Qeios

Peer Review

Review of: "Inequalities Revisited"

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The manuscript focuses on a very current topic, is well-written and structured, and the symbols used are appropriate. I don't detect any plagiarism or self-citations.

The paper introduces a geometrical framework for interpreting inequalities originating from information theory and Shannon entropy studies. It aims to apply this framework, which has led to non-Shannon-type entropy inequalities, to general universally quantified inequalities.

The study develops concepts with simple examples and progresses to more complex ones, presented in a logical order:

The AM-GM Inequality

The AM-GM inequality is used to illustrate the concepts developed in the study.

A geometrical framework is introduced to understand the relation between the arithmetic mean (AM) and the geometric mean (GM).

Theorem 1 states that an ordered pair is achievable if and only if it belongs to a defined region, Υ, completely characterizing the relationship between AM and GM.

Markov's Inequality

Markov's inequality in probability theory is discussed, focusing on the relationship between expectation and probability for a nonnegative random variable.

A geometrical framework is introduced, defining a region, Ψc, to analyze the achievability of ordered pairs representing probability and expectation.

Theorem 4 asserts that an inequality is valid if and only if a specific set inclusion holds, implying the region completely characterizes all valid inequalities of a certain form.

Cauchy-Schwarz Inequality

The Cauchy-Schwarz inequality is examined within real inner product spaces.

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The achievable region is defined, and Theorem 5 states that an inequality set is valid if and only if a specific condition holds.

Theorem 6 states that for an inner product space V, $\Phi* = \Phi$ if and only if $\dim(V) \geq 2$, and provides characterizations for cases where $\dim(V) = 0$ or $\dim(V) = 1$.

Entropy Inequalities

The section applies geometrical formality to inequalities on Shannon entropy, leading to results in the past two decades.

Proposition 3 states that a set of entropy inequalities is valid if and only if the region is included in a specific set.

Proposition 4 provides a constrained version, stating that a set of entropy inequalities is valid under constraint if and only if another inclusion holds.

Connections with Other Fields

The characterization of the region $\Gamma*$ was shown to be related to a distributed coding problem inspired by satellite communication and developed into the theory of network coding.

The general implication problem in probability theory is related to the region $\Gamma*$ and is one of the most basic problems in the field.

A group inequality can be obtained from an entropy inequality.

Concluding Remarks

The paper develops a framework for universally quantified inequalities, providing a geometrical interpretation.

The application of this formality to the study of entropy inequalities has yielded results in subjects related to Shannon entropy.

Applying this formality to different mathematical branches can identify new fundamental inequalities on quantities of interest.

Declarations

Potential competing interests: No potential competing interests to declare.