

# Review of: "Counting Processes with Multiple Randomness: Examples in Queuing Theory"

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**Potential competing interests:** No potential competing interests to declare.

The authors discuss a departure process from a stable, general single-server queue in statistical equilibrium and introduce a counting process with multiple sources of randomness to model the inter-departure time as a random variable. They argue that the inter-departure time cannot be fully described by a single random variable over the entire sample space  $\Omega$ . Instead, they propose dividing  $\Omega$  into subsets  $\Phi_j$  and  $\Psi_j$  to distinguish between the states of the server being idle or busy after the  $j$ -th departure. Using this decomposition, they derive the probability density function (pdf) of the inter-departure time.

The authors criticize the established work by Burke (1956)[2], and they also challenge the results related to queuing networks by Jackson, which are based on Burke's findings.

I disagree with the authors' viewpoint. The inter-departure time  $(\tau_{j+1} - \tau_j)$  can be adequately described as  $I_j \delta_{Q_j, 0} + S_{j+1}$ , with all the variables,  $I_j, \delta_{Q_j, 0}, S_{j+1}$ , being random variables defined over the entire sample space  $\Omega$ . Therefore, there may not be a necessity to introduce a counting process with multiple sources of randomness. As demonstrated by the authors in (3.2), the marginal distribution of  $(\tau_{j+1} - \tau_j)$  exists and is consistent with Burke's results. I cannot agree that Burke's discussion is fundamentally flawed based on the results. The authors seem to offer an alternative perspective on the conditional estimation of the pdf of the inter-departure time.

I do not recommend the publication of the manuscript.