

Review of: "Quaternion Quantum Mechanics: Unraveling the Mysteries of Gravity and Quantum Mechanics within the Planck-Kleinert Crystal"

Quentin Ansel¹

¹ Université de Bourgogne

Potential competing interests: No potential competing interests to declare.

The topic of this paper is interesting and it offers a new point of view to ideas that have been more or less investigated by other means (other mathematical point of view).

I had a quick look at the first version of the paper, and the second version is better. However, I have a few questions/remarks that may be considered by the authors. Since this aim of this paper is to summarize and introduce the ideas of QQM, I'm not going into the mathematical details and the validity of the computations.

- I'm not sure to understand well one of first (and important) sentence of the paper: "Besides that, we try to convince reader that the quaternions $(q_0, q_1, q_2, q_3) = (q_0, q^A)$ can be considered a physical reality in the same sense as the four-dimensional time-space continuum $(t, x, y, z) = (t, \vec{u})$ ". If I understand well, the idea is to change the algebra of the coordinate system such that it becomes non-commutative? If this is correct, I suggest reformulating the sentence, to avoid any misunderstanding.
- The idea of introducing non-commutativity in space-time (or æther, call it as you want) is not new, and it as already been deeply investigated. I can recommend the recent work of David Viennot (UTINAM institut) or the work of Alain Connes (see e.g. "Geometry and the quantum, Alain Connes, Foundations of mathematics and physics one century after Hilbert, pp. 159–196, Springer, Cham, 2018.")
- Equation (3) is not very easy to read, I suggest using the align environment. Moreover, why there is a + in the second line?
- I do not understand the sentences "A topic of primary importance is the Cauchy-Riemann derivative D. This operator *"returns us as the observers"* to R^3 ". A few more comments would be helpful.
- Eq. (36) does not correspond to the Dirac equation, it is non-linear. The similitude is closer to the non-linear Dirac equation. This latter is considered as an effective equation and it is not considered as a fundamental one. A few comments on the domain of the validity of this equation could be interesting. For example, shall we consider that standard regimes are obtained when $m \rightarrow 0$? Can it be tested with current experimental data?
- The fact that gravity can be more or less described by inner modification of a medium is not new (I have no specific reference in mind on the subject, but mathematically speaking, this is just differential geometry, so the analogy is straightforward), however, general relativity has very specific features that make clear distinctions with usual solid-state physics. One of the main points is the diffeomorphism invariance of the equations in space-time. Is this property

conserved by QQM ? Is it always possible to identify the operator D to the covariant derivative of space-time manifold?