Review of: "Horizon and curvature"

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Potential competing interests: No potential competing interests to declare.

Alain,

please find below my comments to your article “Horizon and Curvature”.

General remarks:

C1 – My understanding is that your article is about finding a relation between the curvature of a planar compact curve gamma and the horizon (a set of two points on gamma) that an observer placed at height h can see.

This should be made clear either somewhere in the article or adding more details in the propositions.

For example, to understand theorem 3.3 I had to go through the prove and understand it. The first time I read it I thought gamma was an horizon line and the observer was in a point M somewhere on the concave set for which gamma was the horizon. Moreover when you say “rho(M) is the curvature at point M”, I thought you were talking about Gaussian curvature of the set for which gamma was the boundary.

It would have been enough to say that gamma is a planar curve, that M is on gamma, that rho(M) is the curvature of gamma (mentioning explicitly gamma) in M and I would have understood the proposition immediately.

C2 – You switch between arguments on curves and surfaces (for example you talk about curvature of a surface in corollary 3.4) and although you do not say it explicitly, it seems that your results are applicable to both and results on a curve can be extended to surfaces. I am not sure this is true.

For example given the following setup: you take a cone of height A, you split it In two along any plane containing the height, you glue it to a sphere of radius R>>A, you smoothen the sharp edges to get a smooth surface and you choose a point M on the half cone where to place the observer. For h<< A the curvature of gamma will give you the directional curvature of the surface on the half cone but for h>>A the curvature of gamma will not be related to the curvature of the sphere because it is not a maximum circle on the sphere. This should be mentioned somehow.

Specific remarks:

C3 – The setup and the proof for proposition 3.2 seems too complex to me. I propose a different proof:

Let gamma be the boundary of a square with the four edges of length L. We smoothen the 4 vertices and we get a smooth curve. For any given number, let say 1.5, we can find h such that H<=1.5L for any point on gamma. Since L and Ho are unrelated, we can choose an L such that:

H<2Ho, H<3Ho, H<4Ho or even H<kHo with any k>1.5.

C4 – Proof of proposition 3.2. In F(-L)=F(L)=0, what is L?

C5 – Proof of proposition 3.2. Why the horizon is reduced to 0 for summits? After you smoothen the curve it can be arbitrarily small, before you smoothen it you cannot define a tangent to the curve on the corners and you can choose any tangent between the one on the right and the one on the left edge. Whatever tangent you choose the horizon distance is not zero.

C6 - Proof of proposition 3.2. The sentence “the distance of horizon for a point lying on the smoothed curve remains less of the maximum of lengths of the faces”, does not seems true to me. And therefore the next sentence is not correct and actually you do not need to take epsilon to zero (see also my comment C3).

C7 – Proof of proposition 3.3 is very nice. I like it’. However, the sentence: “we set H^2(h,m=H” is not understood. You do not need it in the rest of the proof and therefore I guess it is a typo.

C8 – Section 4.2 “Which extension can we expect?”. For very small h you get always to a point where your horizon on a surface is a perfect ellipse and by measuring the horizon along the two axis you get the two main directional Gaussian curvatures. So h cannot be too small!!

But if h is grater of any L where the above I said is not true, the curvature of gamma is not related to the directional curvature of the surface (see my comment C2) anymore and, most of all, there will always be small patches of surfaces smaller then L which are flat! The main problem to detect those flat patches is with your proof of proposition 3.2 where you assume the length of the edges of the curve are related to Ho while they are not. See my comment C3. For the above reasons, I see the extension you propose to be very unlikely to be true.