

On the cosmological arrow of time

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The cosmological arrow of time is defined by the initial conditions of the Universe and its subsequent expansion. Wetterich [1] has proposed an alternative picture of the expanding Universe where the observed cosmological redshift can equivalently be viewed as originating from the increase in mass of particles over time. In this article, we address the following problem: How can the cosmological arrow of time be interpreted in a static Universe where the particle masses increase? We take the philosophical point of view that the set of minima in the effective potential of a symmetry-broken quantum field theory represents physically coexisting groundstates. Tunneling of energy, carried by instantons, within the set of groundstates, is suggested to induce the change in particle masses. The directionality in which the instanton energy tends to flow, i.e. from high-energy-density to low-energy-density groundstates, defines then the cosmological arrow of time. It is concluded that the premises on which this article is built, i.e. the assumption that the set of groundstates physically coexists, is invalid if it is found to be impossible to provide a sound argument for the increase in entropy within a contracting spacetime. This is because otherwise the second law of thermodynamics is violated.

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Introduction

The problem of the arrow of time has a long history in theoretical physics and the philosophy of physics. It remains to this day to come up with a convincing argument for its existence that can resolve e.g. why the Universe started in such a low entropy ordered state. From the physics point of view, the most relevant arrows are the thermodynamic arrow, the quantum arrow, and the cosmological arrow. The thermodynamic, or entropic, arrow is defined by the universal increase of entropy from the past to the future. The quantum arrow is given by the build-up of quantum entanglements between the components of a system as time passes. The cosmological arrow is correlated with the expansion of spacetime. This article will consider the cosmological arrow.

In 2013, Wetterich presented a model of the Universe where the expansion of spacetime could equivalently be described by an increase in particle masses with time [1]. The arrow of time was discussed by Wetterich within the cosmological model presented, where it originates from the attractive character of solutions to the cosmological field equations [2]. In this article, we will present a novel interpretation of the concept of the cosmological arrow of time in a static Universe where the mass of particles increases with time. It should, however, be admitted that no specific model is put forth that aims to describe in detail the cosmological dynamics of the Universe. The reason for this is that we feel that the concept of the arrow of time seems to be so fundamental that general qualitative arguments should be possible to make, relying only on well-established principles of physics such as

energy conservation and the second law of thermodynamics, which illuminate the physical existence and origin of the arrow.

The short answer to the problem of how the mass of particles increases with time would simply be to state it as a fundamental law of nature. However, that seems to be quite unsatisfactory. Instead, we will in this article try to argue for a definition that is built on the conservation of energy and a quantum cosmological analog of the second law of thermodynamics. Furthermore, we take for granted the possibility of quantum tunneling through potential barriers that separate classically stable phases, as described by instantons in quantum field theory. These ideas stand on solid ground in theoretical physics. The key element of this article on which the discussion is built is an assumption based solely on a philosophical curiosity. The line of reasoning in stating this assumption goes as follows. Let us suppose that the expansion of spacetime can equivalently be understood by an increase in the mass of particles with time. But how can they increase their mass? That can be accomplished by absorbing energy. To satisfy the conservation of energy, while particles absorb energy everywhere in our four-dimensional spacetime, it seems necessary for the Universe to be larger than what we are familiar with. But larger in what sense? There exist several proposals for higher-dimensional theories, too many to name here. The point of view that we propose in this article is the following: The set of minima appearing in the effective potential, as a function of the field configuration, in a symmetry-broken quantum field theory, do not represent the possible groundstates in which our Universe might exist, whether it be the familiar four-dimensional spacetime or the well-known higher-dimensional spacetimes proposed in the literature. The entire set of minima represents different phases of the Universe that coexist.

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The Universe from this viewpoint is thus not defined by a single groundstate. The enlarged picture of the Universe is that the set of field configurations, for the known fields of the Universe, whose energies are locally minimized in the field space, are all physically manifested within spacetime, in a sense 'on top of each other'. Thus, when we in this article speak of quantum tunneling between groundstates, which is a commonly studied problem in quantum field theory, we do not adopt the contemporary view that the Universe exists in e.g. the false groundstate from which it might suffer a catastrophic decay to the lower-lying possible true groundstate. Rather, we speak of quantum tunneling within the set of different phases, defined by distinct groundstate energies, that are all physically realized in the enlarged picture of the Universe.

The structure of the article is as follows. The philosophy of the article, i.e. that the set of minima in the effective potential represent physically coexisting groundstates, is stated within the simplest possible quantum field theory, i.e. that of a real-valued scalar field. Taking this philosophy as a fact, we try to qualitatively deduce some consequences it might have for the Universe. The concept of groundstate energy and quantum fluctuations about its classical value is briefly discussed. The tunneling between groundstates, as given by the theory of instantons in quantum field theory [3], is then summarized. We propose a quantum cosmological analog of the second law of thermodynamics, which state that the Universe, in which all phases physically coexist, tends to evolve in such a way that the energy separations between the coexisting groundstates become zero, at which the Universe has reached what we will refer to as the state of quantum equilibrium. We then explain how the increase in the mass of particles, suggested to be induced by the tunneling of instantons, gives rise to an apparent cosmological redshift thus giving the appearance of cosmic expansion and acceleration. From there, we discuss how a pair of observers, living at each end of the quantum tunnel connecting the pair of groundstates, perceive the flow of energy within the coexisting phases. Their perspectives are connected with the expansion and contraction of spacetime.

The cosmological arrow of time for the static Universe, where the masses of particles increase with time, is then suggested to be defined by the following statement:

The directionality for the cosmological arrow of time is defined by the tendency of the Universe to evolve in such a way that the energy separations between the physically coexisting groundstates vanish, at which stage the Universe has reached quantum equilibrium.

Groundstates of the Universe

To state the point of view on which this article is built, pretend that the Universe consists of a single real-valued scalar field $\psi(\vec{x}, t)$ in four-dimensional spacetime, with dynamics defined by the Lagrangian density $\mathcal{L}(\psi)$,

$$\mathcal{L}(\psi) = \frac{1}{2} \partial_{\mu} \psi \partial^{\mu} \psi - \frac{1}{2} m^2 \psi^2 - V(\psi).$$
 (1)

Furthermore, consider the situation when the mirror symmetry of the effective potential $V(\psi_c)$, where ψ_c is the vacuum expectation value of the field, also referred to as the classical field or the field condensate, has been broken such that there are two local minima, at $\psi_c = \pm a$, with an energy separation ϵ , see Fig.1. The pair of minima

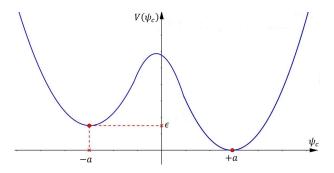


FIG. 1. Effective potential with two energetically distinct groundstates.

correspond to the two distinct classical groundstates associated with the unique pair of solutions to the classical field equations of motion. The consideration of a single real-valued scalar field in this article is for simplification purposes only. The extension to the symmetry-broken field theories describing the actual Universe is straightforward, albeit laborious.

In contemporary field theory, the common perspective is that the Universe, in its groundstate, occupies either the minimum V(+a) or the minimum V(-a). They are referred to as the true and false vacuum, respectively. Both groundstates are stable in classical field theory. In quantum field theory, on the other hand, the false vacuum is rendered unstable due to quantum fluctuations allowing for the possibility that it decays into the true vacuum. This decay process is the analog of nucleation processes in condensed matter physics, e.g. the boiling of a superheated fluid. There, the graph of the free energy of the fluid as a function of density would have the same form as the graph of the effective potential in the symmetry-broken quantum field theory. The lowest energy state for the superheated fluid phase corresponds to the false vacuum, whereas for the vapor phase, it is the true vacuum. Thermal fluctuations cause bubbles of vapor to nucleate in the fluid. During the process of boiling, both the superheated and the vapor phases physically

coexist. Considering this close resemblance between the pictures for the boiling of a superheated fluid and vacuum decay, with the difference that thermal fluctuations are replaced by quantum fluctuations, it is the purpose of the present article to initiate a study on the possible physical implications following the viewpoint that the pair of minima of the effective potential in quantum field theory does indeed physically coexist. Based on this philosophy, the pair of groundstates represent distinct phases of the Universe that are both equally real. It is not the case, then, that the Universe exists either in the false vacuum or in the true vacuum.

Energy of the groundstates

Does the absolute value for the energy of the groundstate of a system make physical sense? Consider, as an example, the groundstate of the hydrogen atom. Its energy is $-2.18 \cdot 10^{-18} J$. This means that an energy of $+2.18 \cdot 10^{-18} J$ is needed to ionize the hydrogen atom to reach the chosen reference state, with zero energy, when it is in its groundstate. This illustrates the point that the absolute value for the energy of any given state carries no physical information. It is the comparison of energy between pairs of states which is physically observable. The concept of energy acts thus as a measure to distinguish between distinct states of a system. The absolute values assigned to individual states have no physical meaning unless they are being compared with a chosen reference state, e.g. the state of ionization in a hydrogen atom.

In quantum field theory, the value for the groundstate energy of the field experiences quantum fluctuations about its classical value due to the Heisenberg uncertainty principle. The amplitude of these fluctuations is controlled by Planck's constant. Given the assumption that Planck's constant has the same value everywhere in the Universe, the strength of the quantum fluctuations for the pair of groundstates, located at the field condensates $\psi_c = -a$ and $\psi_c = +a$, must be the same. The energy associated with these quantum fluctuations represents the energy carried by the so-called quantum vacuum of the field.

The quantum fluctuations of the classical groundstate do not define a new state of the system. The system still occupies the groundstate, albeit with a deviation in its energy as compared to its classical value. This, combined with the definition of the concept of energy as a physical measure for the distinction between distinct physical states, is of importance in understanding the role played by vacuum energy in attempting to relate it with the appearance of dark energy. In contemporary approaches, it is an assumption that the origin of dark energy is somehow due to the energy of the groundstate fluctuations [4–8]. The problem is, however, that when the energies associated with these fluctuations are summed up for the known fields of the Universe, a value for the cosmological constant is obtained which is between 50 and 120 orders of magnitude larger than the value obtained from cosmological observations. This is the so-called cosmological constant, or vacuum catastrophe, problem. The error with this calculation, suggested in this article, is that we should not estimate the absolute value of the energy fluctuations for a specific groundstate. Rather, we should compare the energies associated with the groundstate fluctuations for the pair of groundstates. Given the assumption that the fluctuations are equal in strength for both groundstates, their relative energy difference should be zero. The physically relevant energy to the problem of dark energy is then the difference in energy between the classical groundstates, e.g. ϵ between the pair V(-a)and V(+a). The role of the fluctuations is to initiate the tunneling of energy between the pair of groundstates, in the form of instantons.

Tunneling between groundstates

The flow of energy between the pair of groundstates at $\psi_c = -a$ and $\psi_c = +a$ is described by the theory of instantons in the standard formalism of quantum field theory [3], whose results are quickly summarized here. The shift in the energy δE of the groundstates, per unit volume and unit time, induced by the tunneling of scalar instantons, is proportional to the imaginary part of the instanton energy, i.e.

$$\delta E(\epsilon) \sim \frac{\hbar |K|}{4\pi^2} \cdot \left(2\sqrt{2}a \cdot \sqrt{\epsilon}\right)^2 \cdot e^{-2\sqrt{2}a \cdot \sqrt{\epsilon}}, \qquad (2)$$

where K is an imaginary-valued determinantal factor. This estimate is valid in the thin-wall approximation, i.e. when the difference in the energy density between the groundstates is small. The tunneling amplitude Γ , per unit volume and unit time, i.e. the decay rate, is proportional to the energy shift, i.e.

$$\Gamma(\epsilon) \sim \delta E(\epsilon).$$
 (3)

The value $\epsilon = \epsilon_m$ for which $\Gamma(\epsilon)$ is stationary, i.e. where $d_{\epsilon}\Gamma(\epsilon) = 0$, is given by

$$\epsilon_m = \frac{1}{2a^2}.\tag{4}$$

Since $d_{\epsilon}^2 \Gamma(\epsilon)|_{\epsilon=\epsilon_m} < 0$, the stationary point $\epsilon = \epsilon_m$ maximizes the tunneling amplitude, which becomes

$$\Gamma_m \sim \frac{e^{-2}}{\pi^2} \hbar |K|. \tag{5}$$

The graph of the tunneling amplitude $\Gamma(\epsilon)$ is shown in Fig.2. Consider the scenario when the decay process is initiated at some initial energy separation $\epsilon > \epsilon_m$. As the decay process is underway, the energy separation between

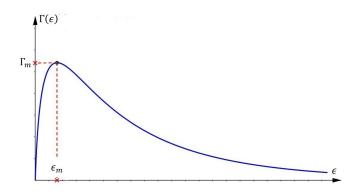


FIG. 2. The tunneling amplitude depends on the energy separation ϵ between the pair of groundstates.

the pair of groundstates decreases. The rate at which instanton energy flows from the groundstate at $\psi_c = -a$ to the groundstate at $\psi_c = +a$ increases until the energy separation between the pair has reached $\epsilon = \epsilon_m$, where it is at a maximum. After this point, the decay rate quickly goes to zero as $\epsilon \to 0$.

Towards quantum equilibrium

Supposing that the total energy is conserved in the Universe, in which the set of groundstates coexists, the total sum of the groundstate energies must remain constant in time. Thus, if the groundstate energy for the field condensate $\psi_c = -a$ decreases by an amount δE , then the field condensate $\psi_c = +a$ will gain an energy δE to its groundstate.

Thermal systems tend to evolve in such a way that they reach thermal equilibrium with their environment. Thermal energy tends to flow from high-density regions to low-density regions. At thermal equilibrium, the thermal fluctuations are, on average, the same everywhere within the system. With the picture presented in this article, with coexisting groundstates with possibly different energies for their groundstates, the following quantum cosmological analog of the second law of thermodynamics is proposed:

The Universe tends to evolve in such a way that the energy separation between the groundstates becomes zero.

When the Universe has reached $\epsilon = 0$, it is in a symmetric state where all groundstates appear identical. They are indistinguishable from each other. This special condition will, in this article, be referred to as the state of quantum equilibrium of the Universe. The energy separation tends to zero by having instanton energy tunneling to groundstates with lower energy density from groundstates with higher energy density, e.g. from

the groundstate at V(-a) to the groundstate at V(+a).

Given the definition of the concept of energy, it is natural to select the state of quantum equilibrium as the state of reference, with zero energy.

Change in particle masses

Due to the influx of instanton energy to the groundstate at $\psi_c = +a$, the particle excitations of the field ψ in this phase will increase in amplitude, i.e. the mass of the particles will increase, per unit volume and unit time, by an amount δm , given by

$$\delta m(\epsilon) = \frac{\delta E(\epsilon)}{c^2}.$$
 (6)

The maximum increase in mass, per unit volume and unit time, occurs when $\epsilon = \epsilon_m$, for which it becomes

$$\delta m_m \sim \frac{e^{-2}}{\pi^2} \frac{\hbar}{c^2} |K|. \tag{7}$$

The graph of $\delta m(\epsilon)$ has the same form as the graph of $\Gamma(\epsilon)$, see Fig. 2. The rate of change for δm increases as the energy separation approach ϵ_m from an initial value $\epsilon > \epsilon_m$. As the pair of groundstates are drawn further closer to each other, the rate of change for δm decreases, i.e.

$$\lim_{\epsilon \to 0} \delta m(\epsilon) = 0. \tag{8}$$

When the pair of groundstates becomes degenerate, there is no further change in the mass of particles.

Frequency-shifted photons

In the discussion above, a real-valued scalar field was considered. This is the simplest possible example. However, the same general conclusions hold when considering e.g. quantum electrodynamics. The mass of the quantum field excitations increases for symmetry-broken effective potentials due to the influx of instanton energy from neighboring groundstates at higher energy density.

As the mass of particles, e.g. electrons, are increased over time, the energy spectra of atoms are directly affected. Consider e.g. the hydrogen atom, which is the most abundant atom in the Universe. Since the proton is about 2000 times heavier than the electron, the proton can be considered as being at rest relative to the electron. Then, the energy of the hydrogen atom is dominated by the kinetic energy of the electron such that the *n*'th energy level in the hydrogen spectrum is proportional to the electron mass m,

$$E_n \sim -\frac{m}{n^2}$$
, $n = 1, 2, 3, ...$ (9)

The energy difference ΔE between two arbitrary levels *i* and *j*, with j > i, is

$$\Delta E = E_j - E_i \sim m \cdot \left(\frac{1}{i^2} - \frac{1}{j^2}\right) > 0.$$
 (10)

Thus, as the mass of the electron increase, the separation ΔE between energy levels in the hydrogen spectrum grows. As a consequence, the energy and frequency of emitted photons, given by

$$E_{\gamma} = \Delta E = h \cdot f, \tag{11}$$

are increased. The emitted photons have become blueshifted. The same general conclusion holds for all types of atoms.

Cosmological redshift

Consider a pair of galaxy clusters, Y and Z, with distances r_Y and $r_Z > r_Y$ from an arbitrarily chosen galaxy cluster X, which is to act as the 'coordinate origin', see Fig.3. The light reaching X from Z has traveled a longer

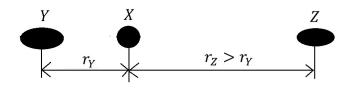


FIG. 3. The galaxy pair Y and Z at different distances from the origin X.

distance as compared to Y. Therefore, the light coming from Z was emitted at an earlier time as compared to Y. The picture of the pair of galaxies, as seen from the lenses of X, shows them at different ages in their evolution. The light coming from Z, reaching X, depicts the state of Z at a 'younger' age as compared to Y, given that both galaxies were born at the same time. This has an interesting consequence when combined with the idea that the masses of particles, and hence the separation in atomic spectra, change with time due to tunneling between the possible groundstates of the Universe. If the galaxy clusters are located within a phase whose groundstate has a lower energy density as compared to its environment, such that it on the net has an inflow of instanton energy, then the light from Z was emitted from atoms at a stage when the separation of atomic energy levels had not had as much time to increase as compared to Y. Therefore, the light from Z, as compared to the light from Y, appears to have lesser energy. In other words, the photons reaching X from Z appear to be red-shifted relative to the photons coming from Y, with the conclusion that the Universe is expanding.

If, on the other hand, the galaxy clusters are located within a phase whose groundstate has a larger energy density as compared to its environment, such that it on the net has an outflow of instanton energy, the situation is the opposite. The masses of particles decrease with time. The light emitted from atoms would be redshifted with the consequence that the light reaching the X from Z would carry more energy as compared to the light coming from Y, since the atoms in Y would have had more time to decrease their energy level spacings. The more distant galaxies would then appear to be blue-shifted with the conclusion that the Universe is contracting.

The Big Bang

At quantum equilibrium, the groundstate energy of the Universe is zero. The conservation of energy within the Universe, however, allows for the possibility that spontaneous fluctuations push the Universe away from its state of quantum equilibrium, as long as the total energy remains zero. In other words, the groundstate at quantum equilibrium, with energy zero, can be split into a set of coexisting groundstates if their energies add up to zero. This means that some groundstates have gained energy whereas others have lost energy, relative to the groundstate at quantum equilibrium. This, of course, does not necessarily imply that the energy of a coexisting groundstate can, at the fundamental level, take negative values. The negative-energy groundstate is nothing more than a statement that there is a net inflow of instanton energy from the environment of coexisting groundstates, as compared to a positive-energy groundstate for which there is a net outflow of instanton energy. In this sense, the concept of energy does have a directionality associated with it in the Universe, in contrast to how the concept is perceived within the four-dimensional spacetime of any given phase of the Universe. This directionality can be compared with the electric charge. That the electric charge can take positive and negative values represent the experimental fact that the electric force can be both repulsive and attractive, causing charged particles to accelerate, respectively, either away or towards each other. The positive and negative energy groundstates represent, respectively, contracting and expanding spacetimes as they approach quantum equilibrium.

The physical event that the Universe spontaneously fluctuates away from its state of quantum equilibrium, is from the perspective of the phase with a negativeenergy groundstate seen as the Big Bang, where spacetime expands from a very dense and hot state. The same event would look quite different for an observer within the phase whose groundstate has a positive energy. There, spacetime would seem to contract, starting from a very dilute and cold state into the Big Crunch.

It is worthwhile to note that the common statement

that the Universe began at an initial singularity with infinite density and temperature is based solely on the classical equations of motion for the gravitational field as given by the theory of general relativity. This statement is based on the famous singularity theorems given by Roger Penrose and Stephen Hawking. The questionable validity of this statement is obvious and well-known, considering the importance that quantum mechanics should play at this stage in the history of the Universe. The initial conditions of the Universe are, within the philosophy of this article, the following. The Universe has no beginning and it has no end. It is without boundaries. It has zero energy at quantum equilibrium and due to energy conservation it will remain at zero energy for all eternity, both into the past and into the future. It is only from the perspective within any given coexisting phase that the Universe seems to have had a beginning. That beginning took place when the four-dimensional spacetime of the given phase appeared to have started expanding, or contracting.

Cosmological arrow of time

The cosmological arrow of time is thus viewed as a statement on the tendency of the Universe to change towards the state of quantum equilibrium, where the set of coexisting groundstates has become physically identical. When this state has been reached i.e. when the expansion, or contraction, of the coexisting phases within the Universe has stopped, the cosmological arrow of time ceases to exist as a physical concept. However, for each spontaneous fluctuation away from the state of zero energy, there are new Big Bangs and Big Crunches, and the concept of time is rebooted with a cosmological arrow that is again pointing toward the quantum equilibrium state of the Universe. In this sense, the Universe appears to be cyclic in a manner that is distinct from wellknown cyclic models, see e.g. the bouncing cosmologies discussed in the reviews [9, 10] and Penrose's conformal model [11, 12].

From the perspectives of both types of coexisting phases of the Universe, i.e. groundstates carrying positive and negative energy relative to quantum equilibrium, time should go from the past to the future. In both types of phases, metaphoric people should grow older and the metaphoric shattered pieces of a dropped glass of red wine should never be witnessed to reassemble and come back to the clumsy hand. This should be the case, from both perspectives, if the second law of thermodynamics is to be valid throughout the entire Universe, with thermodynamic entropy increasing as time passes. For this to be the case, the thermodynamic arrow of time must coincide with the expansion of spacetime within the negative-energy groundstate and with the contraction of spacetime within the positive-energy groundstate. It is thus necessary to provide a valid argument for the increase in entropy as spacetime contracts. The opposite argument, with an expanding spacetime, is well-known. With continued expansion, spacetime becomes more homogeneous and isotropic thus approaching thermal equilibrium where the thermodynamic entropy is at its maximum value. The author has some speculations on the possibility of increasing entropy as spacetime contracts, but they are still immature and therefore left for later.

Conclusion

In this article, we have attempted to address the problem of the cosmological arrow of time in the type of static Universe where particle masses increase with time. The basic assumption on which the discussion was built was that the set of minima in the effective potential for the known fundamental fields of the Universe does indeed physically coexist. Based on this philosophy, the conclusion is that the Universe has no beginning and no end. Its conserved energy is zero and it tends to be driven toward the state of quantum equilibrium, where the set of coexisting groundstates are identical with zero energy. This tendency defines the cosmological arrow of time. Furthermore, each time the Universe spontaneously fluctuates away from its state of quantum equilibrium, the set of negative-energy phases experiences an expanding spacetime from a dense and hot state at the Big Bang, whereas the set of positive-energy phases experiences a contracting spacetime from a dilute and cold state at the Big Crunch. However, to not violate the second law of thermodynamics it is necessary to have a valid argument for the increase in entropy for the contracting spacetime within a positive-energy phase. If this is shown to be impossible, the philosophy of the article falls completely apart.

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