energy can neither be created nor destroyed. It is correctly described as a transformation from an absorption process to an emission process. Therefore all matrix elements in 2), both diagonal

and off-diagonal, need to include non-zero, classical values due to kinetic energy caused by molecular impulses.

2.3 Relativistic versus non-relativistic theory

Einstein's statistically derived quantum theory of radiation evolves continuously in time, whereas the matrix mechanical model describes the physical parameters frequency and spectral line intensity relative to absolute time. Should a non-linear gravitational field intensity be imposed the physical variables of statistical theories derived in continuous time will adapt consistently. However non-relativistic theories will not adapt because they are only valid with respect to absolute time. The question how to incorporate gravitational fields into non-relativistic theory is highly contentious and many theories are devoted to it. Einstein's statistically based theory, ignored because it is not experimentally verifiable, satisfies the conditions of relativity theory due to first principles.

Feynman's sum-over-paths approach of non-relativistic quantum mechanics describes particle dynamics without *explicit* use of a wave function thereby removing the conceptual difficulties of collapse [5]. Predictions are made by summing the probability amplitudes of all possible paths and squaring the result to obtain the probability of an event. As is customary for non-relativistic theory the absorption energy is introduced as an initial condition of experiments and is due, for example, to particle velocity or field potential. The experimental apparatus and initial conditions are described classically in K and the paths of probability amplitudes (the emissions) are described, for example, relative to the nucleus in K'. Transformations of coordinates between K and K' cannot be performed because the equations of motion in K are continuous and in K' they are discrete.

All the paths in K' contribute to the probability amplitude for an event so they are regarded as *possible* trajectories, but they are not like anything anyone has ever experienced for they are unrestricted by the conservation laws. They may form loops, extend to infinity, go backwards in time, or exceed the speed of light; and the use of "all possible paths" is an indication that boundary conditions extend to infinity. Violations of energy and momentum conservation are made possible by rejecting time and space translation symmetries described in the introduction and instead using the coordinates of absolute time in K'.

3. Relativistic quantum mechanics

3.1 Variational principle applied to the action of a conservative system

In order to describe particle motion in a conservative system we apply an action principle, which is the basis for Newton's laws, Hamiltonian mechanics, and Lagrangian mechanics. Each of the paths between the same two end-points will have a different action. The principle of least action asserts that the particle path actually taken is the one for which the action is a minimum. It allows for the local application of conservation laws since for every continuous symmetry of an action it is possible to derive a local conservation law. For a conservative system with constraints of fixed end positions and fixed time travel from t_1 to t_2 the variation of action over a classically defined path is equal to zero.

$$\delta S = \int_{t_1}^{t_2} (T-V)\,dt = 0 \qquad 3)$$

The variation of action is zero because a conservative system *does not exchange energy with the environment*.

3.2 The action of a non-conservative system

An atomic oscillator that absorbs energy from the environment and transforms it into a unit of electromagnetic energy, the photon, is a non-conservative system. Energy exchange by an atomic oscillator can happen by either induced or spontaneous emission. The way an atom emits radiation determines to a large extent how that energy was absorbed, for due to the conservation laws energy must be absorbed before it can be emitted. In the case of induced emission a photon is absorbed and a photon is emitted so there appears to be continuity of energy exchange with the nucleus acting as the source of a conservative force [6]. That is the assumption behind the wave function model of the emission and absorption of radiation. It is why induced emission is mistakenly believed to be due to a path independent *conservative* force. In the case of spontaneous emission, on the other hand, there is no question that the force is non-conservative because there is a time delay between absorption and emission. Photon emission occurs when an electron decays to a lower energy level due to a statistical process that is determined within the atom by structural properties. Not only is there a discontinuity in time between absorption and emission, but they necessarily occur in response to physically distinct statistical laws.

To restore continuity to the equations of motion we use Hamilton's principle which expresses the meaning of the entire set of differential equations describing the paths and calls for minimizing a single physical quantity, the action, in order to obtain the path actually taken. Energy absorption consists of two simultaneously evolving processes; the excitation of an electron and the localization of fields in the creation of a photon. The transition of an electron from the ground state to an excited state is characterized in generalized coordinates with six dimensions, three to describe its position on the electron shells R_1 and R_2, and three to describe its trajectory. The electron initiates its motion at a point on the equipotential surface R_1 of the ground state at time t_1, is excited by linear transverse fields along a path r, and upon arriving at R_2 the fields are localized and assume the experimentally determined orbital angular momentum for the hydrogen atom, $(T-V) = 2\pi E$.

$$S[r(t)] = \int_{R_1}^{R_2}\int_{t_1}^{t_2} (T-V)\,dt \qquad 4)$$

The action, S[r(t)], is a functional that describes the absorption process in four dimensions. It has as its argument an infinite number of functions, the possible electron trajectories r(t). It differs from 3) by requiring physically defined end-points for the path as determined by the electron shells. The energy states $|1\rangle$ and $|2\rangle$ determine the energy and the corresponding

period of a wave cycle. If the initial time t_1 of the excitation is arbitrarily set at zero, then the action integral for one period is evaluated as follows:

$$2\pi E\tau = h$$

Simplifying, we describe the localized fields of a photon bounded in space and time by the atomic orbitals |1> and |2>, as $E\,\tau = \hbar$.

3.3 Relativistic emission of energy by an atomic system

At the relativistic or high end of the energy spectrum in quantum field theory, particles are treated as excited states of the more fundamental underlying quantum fields. As Nobel laureate Frank Wilczek noted [7], "In quantum field theory, the primary elements of reality are not individual particles, but underlying fields." Particles are described by an action functional $S[\psi(x_i)]$ where the fields $\psi(x_i)$ of particles are defined throughout space. Each of the elementary particles has a field and the action depends on all of the fields. We will follow these same practices in order to extend the field interpretation to low energy emissions currently described by non-relativistic theory.

The region of space-time that is of interest is located in K' (see 2.1) and lies between two states of an atomic oscillator. Within the space-time region between the excited and ground states we define a Lagrangian density of the fields and their first derivatives $L(\phi_i, \phi_{i,\mu})$ which allows for a complete accounting of the energy interactions, where ϕ_i is the current density and $\phi_{i,\mu}$ is the combined electromagnetic field strength of electron and nucleus. The action integral for a quantum oscillator with an outer electron that occupies either of two allowable energy states may now be formulated, with emission initiating from the excited state R_2 at time t_2 and finalizing at the ground state R_1 at time t_1. In order to apply Hamilton's principle to the field dynamics we require, following Dirac, that the integral of the Lagrangian density over the region of space-time between the excited and ground states be a minimum for all small variations of the coordinates inside the region [8].

$$S[\phi_i(t)] = \int_{R_2}^{R_1}\int_{t_2}^{t_1} L(\phi_i, \phi_{i,\mu})\,d^3x\,dt = E\tau \qquad 5)$$

In order to describe the complete emission process we solve 5) as a continuation of 4) to obtain an expression for the localized fields of a photon in free space, $E\,\tau = \hbar$. The change in action yields a relativistic formulation of emission that is invariant, the same for all observers. The action $S[\phi_i(t)]$ is a functional, a function of the values of coordinates on the *discrete* boundaries of the space-time surfaces R_2 and R_1 which are in turn functions of the *continuous* space-time variables of the fields within the surface. The field boundaries are uniquely fixed in four dimensions by the volume $\int d^3x$ and the time interval $t_2 - t_1$ causing the photon to be described as a four-dimensional localization of fields. Experimental evidence supporting the idea that two equations of motion, 4) and 5), are necessary to describe the absorption and emission of energy can be found by examining the behavior of the simplest quantum system, an electron cyclotron [9].

3.4 Comparison of the relativistic and non-relativistic models

The principal difference between relativistic and non-relativistic models of the quantization process is in their underlying physical assumptions. To formulate a relativistic theory we use a Lagrangian (T-V) and localize the fields £(ϕ_i, $\phi_{i,\mu}$) four-dimensionally, while in non-relativistic theory quantization is usually described with a Hamiltonian (T+V) as the potential of a point electron. To demonstrate their equivalence we will show how the two methods relate physically. In non-relativistic theory a particular type of wave function or "two-component spinor", is used to describe energy emission. It defies explanation in ordinary space-time for it acts like an ordinary vector for infinitesimal rotations, but transforms to its negative for complete rotations and requires two complete rotations to return to its original state. Attempts to visualize spinor behavior make use of imaginary geometries such as the Möbius strip. Normally rotations and transformations of vector components and the coordinates describing them are carried out continuously. The same cannot be said for spinors. Spinors require two complete rotations to return to the original state, but they are constructed in such a way that they are sensitive to how the gradual rotation of coordinates was carried out to arrive there. In other words, they exhibit path dependence. More specifically, for any final configuration of the coordinates there are two topologically inequivalent continuous rotations of the coordinate system that result in this same configuration. It is impossible to carry out transformations between the two inequivalent rotations of coordinates even though they arrive at the same configuration. Because the spinor follows two independent paths it represents a *non-conservative force*.

A simpler explanation is possible by using physical arguments from the relativistic model. Rather than treat quantum mechanical energy emission as a single event carried out in two steps by a single particle, a 2-spinor, we use equations 3) and 4) to describe it with two field sources, an electron of field ϕ_i and a photon of field $\phi_{i,\mu}$, during two distinct physical processes, absorption and emission. The field ϕ_i of an electron has spin described by Pauli matrices oriented in three-dimensional space and the field $\phi_{i,\mu}$ of a photon is oriented due to polarization in four-dimensional space-time by Maxwell's equations. We interpret the 2-spinor therefore as the juxtaposition of two field geometries that cycle through the physical processes of excitation, localization, and emission. There are two paths possible, spin-up or spin-down, and two rotations are needed to complete a photon emission, where rotations are changes in phase of the photon's electromagnetic field during excitation and decay. Thus non-relativistic 2-spinors are rotations in abstract space and absolute time, which we interpret relativistically as the *time evolution of real particle field geometries*.

4. Conclusion

The wave function describes all possible paths of an electron, and the one that is detected is singled out as the only true path. The true path is not determinable in advance through experimental means, rather it is predicted probabilistically and determined after the fact by measurement. Thus the path with the greatest probability does not necessarily minimize the

action. In contrast to non-relativistic theory, the formulation of quantum mechanics based on Hamilton's principle that is proposed here refers to real time. It singles out paths in the present, not the past; paths that minimize the action independently of the measurement process. In quantum mechanics the only paths that are realized are the ones with action minimum ħ. Electron excitation is one path and decay is another. The fact that two transitions, or paths, cause one result, a photon emission; and that they proceed according to distinct laws of motion is to a large extent what makes quantum theory so different from classical theory and difficult to understand.

The significance of Einstein's 1917 paper is now clear. He begins by noting that when atoms absorb energy from a radiation field in K, it constitutes a causal process, "To this transfer of energy there also corresponds a momentum transfer from radiation bundle to resonator, by momentum conservation." And when the atoms decay, an equal amount of energy is emitted, relative to K'. Einstein concludes that two coordinate systems and two *independently derived statistical equations of motion* are required to describe black body radiation (see 2.1). The experimentally determined 2-spinor from non-relativistic theory requires two independent paths to completely describe a *single* emission process (see 3.4). Due to these closely related supporting arguments, that two statistical equations, two coordinate systems, and two paths are needed; we hypothesize that there are two equations of motion describing the absorption and emission of radiation, the time integral of a Lagrangian (Eqn 4) in K and the time integral of a Lagrangian density (Eqn 5) in K'. When used consecutively in time to describe a quantum oscillator emitting radiation the equations provide a complete description of energy absorption, transformation according to the conservation laws, and photon emission.

References

[1] Einstein, A. "Quantum theory of radiation" *Phys Z* **18**, 121 (1917). W https://www.informationphilosopher.com/solutions/scientists/einstein/1917_Radiation.pdf

[2] Lanczos, C. *The Variational Principles of Mechanics* 4th edn. (Dover, 1970)

[3] Feynman, R. Leighton, R.B. Sands, M. *The Feynman Lectures on Physics* (1963), Chapter 19. https://www.feynmanlectures.caltech.edu/II_19.html#footnote_source_1

[4] Heisenberg W. "Quantum mechanical re-interpretation of kinematic and mechanical relations", *Z Phys* **33**, 879 (1925) in B.L. van der Waerden (ed.), *Sources of Quantum Mechanics*, (Dover, 1968); https://www.mathpages.com/home/kmath698/kmath698.htm

[5] Feynman, R. "A space-time approach to non-relativistic quantum mechanics" *Rev Mod Phys* **20**, 367-387 (1948).

[6] Stenholm (2013) *J. Phys. B: At. Mol. Opt. Phys.* **46** 224013 **DOI** 10.1088/0953-4075/46/22/224013; Haroche, S. Raimond, J. *Exploring the Quantum: Atoms, Cavaties, and Photons* (Oxford, 2006)

[7] Wilczek, F. "Mass Without Mass I: Most of Matter," *Physics Today* **52** (11), 11-13 (1999).
[8] Dirac, P.A.M., “The Lagrangian in quantum mechanics” *Phys Zeit Sow* **3,** 1933, p.1; https://www.informationphilosopher.com/solutions/scientists/dirac/Lagrangian_1933.pdf
[9] Brown, L.S.; Gabrielse, G. (1986). “Geonium theory: physics of a single electron or ion in a Penning trap”*Reviews of Modern Physics* **58** (1): 233–311. doi:10.1103/RevModPhys.58.233;
Oldani, R. “Time evolution of energy states” preprints202305.0952.v6.pdf