Research Article

Relativistic Shedding (RS) as Threshold Emission into Hidden Sectors

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1. Independent researcher

"Relativistic Shedding" (RS) is the threshold turn-on of emission into a weakly coupled eigenmode when a relativistic charge crosses a phase-velocity condition. The original RS motivation was that a missing-momentum experiment (LDMX-like) could bracket such a threshold by toggling a dielectric configuration and searching for an on/off excess. In Version 5 we keep that LDMX language as the historical motivation and as an analysis pattern (ABAB threshold bracketing), *but* we correct the physics mapping: for the canonical MeV–GeV dark-photon mediator regime, an ordinary dielectric does not furnish a subluminal propagating eigenmode at $\omega \gtrsim m$, so the "dielectric light-switch" implementation is kinematically closed. This closure is a mismatch of scales (medium response vs m^2), not an exposure problem.

RS remains a well-defined and testable laboratory mechanism for *light* hidden states (sub-eV) and, more generally, for any weakly coupled eigenmode whose phase velocity can be tuned below c in an engineered structure. Accordingly, this complete Version 5 specifies a stand-alone RS laboratory program: a *thresholded source* built from a tunable slow-wave/dielectric eigenmode excited by a relativistic beam, combined with a resonant "light shining through a wall" (LSW) receiver in the spirit of CROWS/Dark SRF. The RS-specific element is the physics-driven threshold modulation variable (phase-velocity bracketing) and the associated falsifier map.

We provide: (i) a consolidated operational definition of RS and its minimal experimental signatures; (ii) expanded derivations of threshold conditions in both homogeneous-media and guided-mode language; (iii) the in-medium effective mixing formalism and its relevant limits; (iv) a concrete "where to look" procedure in a CROWS/Dark SRF-compatible receiver; (v) a worked sensitivity estimate (radiometer-limited) with an illustrative reach curve; and (vi) built-in falsifiers designed to separate RS-correlated signals from electromagnetic leakage and instrumental lines.

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Update note: Superconducting RF cavity LSW searches have demonstrated kinetic-mixing reach at the $\epsilon \sim 10^{-9}$ level in the μeV mass range [1], with refined modeling of frequency instability/microphonics indicating that existing pathfinder data can imply an order-of-magnitude stronger constraint [2][3]. Longitudinal-mode transmission can parametrically enhance LSW sensitivity compared to the commonly quoted transverse $(m/\omega)^8$ suppression, provided the emitter/receiver geometry is optimized [4][5][6]. Recent "thin-wall" proposals extend regeneration-style searches into the off-shell/evanescent regime $m > \omega$ [7][8].

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1. Motivation and pivot narrative: why LDMX looked ideal, and why a stand-alone lab program is required

1.1. RS as a threshold handle

A relativistic charge traversing matter or a structure can radiate into any eigenmode whose phase velocity is below the charge velocity. For ordinary electromagnetism this yields Cherenkov radiation. RS generalizes the same idea to *weakly coupled eigenmodes*—e.g., a kinetically mixed hidden photon mixed with the in-medium photon—and emphasizes an experimental handle: *threshold bracketing*. If a control parameter can move a system from $\beta < v_{ph}$ to $\beta > v_{ph}$ for a hidden-coupled eigenmode, then the hidden-channel emission should turn on sharply.

1.2. Why LDMX looked like the cleanest "first home"

The original RS draft was structured around a missing-momentum implementation motivated by LDMX. LDMX is an 8 GeV electron-beam, thin-target, missing-momentum experiment designed for high-rate data-taking and strong vetoing [9]. Conceptually this makes it attractive for RS:

1. **ABAB lever arm:** one can toggle a radiator/structure parameter A/B while holding beam and detector conditions fixed, and search for a differential excess.
2. **High statistics:** if the RS-emitted quanta carry event-level missing momentum, the signature can be expressed in the same language as existing missing-momentum analyses.

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3. **Staged scans:** LDMX emphasizes staged operation and scanning strategies ("probe new territory along the way") [9], which matches the RS bracketing mindset.

1.3. Where the original LDMX mapping fails (not sensitivity: kinematics + scales)

The LDMX-motivated mapping implicitly assumed a *heavy* invisible quantum (MeV–GeV scale) so that individual RS emissions could produce detectable missing momentum. In that regime, one needs on-shell propagation:

$$\omega \gtrsim m \qquad \text{(on-shell hidden vector)}. \tag{1}$$

A Cherenkov/RS threshold requires a subluminal phase velocity at that same frequency,

$$\beta > v_{ph}(\omega) \quad \Longleftrightarrow \quad \beta n_{eff}(\omega) > 1. \tag{2}$$

For ordinary dielectrics, substantial refractivity $n(\omega) - 1 \sim \mathcal{O}(10^{-1})$ occurs in the optical/IR. At MeV–GeV energies, $n(\omega) \to 1$ and the medium polarization scale is controlled by plasma/atomic response energies (eV–keV), not by $m \sim MeV{-}GeV$. This can be made quantitative in a minimal way: in an isotropic medium the photon polarization tensor satisfies $\Pi_T(\omega) \simeq \omega^2(1 - \varepsilon_r(\omega))$ (Appendix A). In ordinary matter, $|\Pi|$ is set by ω_p^2 or by atomic form factors, typically $\mathcal{O}(eV^2){-}\mathcal{O}(keV^2)$, whereas for a MeV–GeV mediator $m^2 \sim 10^{12}{-}10^{18}eV^2$. Thus even without invoking small kinetic mixing, the medium response is far too small to reshape the dispersion of an on-shell heavy vector into a subluminal branch.

A complementary statement uses the in-medium effective mixing [10]:

$$\kappa_{T,L}^2(\omega, \mathbf{k}) = \frac{\kappa^2 m_V^4}{\left(m_V^2 - Re\Pi_{T,L}(\omega, \mathbf{k})\right)^2 + \left(Im\Pi_{T,L}(\omega, \mathbf{k})\right)^2}. \tag{3}$$

For $m_V^2 \gg |\Pi|$ one has $\kappa_{T,L} \simeq \kappa$ and the hidden-like eigenmode remains vacuum-like: $n_V(\omega) = |\mathbf{k}|/\omega < 1$ (superluminal phase velocity). Therefore, the specific "ordinary dielectric as a tunable RS radiator for heavy on-shell mediators" is closed. This is a correction to the original mapping; it does *not* invalidate RS as a threshold concept.

1.4. The correct laboratory target space

The RS threshold condition is extremely sensitive for ultra-relativistic charges. Near $\beta \simeq 1$ write $\beta \simeq 1 - 1/(2\gamma^2)$, and for a mode with $n_{eff} = 1 + \delta n$,

$$\delta n_{th} \simeq \frac{1}{2\gamma^2}. \tag{4}$$

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For multi-GeV electrons ($\gamma \sim 10^4$), $\delta n_{th} \sim 10^{-9}$. The experimentally realistic way to access such tiny phase-index offsets is *not* to rely on bulk optical dielectrics at MeV energies, but to use engineered eigenmodes (slow-wave structures, coupled-resonator bands) and narrowband receivers.

Consequently, this Version 5 defines RS as a stand-alone laboratory program matched to *sub-eV* hidden states and engineered dispersion. Resonant LSW experiments provide a demonstrated sensitivity scale in this regime: CROWS reported $\chi = 4.1 \times 10^{-9}$ at $m_{\gamma'} = 10.8\mu eV$ [11]; Dark SRF established $\epsilon \simeq 1.6 \times 10^{-9}$ in the 2.1×10^{-7}–5.7×10^{-6} eV mass range [1], with refined modeling implying an order-of-magnitude stronger bound from the same dataset [2][3]. The role of RS is to contribute a *thresholded source modulation* and falsifier map that is orthogonal to simply turning the RF drive on/off.

1.5. The 10^{-9} phase-index scale: what it means, what it does not mean, and how to bracket it in practice

Equation (4) is often emotionally "too small" on first reading, so it is worth being explicit about interpretation. For $\gamma \gg 1$ the Cherenkov/RS threshold for a mode whose phase index is close to unity is set by a *tiny* refractivity offset: $n_{eff} - 1 \gtrsim 1/(2\gamma^2)$. At $8GeV$, $\gamma \simeq 1.57 \times 10^4$ and the threshold corresponds to $n_{eff} - 1 \simeq 2.0 \times 10^{-9}$.

What this does not mean.

It does *not* mean that RS intrinsically requires the experimenter to *tune* a bulk material index at the 10^{-9} level. It means that if one insists on multi-GeV electrons *and* one insists on a mode with $n_{eff} \approx 1$, then the turn-on is a knife-edge. In a stand-alone RS laboratory program one is free to choose the beam energy and the structure dispersion so that the threshold occurs at a tunable, well-measured detuning.

A practical bracketing strategy (beam energy and dispersion are both knobs).

If the beam is less relativistic, the required index offset is much larger. For example: at $E_e \sim 60MeV$ (a scale used in dielectric wakefield tests [12]), $\gamma \sim 120$ and $1/(2\gamma^2) \sim 3 \times 10^{-5}$. Thus the RS ABAB bracketing variable can be made experimentally accessible by choosing a moderate-energy beam and a guided/Bloch mode engineered to have $n_{eff}(\omega)$ near $1/\beta$. Equivalently, one can hold the structure fixed and scan beam energy (or bunch spacing) across synchronism.

 doi.org/10.32388/S6SGLM

Why Version 5 does not rely on bulk γ-optics.

At hard X-ray and γ-ray energies the refractive index deviation $\delta \equiv n - 1$ is known to be extremely small ($|\delta| \sim 10^{-6}$–10^{-9}) and experimentally challenging. There has been an instructive history in the MeV regime: Habs *et al.* reported a positive refractive index contribution in silicon up to 2MeV and discussed Delbrück-scattering effects [13]. Subsequent work reanalyzed the earlier campaign and performed new measurements, finding consistency with the classical scattering model and attributing the earlier apparent sign change to systematic diffraction effects of the prism [14][15]. Regardless of the final sign story, the lesson for RS is that *bulk* refractivity at MeV energies is a difficult substrate for a decisive threshold experiment.

Therefore, the laboratory RS program in this paper is built on *engineered* eigenmodes at RF/THz frequencies, where dispersion can be measured and tuned with standard RF techniques, and where beam-driven dielectric structures have a strong literature base [16][12][17]. Coupled-resonator waveguides (CROWs) provide an additional route to slow-light/slow-wave dispersion engineering in integrated platforms [18][19].

2. Notation, conventions, and scope

2.1. Units and frequency conventions

We use natural units $\hbar = c = 1$ unless stated. We distinguish:

$$\omega \equiv 2\pi f \quad (\text{angular frequency}), E = \hbar\omega \quad (\text{energy}). \tag{5}$$

When writing ratios like (m/ω) we mean m and ω expressed in the same energy units (eV). Appendix D lists common conversions (e.g. 1 GHz $\approx 4.14\mu$eV in hf units, and $\hbar\omega = 2\pi\hbar f$).

2.2. Parameter space and what RS does (and does not) claim

Throughout we use κ or χ interchangeably for the vacuum kinetic mixing of a hidden photon; in the LSW literature χ is common. RS is an *emission/measurement concept*:

- RS does *not* claim a new fundamental interaction beyond standard hidden-photon kinetic mixing.
- RS does *not* claim to supersede the conversion formalism used by LSW experiments; we cite and use it.
- RS *does* claim a new *threshold-modulated source architecture* and the corresponding falsifier/analysis strategy.

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3. Definition of RS and minimal experimental signatures

3.1. Operational definition (one-sentence + one observable)

Relativistic Shedding (RS) is defined as:

> A relativistic charge emits into a weakly coupled eigenmode whose phase velocity is below the charge velocity; RS is the *threshold turn-on* of that hidden-channel emission when a control parameter is tuned across the phase-velocity condition.

In the stand-alone laboratory program of this paper, the primary observable is a *receiver power excess* at a narrowband frequency f_{sys},

$$\Delta P \equiv P_B(f_{sys}) - P_A(f_{sys}), \tag{6}$$

where (A, B) are below/above-threshold configurations for the *source eigenmode*.

3.2. Minimal RS signature set

A credible RS signal must satisfy *all* of the following:

1. **Threshold correlation:** the excess appears only when the independently measured phase-velocity condition $\beta n_{eff}(f_{sys}; \lambda) > 1$ is satisfied.
2. **ABAB reversibility:** in an $A \to B \to A \to B$ sequence the effect tracks the state, not time.
3. **Resonant selectivity:** detuning the receiver by $\Delta f \gg f/Q_{\mathrm{det}}$ removes the line; retuning restores it.
4. **Leakage discrimination:** at least one shielding/geometry falsifier behaves as predicted for hidden-sector transmission and not as generic EM pickup (Section 8).

3.3. Why RS is not "just lock-in LSW"

Conventional LSW searches modulate the source power ("RF on" vs "RF off") and search for regenerated photons in a receiver cavity [20][11][1]. RS instead proposes:

> hold the hardware and beam present, but modulate only whether a specific source eigenmode is ***kinematically excitable*** by crossing a phase-velocity threshold.

That is a physics-driven modulation variable: the toggle is predicted from independently measured dispersion, and it should produce a sharp turn-on with a predictable near-threshold scaling (Section 4.4). This is the RS-unique contribution.

doi.org/10.32388/S6SGLM

3.4. What is new here and what is not (attribution and independence)

RS sits on top of well-established pieces of physics and experimental technique. To avoid ambiguity about novelty and attribution, we separate the ingredients:

- **Not new: kinetic mixing and in-medium mixing formalism.** The hidden-photon portal, its in-medium effective mixing, and the resonant regeneration idea are standard [20][10][21].
- **Not new: resonant LSW implementation.** CROWS and Dark SRF (and related SRF proposals) established the experimental language, geometry factors, and shielding practices for cavity regeneration searches [11][1][22].
- **Not new: slow-wave / dielectric beam structures.** Dielectric-lined waveguides and wakefield structures have long been studied and demonstrated as coherent narrowband sources and accelerators [16][12][17]. Coupled-resonator optical waveguides (CROWs) are a mature platform for dispersion engineering and slow light [18][19]. (*Note:* CROWs are not to be confused with CERN CROWS.)
- **New (RS contribution): threshold-modulated hidden-channel search architecture.** RS proposes to use a *kinematically defined* on/off condition (phase-velocity bracketing or synchronism detuning), measured independently from the receiver, as the primary modulation variable for a hidden-sector regeneration search. This yields a falsifier-rich ABAB program that is distinct from simply modulating RF power.

4. Threshold emission: from Cherenkov in a medium to synchronism in a structure

4.1. General threshold condition

For an eigenmode with dispersion relation $\omega(\mathbf{k})$, define its phase velocity

$$v_{ph}(\omega) \equiv \frac{\omega}{|\mathbf{k}(\omega)|}, n_{eff}(\omega) \equiv \frac{|\mathbf{k}(\omega)|}{\omega}. \tag{7}$$

For a charge with speed $v = \beta$ (in $c = 1$ units), RS emission requires

$$\beta > v_{ph}(\omega) \quad \Longleftrightarrow \quad \beta n_{eff}(\omega) > 1. \tag{8}$$

4.2. Homogeneous medium language (Cherenkov-like)

In a homogeneous medium approximated by a refractive index $n(\omega)$, the angular condition becomes

doi.org/10.32388/S6SGLM

$$\cos\theta(\omega) = \frac{1}{\beta n(\omega)}, \tag{9}$$

and emission exists only if the right-hand side is ≤ 1. Near threshold one has the Frank–Tamm scaling (rescaled by an effective coupling) [20]:

$$\frac{d^2N}{dx d\omega} \propto \left(1 - \frac{1}{\beta^2 n^2}\right)\Theta(\beta n - 1). \tag{10}$$

4.3. Guided/periodic structure language (synchronism and detuning)

For a guided or periodic slow-wave structure, the natural language is synchronism with the beam line. Write the mode dispersion as $\omega = \omega(k_z; \lambda)$, where k_z is the longitudinal wave number and λ is a tunable parameter (geometry, dielectric constant, temperature, etc.). A charge moving along z with speed v has a Fourier component that enforces the phase-matching condition (Appendix B)

$$\omega = k_z v. \tag{11}$$

Efficient excitation occurs when the beam line intersects the mode dispersion, equivalently when $v_{ph}(\omega) = \omega/k_z \leq v$. Detuning away from synchronism suppresses excitation over the formation length.

This is the engineering form of RS: the "index" is an effective phase index of a guided/Bloch mode, not the bulk index of a homogeneous material.

4.4. Near-threshold scaling and the RS control parameter

Define a near-threshold detuning parameter

$$\Delta(\omega; \lambda) \equiv \beta n_{eff}(\omega; \lambda) - 1. \tag{12}$$

For $|\Delta| \ll 1$ the standard Cherenkov factor linearizes:

$$1 - \frac{1}{\beta^2 n_{eff}^2} \simeq 2\Delta + \mathcal{O}(\Delta^2). \tag{13}$$

Thus, if a control parameter λ moves Δ from negative to positive while holding other conditions fixed, the signal turns on with a predictable linear scaling in $(\lambda - \lambda_{th})$ up to damping/absorption effects (Appendix A).

5. Photon–hidden photon eigenmodes and effective mixing

5.1. Minimal kinetic-mixing model

We take the standard low-energy Lagrangian for a hidden photon V_μ with kinetic mixing κ and mass m_V,

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{\kappa}{2}F_{\mu\nu}V^{\mu\nu} + \frac{m_V^2}{2}V_\mu V^\mu + eJ^\mu_{em}A_\mu. \quad (14)$$

General background and conventions can be found in dark-photon reviews [23][24].

5.2. In-medium effective mixing and useful limits

In a medium, photon propagation is modified by the polarization tensor $\Pi_{T,L}(\omega, \mathbf{k})$. Production/absorption rates can be expressed in terms of an *effective in-medium mixing* [10]:

$$\kappa^2_{T,L}(\omega, \mathbf{k}) = \frac{\kappa^2 m_V^4}{\left(m_V^2 - Re\Pi_{T,L}(\omega, \mathbf{k})\right)^2 + \left(Im\Pi_{T,L}(\omega, \mathbf{k})\right)^2}. \quad (15)$$

Three regimes:

1. **Vacuum-like / heavy mass:** $m_V^2 \gg |\Pi| \Rightarrow \kappa_{T,L} \simeq \kappa$.
2. **Screened / light mass:** $m_V^2 \ll |\Pi| \Rightarrow \kappa_{T,L} \simeq \kappa m_V^2/|\Pi|$.
3. **Resonant mixing:** $m_V^2 \simeq Re\Pi$ can enhance mixing (limited by $Im\Pi$).

5.3 Hidden-like phase index shift at small mixing

In vacuum, the massive hidden photon has $k^2 = \omega^2 - m_V^2$, implying a phase index $n_{vac} = k/\omega < 1$. In a medium, mixing repels eigenvalues and the hidden-like eigenmode inherits a small fraction of the ordinary refractivity. Away from resonance and for $\kappa \ll 1$, an engineering-level approximation is

$$n_V(\omega) - 1 \sim \mathcal{O}\left(\kappa_T^2(\omega)\right)(n_\gamma(\omega) - 1), \quad (16)$$

with the proportionality controlled by Eq. (15). This is the quantitative statement behind the RS intuition: a very weakly coupled eigenmode can be *slightly* slowed, and ultra-relativistic charges are sensitive to extremely small slowing.

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6. Retaining the LDMX interface for context (and documenting the corrected conclusion)

6.1. What an LDMX-style RS search would look like (if the channel existed)

An LDMX-style RS search would: (i) choose a tunable parameter λ that moves a hidden-channel threshold across the beam energy, (ii) collect alternating A/B datasets, and (iii) search for a missing-momentum excess in B only. The strength is cancellation of smooth backgrounds in $B - A$.

6.2. Why the canonical MeV–GeV mediator benchmark does not match "RS-in-ordinary-dielectric"

For RS-in-ordinary-dielectric emission to generate a canonical LDMX mediator one needs simultaneously: (i) on-shell emission $\omega \gtrsim m$, (ii) a subluminal eigenmode at that same ω, and (iii) a medium with $n_{eff}(\omega) - 1$ large enough to satisfy Eq. (4). Ordinary dielectrics cannot supply this at MeV–GeV frequencies; their significant refractivity sits in the eV regime. Therefore the originally proposed dielectric Cherenkov interpretation does not apply to the standard LDMX heavy-mediator target. This is the corrected conclusion that motivates the laboratory pivot.

7. A stand-alone RS laboratory program: thresholded slow-wave source + resonant regeneration receiver

7.1. Why resonant regeneration is the right laboratory sensitivity scale

Cavity LSW experiments are designed to probe extremely small couplings using high quality factors and narrowband detection [20][11][1]. Modern SRF technology and frequency-stability modeling have made $\epsilon \sim 10^{-9}$ a demonstrated laboratory scale [1][2][3]. SRF cavity technology is also broadly recognized as a platform for new-particle searches [22][5].

7.2. Regenerated power: transverse vs longitudinal scaling

A key update since early LSW discussions is that the scaling with hidden-photon mass depends on polarization and geometry. For generic transverse-to-transverse configurations (often conservative), the regenerated power scales schematically as

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$$P_{\text{det}}^{(T)} \sim \chi^4 \left(\frac{m_{\gamma'}}{\omega}\right)^8 |G_T|^2 Q_{em} Q_{\text{det}} P_{em}, \tag{17}$$

where $\omega = 2\pi f_{sys}$, Q_{em}, Q_{det} are loaded quality factors, P_{em} is emitter power, and G_T is a geometry factor [11].

Longitudinal-mode transmission can yield parametric enhancement. In a "well-positioned" geometry, the regenerated field can scale as $E_{rec} \sim \epsilon^2 (m_{\gamma'}/\omega)^2 E_{em}$ [4][6], implying a milder power suppression,

$$P_{\text{det}}^{(L)} \sim \chi^4 \left(\frac{m_{\gamma'}}{\omega}\right)^4 |G_L|^2 Q_{em} Q_{\text{det}} P_{em}, \tag{18}$$

while off-axis placements revert toward the stronger m^8 behavior [5].

7.3. RS-specific source concept: engineered eigenmode with a phase-velocity threshold

To turn an LSW receiver into an RS search, the source must be *threshold-modulated* by physics rather than by simply turning RF on/off. We propose a source based on an engineered eigenmode with tunable phase velocity, excited by a relativistic beam:

1. **RS radiator/source:** a dielectric-lined waveguide, periodic slow-wave structure, or coupled-resonator chain traversed by a relativistic electron beam. These structures support narrowband eigenmodes with tunable dispersion and phase velocity. Beam excitation is efficient only when the synchronism condition $\omega = k_z v$ is satisfied (Appendix B); detuning suppresses excitation.
2. **Wall/shield:** electromagnetic shielding blocks ordinary EM propagation.
3. **Receiver:** a high-Q cavity (CROWS/Dark SRF class) tuned to f_{sys} measures regenerated photons from hidden-sector propagation through the wall.

In this architecture, RS is implemented by tuning a control parameter λ (geometry, temperature, dielectric constant, or beam energy) across the threshold $\beta n_{eff}(f_{sys}; \lambda) = 1$ for the *source eigenmode*.

7.4. Where to look for RS in a CROWS/Dark SRF-compatible receiver

In a resonant receiver, RS manifests as a narrowband excess at f_{sys} that is *correlated* with the source eigenmode crossing threshold. A concrete, publishable procedure:

Step 1: dispersion calibration (independent of hidden physics).

Identify a candidate source eigenmode at f_{sys} and measure/fit its dispersion $\omega(k_z; \lambda)$ or equivalently its phase index $n_{eff}(f_{sys}; \lambda)$ via standard RF/beam diagnostics.

Step 2: define bracketed settings.

Choose (λ_A, λ_B) such that

$$\beta n_{eff}(f_{sys}; \lambda_A) < 1, \beta n_{eff}(f_{sys}; \lambda_B) > 1, \tag{19}$$

with all other operating conditions fixed. This is the RS "light switch": it is determined by dispersion, not by instrument convenience.

Step 3: ABAB acquisition and lock-in analysis.

Alternate A/B states in time (ABAB...) and measure the receiver power spectral density around f_{sys}. Construct a demodulated estimator $\Delta\hat{P}$ at the A/B modulation frequency (Appendix C).

Step 4: apply falsifiers.

Perform at least two orthogonal falsifiers (receiver detune + shielding augmentation, or geometry flip + frequency scan) and require the RS-consistent responses (Section 8).

7.5. Worked sensitivity estimate (radiometer-limited) and illustrative reach

A complete proposal must translate receiver noise into a projected $\chi(m)$ reach. For a narrowband measurement with system noise temperature T_{sys}, effective measurement bandwidth Δf, and integration time τ, the radiometer equation gives a noise power uncertainty [25]

$$\sigma_P \simeq k_B T_{sys} \sqrt{\frac{\Delta f}{\tau}}, \tag{20}$$

so a n_σ detection threshold is $P_{\min} \simeq n_\sigma \sigma_P$. Setting $P_{\det} = P_{\min}$ in Eq. (17) or (18) yields

$$\chi_T(m) \simeq \left(\frac{P_{\min}}{Q_{em} Q_{\det} P_{em} |G_T|^2}\right)^{1/4} \left(\frac{\omega}{m}\right)^2, \tag{21}$$

$$\chi_L(m) \simeq \left(\frac{P_{\min}}{Q_{em} Q_{\det} P_{em} |G_L|^2}\right)^{1/4} \left(\frac{\omega}{m}\right). \tag{22}$$

These expressions make clear (i) the Q and power leverage, and (ii) the importance of longitudinal optimization.

Illustrative numerical example (not a design claim).

Take a representative SRF-scale setup at $f_{sys} = 1.3 GHz$ ($\hbar\omega \simeq 5.4\mu$eV), $Q_{em} = Q_{\det} = 10^{10}$, $P_{em} = 1$ W circulating, $|G|^2 = 10^{-2}$, $T_{sys} = 2$ K, $\Delta f = 1$ Hz, $\tau = 10^5$ s, and $n_\sigma = 5$. Figure 1 shows the resulting illustrative $\chi(m)$ scaling curves for transverse and longitudinal assumptions, alongside the published Dark SRF sensitivity band as a benchmark.

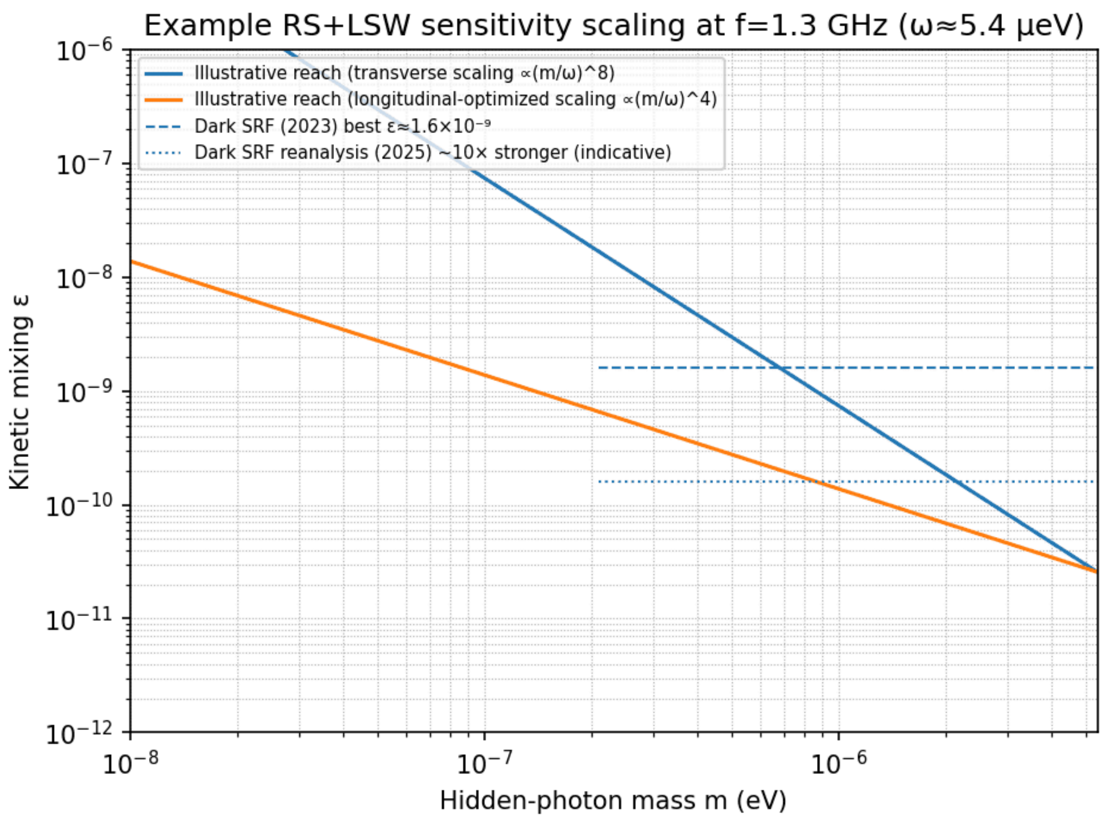

Figure 1. Illustrative RS+LSW reach scaling at $f_{sys} = 1.3GHz$ ($\hbar\omega \simeq 5.4\mu$eV) using the radiometer-limited estimate described in Section 7.5. The transverse curve follows $P_{\rm det} \propto \chi^4(m/\omega)^8$, while the longitudinal-optimized curve follows $P_{\rm det} \propto \chi^4(m/\omega)^4$. Dashed segments indicate the published Dark SRF mass window and best reported mixing scale [1]; the dotted segment indicates the order-of-magnitude tightening inferred in Ref. [2] (schematic). This figure is intended to show scaling and parameter leverage, not to reproduce any published exclusion curve.

7.6. Constraints landscape (laboratory-first)

A concise laboratory constraint summary relevant to the RS program:

- **CROWS (2013):** $\chi = 4.1 \times 10^{-9}$ at $m_{\gamma'} = 10.8\mu$eV [11].
- **Dark SRF (2023):** $\epsilon \simeq 1.6 \times 10^{-9}$ in 2.1×10^{-7}–5.7×10^{-6} eV [1].

- **Dark SRF refined (2025):** order-of-magnitude stronger constraint inferred from improved frequency-instability modeling [2][3].
- **Thin-wall/off-shell extensions:** LSthinW-style concepts extend sensitivity to $m > \omega$ by exploiting evanescent hidden-photon fields across a thin barrier [7][8].

Broader reviews of axion and dark-photon search technologies, including quantum-limited receivers and SRF opportunities, are given in Refs. [22][5].

8. Built-in falsifiers and systematic control

A threshold claim must come with built-in falsifiers. Table 1 lists executable tests in a single apparatus.

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Falsifier / constraint	Expected RS behavior
ABAB threshold scan (λ_A below, λ_B above)	Receiver line at f_{sys} appears only in B; turn-on tracks the independently measured phase-velocity threshold.
Receiver detuning	Detune by $\Delta f \gg f/Q_{\mathrm{det}}$: line disappears. Retune: line returns with consistent scaling.
Shielding augmentation	Add extra EM shield/seal without changing hidden geometry: true hidden signal unchanged; EM leakage decreases.
Geometry flip / alignment	Rotate/translate receiver relative to source: RS changes as predicted by $G_{T,L}$; spurious pickup need not follow.
Distance dependence	Vary separation within allowed range: hidden signal follows the predicted propagation factor and overlap; EM leakage has different scaling and phases.
Source-mode pickup	Monitor source eigenmode amplitude: RS-correlated receiver power scales with P_{em} and with the threshold detuning parameter Δ.
Frequency scan	Sweep f_{sys} within tuning range: RS line tracks tuned resonance; stationary lines indicate electronics interference.
Beam-off null	With beam off (or detuned so the source mode is not excited) the RS-correlated excess vanishes.

Table 1. Built-in falsifiers and constraints for an RS laboratory search using a thresholded source and a resonant LSW receiver.

9. Decisive outcomes: discovery claim vs null constraint

9.1. Positive detection criteria

A credible RS detection requires: (i) a narrowband signal at f_{sys} in the receiver, (ii) ABAB threshold correlation with independently calibrated dispersion, (iii) resonant selectivity under receiver detuning, and (iv) at least one geometry/shielding falsifier behaving as expected for hidden-sector transmission. A threshold-correlated signal that fails detuning or shielding tests should be treated as leakage.

9.2. Null result and reporting standard

A null RS result should report:

- the calibrated threshold condition and the bracketing parameters (λ_A, λ_B),
- the measured $P_{\min}(f_{sys})$ (or equivalently $T_{sys}, \Delta f, \tau$),
- the assumed geometry factor(s) $G_{T,L}$ and whether longitudinal optimization was targeted,
- and the resulting $\chi(m)$ constraint in the relevant mass window.

10. Cosmological/astrophysical relevance (brief)

RS was originally posed as a general threshold emission mechanism and could be relevant in early-universe or astrophysical plasmas, where $\Pi_{T,L}$ and dispersion relations differ dramatically from laboratory dielectrics. The in-medium mixing framework (Eq. (15)) is directly applicable in those environments. This Version 5 focuses on laboratory realizations and does not attempt a full cosmological phenomenology.

11. Conclusions

RS is a threshold concept: when a weakly coupled eigenmode becomes subluminal, a relativistic charge can shed energy into it. LDMX-like missing-momentum experiments motivated the ABAB methodology, but the specific RS-in-ordinary-dielectric implementation does not generate the canonical MeV–GeV mediator on-shell. The correct laboratory target is instead light hidden states and engineered eigenmodes.

A complete RS laboratory program can be constructed by combining: (i) a tunable slow-wave/dielectric source eigenmode that turns on only above a phase-velocity threshold, and (ii) a resonant LSW receiver (CROWS/Dark SRF class) for hidden-sector regeneration. This architecture comes with strong falsifiers and leverages demonstrated $\chi \sim 10^{-9}$ laboratory sensitivity.

Appendix A. From dielectric response to the polarization tensor

In an isotropic, non-magnetic medium ($\mu_r \simeq 1$), the complex permittivity $\varepsilon_r(\omega)$ determines the photon dispersion. For a transverse wave in a homogeneous medium,

$$k^2 = \varepsilon_r(\omega)\omega^2, \tag{23}$$

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so $n_\gamma^2(\omega) = \varepsilon_r(\omega)$ and $n_\gamma(\omega) = k/\omega$. Equivalently, the transverse polarization can be written as

$$\Pi_T(\omega) \simeq \omega^2 \left(1 - \varepsilon_r(\omega)\right), \tag{24}$$

which connects directly to Eq. (15). Absorption is encoded in $Im\varepsilon_r$ (or $Im\Pi$), which smears sharp thresholds when $|\Delta| \lesssim n''$.

Appendix B. Synchronism and threshold bracketing in slow-wave structures

Consider a mode with dispersion $\omega = \omega(k_z; \lambda)$ and a charge moving at speed v along z. The beam current has Fourier support enforcing the phase-matching condition $\omega = k_z v$ (Eq. (11)). Define the detuning at a chosen frequency ω_0:

$$\delta k_z(\lambda) \equiv k_z(\omega_0; \lambda) - \frac{\omega_0}{v}. \tag{25}$$

When $\delta k_z = 0$ the beam is synchronous and excites the mode efficiently. For $\delta k_z \neq 0$ the excitation is suppressed over a formation length $\ell_{form} \sim 1/|\delta k_z|$. An RS ABAB toggle is obtained by choosing λ_A, λ_B such that δk_z changes sign across zero.

Appendix C. ABAB statistics and lock-in style estimators

For a narrowband receiver measurement, one can form a demodulated estimator of the ABAB-correlated excess by toggling the state at a modulation frequency f_{mod} and measuring the Fourier component of the received power time series at f_{mod}. This is analogous to lock-in detection: it suppresses stationary spectral lines and slow drifts. In a conservative approach, one may instead treat each A and B segment as an independent power estimate and form $\Delta \hat{P} = \overline{P}_B - \overline{P}_A$ with uncertainty determined by the radiometer equation.

Appendix D. Unit conversions

Useful conversions:

$$h = 4.135667 \times 10^{-15} eVs, \hbar = 6.582120 \times 10^{-16} eVs. \tag{26}$$

Thus

$$E = hf \approx (4.14\mu eV)\left(\frac{f}{GHz}\right), E = \hbar\omega(\omega = 2\pi f) \approx (4.14\mu eV)\left(\frac{f}{GHz}\right). \tag{27}$$

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Be explicit about conventions: if you quote f in Hz, convert to energy with $E = hf$; if you quote angular frequency ω in rad/s, convert with $E = \hbar\omega$. In this paper ω denotes angular frequency, and when forming ratios like m/ω we mean the corresponding energy units with $\hbar = 1$.

Notes

Version note (what Version 5 standardizes)

- **Keeps the LDMX-motivated origin, but fixes the mapping.** LDMX is retained as the motivating context for ABAB threshold bracketing, while the dielectric Cherenkov interpretation is corrected for the MeV–GeV mediator regime.
- **Defines RS uniquely and operationally.** RS is a threshold-modulated hidden-channel emission mechanism; it is not a rebranding of LSW, kinetic mixing, or Cherenkov radiation.
- **Adds a complete lab program.** We specify a tunable slow-wave source + resonant regeneration receiver, with a concrete "where to look" procedure.
- **Adds quantitative completeness.** A worked sensitivity estimate and an illustrative reach curve are included; all key unit conventions are stated.
- **Adds constraints and falsifiers.** Existing laboratory limits are summarized; built-in null tests are provided in an executable form.

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