

Review of: "An Analysis of the Continuum Hypothesis"

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Potential competing interests: No potential competing interests to declare.

Summary: It was a pleasure to read this article. It is original and well written. Therefore, it can be published with just a few modifications.

The only reservation I have is with respect to the last remark (page 7). In particular, the sentence "CH* is really a claim that CH* is the same as CH*- " ought to be rewritten. It is also not obvious why CH- should imply CH.

In more detail:

The introduction is excellent and should remain exactly as it is. The author motivates his original ansatz, which focuses on the question whether or not CH is a reasonable assumption. In order to answer the latter question, CH is interpreted as a search algorithm that treats all elements of a given set uniformly (i.e., the algorithm does not depend on any particular element). Consistently, CH is a very strong assumption that implies the Axiom of Choice (AC).

Section 2 is very dense, and lists many definitions that are rather technical. In other words, the manuscript would improve if those definitions were explained in more detail. Moreover, the contemporary edition of Li and Vitanyi's standard work should be cited (3rd, 2008).

Section 3: Information should be emphasized as a crucial concept. There is no need for two remarks, simple text would suffice.

Section 4:

- There does not seem to be a "definition" at the beginning.
- The last sentence in the proof of Proposition 4 actually seems to summarize that proposition. ("Formalization of CH as a search algorithm.")
- Corollary 5 should be reformulated. "Given... then..."
- Proposition 6 is a major result of the article. Thus it should be a "theorem".
- Remark 7 is the only remark with a number.

This reader thinks that it would be useful to present and summarize the main propositions in a concise form:

1. Proposition 1: $CH = \Leftrightarrow CH$
2. Proposition 3: $CH \Leftrightarrow CH^*$
3. Last Remark, p. 4: $CH \Rightarrow AC$
4. Remark 7: $AC \Rightarrow CH^*-$

Therefore, in a single line: $CH = \Leftrightarrow CH^* \Leftrightarrow CH \Rightarrow AC \Rightarrow CH^*$

The last line also demonstrates that "CH* is stronger than CH*" (p. 7). Unfortunately, I do not see why the equivalence $CH \Leftrightarrow CH^*$ should hold.

Moreover, despite Proposition 4, and although CH* is stronger than AC, the author writes: "Thus with CH* there is no [??] natural way to map... The choice function in AC may provide such a mapping."

Typos:

- Page 4, penultimate line: ...that is at least as strong...
- Page 5: the last index before Proposition 4 is rather strange.