Research Article

The Monty Hall Problem: Does Performance Improve After the Acquisition of Some Probability Knowledge?

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The Monty Hall Problem (MHP) is a classic probability dilemma that is characterized by being counterintuitive even for experts in the field. The aim of this work was to study how a group of people assesses the chances of winning in the MHP and whether the decision they would make is consistent, with and without some knowledge of computing probability. The MHP was proposed to a group of students from a postgraduate program, before and after a class on probability. The students had to respond to two questions on each occasion: the first about the choice that they believed had more chances of winning and the second about what they would do. The results suggest that when participants had little knowledge of probability, they were more likely to rely on intuition, i.e., keep the selected door, whereas when they had more knowledge, they tended to consider both options as equally likely. Nevertheless, independently of which alternative they believed was more advantageous, most respondents would make the decision to stick to their first choice, probably evidencing biases such as the endowment effect, illusion of control, status quo, loss aversion, and anticipated regret.

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1. Introduction

The Monty Hall Problem (MHP) or Monty Hall Dilemma is one of the most famous brain teasers in the field of probability. Just in terms of the mathematical problem, its first appearance was in[1] as the three prisoners dilemma. As we know it today, although with subtle differences, it was presented and solved

in[2][3]. Nevertheless, it became popular when a reader of Parade magazine proposed it to Marilyn vos Savant in the Ask Marilyn column[4][5]. The letter from the reader said:

> Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

Marilyn vos Savant answered that switching had a greater probability of winning, which generated a real avalanche of letters (on the order of ten thousand), most of them pointing out her "mistake"[5]. It is worth noting that among these people indicating that the answer was erroneous, there were several scholars, mostly men, who referred to her in unflattering terms.

Beyond the anecdote, the correct answer is, in fact, that it is more advantageous to switch the door, a choice that has a probability of 2/3 vs. the original 1/3 if door 1 was kept. There are several possibilities to solve this problem, such as Bayes' theorem, simulations, and other strategies. Most people believe the probability of winning is 50% for the initially chosen door and 50% for the other closed door, but this is a dilemma in which the correct answer goes against intuition, even for experts in computing probability.

What makes this problem interesting is its counterintuitive nature and the systematic occurrence of an erroneous response from individuals. Therefore, its study has extended beyond the fields of mathematics and statistics[6]. This has led numerous researchers from fields such as psychology and behavioral economics to use it in experiments, providing evidence for various cognitive biases of humans.

It is possible to identify two processes that occur in response to this dilemma. The first is cognitive, and it involves an incorrect assessment of the probability of winning when the host opens a door with a goat behind it. This difficulty is directly related to mathematical reasoning in certain situations, particularly when conditional probabilities are involved[7]. The other is the behavioral one, manifested in the decision the subject makes or would make. Based on research, both processes happen quickly and are often hard to tell apart, making it seem like they occur at the same time. However, it has been shown that they are actually distinct processes influenced by different biases. There is a dissociation between the two processes, and they are not necessarily consistent with each other[8][9][10][11].

In this study, we share the outcomes of an experiment where the MHP was introduced to a group of participants. The experiment included a pre- and post-class on probability, where participants had to

answer questions related to assessing advantageous decisions and the decisions they would actually make.

The objectives of this study are threefold:

1. to analyze whether there is a change in the assessment after individuals acquire some probability knowledge,
2. to examine whether there is a change in potential decision-making after the acquisition of some probability knowledge, and
3. to assess the consistency between the assessment and decision-making processes, both separately and by comparing before and after the acquisition of some probability knowledge.

2. Methods

The participants were students of the Introduction to Biostatistics course, which was part of the Postgraduate Master of Public Health program offered by the School of Public Health at the University of Chile, during the first semester of 2017. They were exposed to the MHP on two occasions, before and after studying probability in the course. The problem was presented and explained to the entire class, and the responses were collected using a Google Form.

2.1. First occasion

During the first class of the semester, on April 29, there were 48 students present in the classroom. All of the students agreed to participate. However, one student indicated that they were already familiar with the problem being discussed and was therefore not included in the responses. The survey included 47 respondents with a median age of 32 and a range from 23 to 61 years old. Of these respondents, 26 (55%) were women. The students were presented with a slide containing the following statement and questions in class (see section A of the supplementary material for the original Spanish version):

> You're on a game show, and you're given the choice of three doors: behind one door is a car and behind the others, consolation prizes[1]. You choose the first door, and the host (he knows what door the car is behind) opens the third one, which has a consolation prize. The host asks to you "Do you want to change to the second door?"
>
> Question 1: Which of the following alternatives you think has more chances of winning?
>
> - Alternative 1: Keep the first door

- Alternative 2: Change to the second door
- Alternative 3: It doesn't make any difference

Question 2: What would you do?

- Alternative 1: Keep the first door
- Alternative 2: Change to the second door

In order to participate, students were required to fill out a Google Form individually (see section B of the supplementary material for the original Spanish version). The form asked whether or not they accepted to participate. If they agreed, they were instructed to provide information about their date of birth, gender, nationality (Chilean or Other), age, and the last four digits of their identification document. After providing this information, they were asked to select the appropriate answers based on the former questions. Before the two questions, the following indication was added:

> Listen carefully to the presented issue and choose the alternative according to the decision you would make.
> IMPORTANT: Don't overthink it, imagine that you have a few seconds to answer, so trust your intuition. If you already know this problem, please refrain from answering and instead select the appropriate option.

The participants responded in less than five minutes during the activity that lasted less than 30 minutes.

2.2. Second occasion

The second part was conducted on June 8. On this occasion, 49 students attended, and all agreed to participate. None indicated prior knowledge of the problem, and all responded.

The median age was 32 years old, in a range from 25 to 61 years old. Twenty-seven (55%) participants were female.

The statement was the same as on the first occasion, except that here the concept of "probability" was included in question 1 and in its alternative 3 (for the original Spanish version, see section C of the supplementary material):

> Question 1: Which of the following alternatives do you think has a higher probability of winning?

- Alternative 1: Keep the first door
- Alternative 2: Change to the second door
- Alternative 3: Both doors have equal probability

Question 2: What would you do?

- Alternative 1: Keep the first door
- Alternative 2: Change to the second door

In a similar manner to the first occasion, the participants filled out a Google Form (refer to section C of the supplementary material) providing the same information. The only difference was that the problem statement was displayed in the form, rather than on the classroom screen. The entire activity took less than 30 minutes, and the participants completed the form in under five minutes. After the statement and before the questions, the following indication was added:

> IMPORTANT: Apply what you know about probability to answer. If you are already familiar with the question, refrain from answering and choose the corresponding option.

3. Results

Table 1 shows the global results for both occasions. During the first occasion, out of a total of 47 individuals, 27 believed that keeping the first door had a greater chance of winning, 5 indicated that switching doors was more favorable, and 15 stated that the choice did not matter. The results obtained from the responses show a significant deviation from the expected one-third proportion for each alternative (multinomial test, $p < 0.01$). This indicates that the results were not by chance. Regarding the second question, out of the 47 participants, 43 (91%) said that they would keep the first door. The probability of this occurring by chance was extremely low (one-sided binomial test, $p < 0.01$).

Question	Answer	Occasion	
		First	Second
Which of the following alternatives do you think has more chances/probability of winning?	Keep the first door	27	3
	Change to the second door	5	4
	It doesn't make any difference / Both doors have equal probability	15	42
What would you do?	Keep the first door	43	45
	Change to the second door	4	4

Table 1. Answers to questions on both occasions.

On the second occasion, out of 49 subjects, 42 (86%) responded that both options have an equal probability of winning, while 3 and 4 responded that keeping and changing the door have more chances to win, respectively. The proportion of responses for the first and third alternatives showed a significant difference compared to the first occasion (Fisher's test, $p < 0.01$). However, there was no significant difference in the second (and correct) alternative (Fisher's test, $p = 0.738$). Overall, the distribution of responses across all three alternatives showed a significant difference between both occasions (Fisher's test, $p < 0.01$). For the second question, 45 out of 49 subjects (92%) indicated that they would stick with the first door. This result was not attributed to chance (binomial test; $p < 0.01$), and there was no significant difference from the first occasion (Fisher's test; $p = 1$).

The results for questions from respondents who were present on both occasions (paired sample) are presented in Table 2. The changes in the distribution of responses had the same direction as those assuming independent samples, and there were significant differences (as determined by McNemar's test; $p < 0.01$).

Question	Answer	Occasion	
		First	Second
Which of the following alternatives do you think has more chances/probability of winning?	Keep the first door	22	2
	Change to the second door	3	3
	It doesn't make any difference / Both doors have equal probability	10	30
What would you do?	Keep the first door	33	32
	Change to the second door	2	3

Table 2. Answers to questions from respondents who were present on both occasions.

To compare how each group responded in the same opportunity, Tables 3 and 4 present responses to the same questions asked on the first and second occasions, respectively. In Table 3, we observe that all individuals who answered that keeping the first door had more chances of winning also stated that they would stick to their initial choice. Two out of the 5 respondents who reported that it was more convenient to change their choice actually changed their decision. Out of the 15 subjects who responded that it did not make any difference, 13 stated that they would keep the first door. This indicates that there is evidence to suggest that this proportion is not by chance (two-sided binomial test, $p < 0.01$).

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Which of the following alternatives do you think has more chances of winning?	What would you do?	
	Keep the first door	Change to the second door
Keep the first door	27	0
Change to the second door	3	2
It doesn't make any difference	13	2

Table 3. Answers to both questions on the first occasion.

Table 4 indicates that on the second occasion, all participants who initially chose the first or second alternative in the first question also chose the corresponding alternative in the second question. Additionally, all respondents who believed that both doors had an equal probability of winning indicated that they would stick with their initial choice of the first door.

Which of the following alternatives do you think has a higher probability of winning?	What would you do?	
	Keep the first door	Change to the second door
Keep the first door	3	0
Change to the second door	0	4
Both doors have equal probability	42	0

Table 4. Answers to both questions on the second occasion.

4. Discussion and conclusions

The counterintuitive nature of the MHP is reflected in the answers to the first question, on both the first and second occasions. On the first occasion, only 5 out of 47 subjects answered correctly that it would be

more convenient to change the door. This proportion did not significantly change on the second occasion, where 4 out of 49 respondents answered correctly. These proportions are consistent with those obtained in other research on this problem[6][8][11][12][13].

Incorrect answers show significant differences between both occasions. It is striking that when the contents of probability had not been taught in class, more than half (57%) of the subjects responded that it was more advantageous to stick with the first door, while approximately one-third (32%) stated that either option was acceptable. However, the latter answer increased to more than twice that (86%) after the contents were introduced. This would indicate that when people have less knowledge about probability, they tend to incorrectly assess the chances of winning by sticking to their first choice, which is the most intuitive inclination[4][14]. After acquiring some knowledge for computing the probability of winning, participants tend to evaluate both alternatives as equally probable, which is consistent with other studies[8][11]. In other words, this suggests that individuals without much knowledge of probability are more likely to rely on intuition (i.e., think that sticking to the first choice is more advantageous), whereas those with more knowledge tend to consider both options as equally likely. As an example of an extreme case of knowledge about probability, we can recall the avalanche of letters, many of which were quite offensive, strongly criticizing Marilyn vos Savant after she gave the right answer in Parade magazine[4][5], where several experts in the area stated that both choices had a 50% probability of winning.

Regardless of the assessment of the more advantageous alternative, on both occasions, more than 90% of the subjects pointed out that they would stick to the first door, without a statistical difference. Furthermore, there were some differences in the consistency of responses between the two occasions. In the first instance, those who believed that staying with the first door had more chances of winning said they would do so, while those who thought that changing was more advantageous were divided between both choices. This behavior was similar in the second instance, although individuals who answered one of the first two alternatives in the first question were consistent with the decision they would make. Not surprisingly, on the first occasion, most of the participants (13 out of 15) who believed that sticking to or changing the door had an equal chance of winning responded that they would stick with their first choice. On the second occasion, all 42 participants who stated an equal probability of winning responded that they would not switch doors.

These findings align with the typical behavior of individuals as reported by other researchers. The tendency to stick with the initial decision, even if switching would be more beneficial, is indicative of

various cognitive biases such as the endowment effect, illusion of control (or gambler's fallacy), *status quo* bias, loss aversion, and anticipated regret. The endowment effect is a cognitive bias in which individuals tend to value something more highly simply because they own it. This phenomenon has been proposed as an explanation for the MHP, where participants may mistakenly believe that the initially chosen door has a higher probability of winning, even after the host opens an empty door. This belief is often rooted in an illusion of control and a sense of ownership over the initial decision[9][13][14].

The *status quo* bias is a phenomenon where people tend to resist change and stick to their initial decision even if it is not the most rational or advantageous one. This bias is often linked with the endowment effect, and some authors treat them together[4][9].

Participants often experience anticipated regret when they imagine the possible outcomes of opening the second door, which can be more distressing if they had initially chosen the first door and the prize was behind it. This effect is known as "a negative consequence incurred by inaction causes less regret than the same negative consequence incurred by action"[6]. This may even lead people, after making the decision to act, to unconsciously strategize ways to convince themselves that the result is not so bad, in order to reduce dissonance[14][15].

In connection with the previous discussion, particularly with regard to regret and how to avoid it, the higher the value of what a person possesses (or could possess), the greater the subjective cost of its potential loss. This means that changing doors and losing the prize is perceived as a greater loss than not changing doors and losing anyway. As a result, the effect of loss aversion comes into play, where the decision is based on avoiding an outcome that would result in a higher subjective loss[9][16][17].

It could be considered a strength of this research that a group was evaluated in two different conditions: once before and once after acquiring basic knowledge of probability within a period of just over one month. This would allow for evidence to be obtained regarding the incorrect assessment of the probability of winning while staying or changing doors in both conditions. However, in the first condition, the error was apparently driven by intuition, while in the second condition, it would have been due to a classical incorrect computation of equiprobable alternatives. The results indicate that the decision to stick with a particular choice remains unchanged despite prior knowledge (or belief) about the probabilities of winning associated with each option. This aligns with other research studies that suggest a general tendency to stick with one's initial choice.

This study has important limitations that need to be taken into consideration. Firstly, the sample size is small in comparison to other studies in the same field, which can lead to the known possibility of a non-significant result or a "false rejection" despite the low p-values. Secondly, the subjects who participated in the study had a particular profile: all of them were postgraduate students, professionals, and had a median age of 32. It is expected that these two characteristics may have impacted the external validity of the results and the possibility of generalization. This is because there is no certainty regarding the representativeness of the general population. The MHP is a well-known probability puzzle. However, one limitation of conducting experiments on this problem is the possibility that some participants may already know the answer. To control for this, it was checked, and those who knew the problem beforehand were excluded. Only one person indicated knowing the problem on the first occasion and did not answer the question. While it is possible that some participants may have searched for the problem after the first occasion and knew the answer on the second occasion, no one declared knowing it. Both occasions were conducted by surprise, without prior notice, and participants were not informed that there would be a second occasion. Despite the limitations, the results are consistent with the findings of other researchers regarding probability assessment and decision behavior, as well as the biases that explain them.

Statements and Declarations

Funding

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Conflicts of Interest

No potential competing interests to declare.

Ethics

This study was conducted in accordance with the ethical principles of the Declaration of Helsinki. Informed consent was obtained from all individuals who participated in the study by means of a consent question in the response form. Given that the research involved a low-risk, non-invasive educational activity conducted as part of a regular course curriculum with consenting adults, formal ethics committee approval was not required under the institutional guidelines at the time of the study.

Data Availability

The anonymized dataset generated and analyzed during the current study is available at https://github.com/MFuentesAl/Monty_Hall.

Author Contributions

M.F-A. was the sole author and was responsible for the conceptualization, methodology, formal analysis, investigation, and writing of the manuscript.

Footnotes

[1] It is known that in the original problem there is a goat behind doors without a prize, but that term is not familiar to Chilean people. The equivalent in Chile for a goat in a contest is a “consolation prize”.

References

1. ^*Gardner M (1959). "Mathematical Games: Problems Involving Questions of Probability and Ambiguity." Sci Am. **201**(4):174–182. doi:10.1038/scientificamerican1059-174.*
2. ^*Selvin S (1975). "A Problem in Probability." Am Stat. **29**(1):67.*
3. ^*Selvin S (1975). "On the Monty Hall Problem." Am Stat. **29**(3):134.*
4. ^a, b, c, d*Granberg D, Brown TA (1995). "The Monty Hall Dilemma." Pers Soc Psychol Bull. **21**(7):711–723. doi:10.1177/0146167295217006.*
5. ^a, b, c*vos Savant M (1996). The Power of Logical Thinking: Easy Lessons in the Art of Reasoning, and Hard Facts About Its Absence in Our Lives. New York: St. Martin’s Press.*
6. ^a, b, c*Rosenhouse J (2009). The Monty Hall Problem: The Remarkable Story of Math’s Most Contentious Brain Teaser. New York: Oxford University Press.*
7. ^*Kahneman D, Slovic P, Tversky A (1982). Judgement Under Uncertainty: Heuristics and Biases. New York: Cambridge University Press.*
8. ^a, b, c*Franco-Watkins A, Derks P, Dougherty M (2003). "Reasoning in the Monty Hall Problem: Examining Choice Behaviour and Probability Judgements." Thinking & Reasoning. **9**(1):67–90. doi:10.1080/13546780244000114.*
9. ^a, b, c, d*Morone A, Caferra R, Casamassima A, Cascavilla A, Tiranzoni P (2021). "Three Doors Anomaly, “Should I Stay, Or Should I Go”: An Artefactual Field Experiment." Theory Decis. **91**(3):357–376. doi:10.1007/s11238*

-021-09809-0.

10. ^Saenen L, Heyvaert M, Van Dooren W, Schaeken W, Onghena P (2018). "Why Humans Fail in Solving the Monty Hall Dilemma: A Systematic Review." Psychol Belg. **58**(1):128–158. doi:10.5334/pb.274.
11. a, b, cSaenen L, Van Dooren W, Onghena P (2015). "A Randomised Monty Hall Experiment: The Positive Effect of Conditional Frequency Feedback." Thinking & Reasoning. **21**(2):176–192. doi:10.1080/13546783.2014.918562.
12. ^Bennett KL (2018). "Teaching the Monty Hall Dilemma to Explore Decision-Making, Probability, and Regret in Behavioral Science Classrooms." Int J Scholarsh Teach Learn. **12**(2). doi:10.20429/ijsotl.2018.120213.
13. a, bFriedman D (1998). "Monty Hall's Three Doors: Construction and Deconstruction of a Choice Anomaly." Am Econ Rev. **88**(4):933–946.
14. a, b, cGranberg D, Dorr N (1998). "Further Exploration of Two-Stage Decision Making in the Monty Hall Dilemma." Am J Psychol. **111**(4):561–579. doi:10.2307/1423551.
15. ^Gilovich T, Medvec VH, Chen S (1995). "Commission, Omission, and Dissonance Reduction: Coping With Regret in the "Monty Hall" Problem." Pers Soc Psychol Bull. **21**(2):182–190. doi:10.1177/0146167295212008.
16. ^Kahneman D (2011). Thinking, Fast and Slow. New York: Farrar, Straus; Giroux.
17. ^Kahneman D, Tversky A (1979). "Prospect Theory: An Analysis of Decision Under Risk." Econometrica. **47**(2):263–291. doi:10.2307/1914185.

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Declarations

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