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Peer Review

Review of: "Primary Hairs May Create Echoes"

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In the current manuscript, the authors study a novel mechanism for echoes in ringdown signals of perturbed compact objects. The traditional interpretation of echoes entails the existence of trapping regions where perturbations can be trapped, and through continuous excitation of the vibrational spectra of the compact object, they produce a train of echoes in the late-time ringdown signal with an occurrence timescale proportional to the scale of the trapping region. Trapping regions appear traditionally when two peaks in the effective potential of an object exist. This occurs in wormholes, where the light ring in our Universe is mirrored onto the other Universe through the throat of the wormhole, in exotic compact objects with reflective surfaces, such as gravastars, firewalls, fuzzballs, and in general when horizon-scale structures are present. These include even dark matter overdensities and astrophysical environments, such as plasma, which works as a high-pass filter. Even though some of the above trapping regions are quite astrophysical, the authors present yet another way for echoes to arise without additions to the effective potential by hand, or changes in the boundary conditions. This is achieved in black holes with Proca-Gauss-Bonnet hair that arise from modified gravity.

The analysis in both the time domain and the frequency domain of scalar and Dirac perturbations performed in this article seems correct and covers a wide part of the parameter space available in these black holes. There are some comments and questions that I would like to pose to the authors, that might have escaped their literature search, but in general, the article is in very good condition and is definitely publishable in Qeios. The comments follow below.

1. The main Lagrangian in equation (1) involves GR, coupled with a vector tensor component and a scalar tensor component. The scalar tensor sector in equation (3) includes coupling with the Gauss-Bonnet invariant, while the vector tensor sector in equation (2) includes the Proca vector field, which involves a coupling of the vector field to the Einstein tensor.

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Of utmost importance is the coupling to the Einstein tensor that leads to novel black hole solutions with Proca hair, and in this case, also the scalar field is coupled to the Einstein tensor. First, I would like to point out to the authors the following works:

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.103.044042

https://link.springer.com/article/10.1007/s10714-022-02929-0

In the above articles, a scalar field theory coupled to the Einstein tensor leads to a change in the asymptotic behavior of the effective potential, thus naturally leading to echoes (scalar and gravitational perturbations are studied there). So it seems that the coupling to the Einstein tensor is the key here. This might be an interesting aspect to discuss in the Introduction.

2. The authors state that "such black holes possesses a remarkable property: within certain ranges of parameters it can produce echoes without the need for any environmental effects in the far zone" and "Typically, an additional peak is introduced artificially, for instance, to model the influence of the astrophysical environment..". While indeed, a large number of papers in the literature use by-hand additions to the effective potential in order to mimic environmental effects like the following

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.128.111103

Recent progress in the environmental effects around black holes has shown that one can use the so-called Einstein cluster formalism to generate a solution to the Einstein field equations with the inclusion of a stress-energy tensor representing an anisotropic fluid. This is an exact methodology and essentially embeds black holes, static or spinning, into astrophysical environments. Yet, these solutions are black holes in astrophysical environments, i.e., we need to solve the non-vacuum Einstein equations, which means that we need to choose an energy density for the fluid that will give rise to the rest of the ingredients and finally result in a black hole surrounded by the energy density profile chosen from various observations of galaxies, for example, like the Hernquist matter halo, or even dark matter overdensities that are better described by the Navarro-Frenk-White density profile. In any case, the authors are correct up to this point; their modified theory does not need a non-zero right-hand side in the Einstein equations but rather appropriate modifications of the geometrical sector.

BUT, recently, a degeneracy was found that directly connects vector Proca fields coupled to the Einstein tensor to the Einstein cluster formalism that leads to environmental effects. This result can be found in

https://arxiv.org/abs/2505.00563.

If the authors have a look at equation (6) in the aforementioned article, they should instantly realize that this theory is very close to theirs, especially because of the part where the Proca vector field is coupled with the Einstein tensor. Again, the action (6) can lead to the spacetime described by equations (1)-(5), which is in parallel with the Einstein cluster formalism that creates black holes embedded in matter halos and has been quite extensively used:

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.105.L061501

https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.129.241103

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.111.064026

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.111.024017

https://www.sciencedirect.com/science/article/pii/S0370269321006742?via%3Dihub

https://iopscience.iop.org/article/10.3847/1538-4357/ac76bc

https://www.sciencedirect.com/science/article/pii/S2214404824001113?via%3Dihub

https://arxiv.org/abs/2509.03301

https://arxiv.org/abs/2508.19333

https://arxiv.org/abs/2508.20191

Notice that some of the above publications and preprints include the authors of the current manuscript.

Of course, as I mentioned, the Lagrangian in the current manuscript is much more complicated than the one that reproduces black holes surrounded by matter, and the extensive analysis performed by the authors here is extremely helpful to elucidate this degeneracy. What I would like is for the authors to comment on the fact that the modification of gravity can sometimes correspond to classical GR with matter and vice versa. This degeneracy is very important when building theories of gravity.

Furthermore, the comments regarding the absence of environmental effects should be smoothed appropriately since here we deal with a very similar modified theory that can lead to GR black holes with matter. The citation of the above references will help the authors guide the discussion, most probably in the conclusions, with some small hints in the main text when environmental effects are stated to be absent, even though they indirectly exist well within the theory in equation (1).

After the addition of the above two discussions in the current manuscript, I will happily recommend it for publication.

Declarations

Potential competing interests: No potential competing interests to declare.