Review of: "Measurement Mechanics — Resolves QM Measurement Discrepancies"

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Potential competing interests: No potential competing interests to declare.

Review by Harish Parthasarathy.

The revised version of the paper contains many additional interesting details about the process of measurement. It says that the process of measurement consists of two
objects. The first is an observable or a collection of observables and the second is a unit which can be varied according to our requirements depending on the magnitude of variations of the observable. The fact that the unit can be varied implies that the result of all measurements is relative. In the case of quantum measurements, the units chosen should be such that the corresponding phase volume is $>> h/2\pi$, h being Planck's constant in order that variations of the measurement outcome due to Heisenberg's quantum uncertainty do not
count as true variations of the values of the observables. For example, if we choose to measure the position and momentum of an object, and $\Delta Q, \Delta P$ are the uncertainties in the measurement caused by perturbing the object with a light signal, or more generally, by the signal generated by the measurement apparatus, then $\Delta Q.\Delta P\approx h$. If the units chosen to measure Q is λ and that chosen to measure P is μ, then we require that $\mu>h/\lambda$ in order that that variations in P due to the uncertainty arising from the quantum measurement process do not count as genuine variations in P. For such a measurement to be successful, of course, we require that Q,P correspond to macroscopic bodies relative to the quantum scale, or more precisely, at least one of the variables Q,P assumes macroscopic values. In other words, the units chosen to measure non-commuting observables should be such that their product is much larger than a quantum threshold, and hence at least one or both of the observables should be macroscopic, because if both of these observables are of the quantum scale, the product of the uncertainities in their measurement should be much smaller than h and hence both cannot be simultaneously measured with complete accuracy.

The second point is that if there are random errors δx_i in the measurement outcome x and also random errors δu_i in the implementation of the unit u, then the random error $\delta\xi_i$ in the measured value $ux=\xi$ of the observable will be approximately,

$$\delta\xi_i = u.\,\delta x_i + x.\,\delta u_i$$

so if our measurements are iid (identically distributed independent random variables), the sample-averaged value of the

observable measurement error, given by

$$\bar{\delta}_\xi = \frac{1}{n}\sum_{i=1}^{n}\delta\xi_i = \frac{u}{n}\sum_{i=1}^{n}\delta x_i + \frac{x}{n}\sum_{i=1}^{n}\delta u_i$$

will have a variance of

$$Var(\bar{\delta\xi}) = u^2\sigma_x^2/n + x^2\sigma_u^2/n$$

where

$$\sigma_x^2 = Var(\delta x_i), \sigma_u^2 = Var(\delta u_i)$$

which means that if the unit u is too large or if the relative measurement outcome x is too large, then the variance in the Monte-Carlo-averaged error in the true value ξ of the observable will also be large. On the other hand, if the unit u is too small, as in the quantum scenario, the variations in the measured values of two observables relative to the unit u, would be attributed, according to the Heisenberg uncertainty principle, to perturbations caused by the measuring instrument and not due to genuinely different values assumed by the observables. Moreover, suppose we sequentially measure non-commuting quantum observables $O_1, O_2,... O_n$ with the initial state of the system being ρ_0. Suppose that the eigenvalues of these n observables are of different orders of magnitude. Let $E_1(\lambda),..., E_n(\lambda)$ denote respectively the spectral projections of these n observables.
Then, according to the collapse postulate, the joint probability of getting values $\lambda_{\sigma j}, j=1,2,..., n$ for the observables measured in the order $O_{\sigma j}, j=1,2,..., n$, where σ is a permutation of the index set $\{1,2,..., n\}$, would be given by

$$P(\lambda_1, \ldots, \lambda_n|\,\sigma) = Tr(E_{\sigma n}(\lambda_{\sigma n})\ldots E_{\sigma 1}(\lambda_{\sigma 1})\rho_0 . E_{\sigma 1}(\lambda_{\sigma 1})\ldots E_{\sigma n}(\lambda_{\sigma n}))$$

which would be generally different for different permutations σ. Of course, if the observables commute, these probabilities would be the same for all permutations σ because all the n spectral projections would then mutually commute. Thus, non-commutativity and the collapse postulate would give us different joint probabilities for different sequential orders in which the measurements are made. Now, since the λ_i's are of different orders of magnitude, we must use different units $u_1,..., u_n$ respectively for making joint measurements. Thus, writing $x_i=\lambda_i/u_i$ for the value of the measurement of O_i relative to the unit u_i, we get the joint probability of getting the x_i's in the order $1,2,..., n$ as

$$P_{O_1\ldots O_n}(x_1, \ldots, x_n|\,u_1, \ldots, u_n) =$$

$$Tr(E_{\sigma n}(u_{\sigma n}x_{\sigma n})\ldots E_{\sigma 1}(u_{\sigma 1}x_{\sigma 1})\rho_0 . E_{\sigma 1}(u_{\sigma 1}x_{\sigma 1})\ldots E_{\sigma n}(u_{\sigma n}x_{\sigma n}))$$

$$Tr(E_n(u_n x_n)\ldots E_1(u_1 x_1)\rho_0 . E_1(u_1 x_1)\ldots E_n(u_n x_n))$$

Then, if there are small shifts $\Delta u_1,..., \Delta u_n$ in the chosen units, the sensitivity or change in the joint probability can be very large owing to sharp discontinuities in the spectral projections, especially at an eigenvalue of one of the observables:

$$P_{O_1\ldots O_n}(x_1,\ldots,x_n|\,u_1+\Delta u_1,\ldots,u_n+\Delta u_n) - P_{O_1\ldots O_n}(x_1,\ldots,x_n|\,u_1,\ldots,u_n) =$$

$$\sum_{k=1}^{n} \Delta u_k . \frac{\partial}{\partial u_k} Tr(E_n(u_n x_n)\ldots E_1(u_1 x_1)\rho_0 . E_1(u_1 x_1)\ldots E_n(u_n x_n))$$

More generally, if there are random fluctuation errors $\Delta u_k,k=1,2,..., n$ in the chosen units, then the average shift in the joint measurement probabilities would be given by the formula

$$\sum_{r_1,\ldots,r_n\geq 0, r_1+\ldots+r_n\geq 1} \frac{<\Delta u_1^{r_1}\ldots\Delta u_n^{r_n}>}{r_1!\ldots r_n!} . \frac{\partial^{r_1+\ldots+r_n}}{\partial u_1^{r_1}\ldots\partial u_n^{r_n}} Tr(E_n(u_n x_n)\ldots E_1(u_1 x_1)\rho_0 . E_1(u_1 x_1)\ldots E_n(u_n x_n))$$

and the multiple partial derivatives here acting on the spectral projections could result in very large values of this sensitivity.

Moreover, as Roger Penrose has stated, it should be noted that quantum mechanics is incompatible with the notion of measurement because the measurement apparatus is always classical and hence can never be a part of any quantum mechanical system. The apparatus perturbs the system and leads to state collapse after the measurement has been made, and subsequent measurements are made on the collapsed state. Such a phenomenon is not a part of Schrödinger dynamics, and the two processes of Schrödinger evolution and measurement can never be made compatible with each other. Measurements on a continuously evolving quantum system are made at discrete times, and in between two measurements, the state of the system evolves according to Schrödinger's equation; at the time of a measurement, the state evolves discontinuously according to the state collapse postulate. Such a discontinuous evolution at the time of state collapse is not unitary and can never be reconciled with Schrödinger dynamics, neither with noisy Schrödinger dynamics described by the quantum master equation or, equivalently, by a quantum dynamical semigroup.

However, quite recently, as pointed out by V.P. Belavkin, in the context of quantum filtering, it is possible in many cases to make "non-demolition measurements" on a quantum system by allowing quantum noise to interact with the system, thereby causing the noise to get deflected by the system without affecting the future state of the evolving system. The state is not affected because we are perturbing it by quantum noise, not classical noise, and moreover, the time samples of this quantum noise form a commutative algebra which also commutes with the present and future state of the system, thereby enabling us to obtain reliable estimates of the system state without perturbing it. It should be noted that such an

estimate of the state will still have small errors because it is obtained by using a conditional expectation operation on the current system observable given all measurements up to the current epoch; the very fact that we are able to construct such a conditional expectation is because of the commutativity of the measurement noise samples and the commutativity of this noise with the system state, in short, because of the non-demolition property. Commutativity leads to zero Heisenberg uncertainty, thereby enabling us to define joint classical probability distributions and conditional expectations. In other words, the absence of Heisenberg uncertainty does not mean the absence of classical randomness; it only means that quantum randomness has been reduced to classical randomness, enabling us to describe the joint probability law of two or more observables.
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Finally, I would like to point out one important result related to independent classical measurements. If $e_1,e_2,..., e_n$ are iid measurement errors with zero mean and fixed variance σ_e^2, then according to the law of large numbers,

$$(e_1 + \ldots + e_n)/n \to 0 a.s. P$$

while

$$(e_1 + \ldots + e_n)/\sqrt{n} \to N(0, \sigma_e^2)$$

in distribution, where $N(0,\sigma^2)$ is the normal distribution with zero mean and variance σ^2. It should be noted that such sample average errors appear because we generally form sample averages of our measurements in order to reduce the variance of the error. Both of these two limit formulae are independent of the probability distribution of the individual errors. However, in practice, the number of measurements n is large but not infinity. Thus, we are interested in the rate at which the sample average error $(e_1+...+e_n)/n$ converges to zero. This rate will depend upon the individual probability distribution of the errors and is computed using Cramer's large deviation principle:

$$P[|(e_1 + \ldots + e_n)/n| > \delta] \approx exp(-nI(\delta)), n \to \infty$$

or, more precisely,

$$\frac{1}{n}. log(|(e_1 + \ldots + e_n)/n| > \delta] - I(\delta) \to 0$$

where

$$I(\delta) = sup_{\lambda>0}[\lambda. \delta - log[\mathrm{E}[exp(\lambda e_1)]]]$$

In particular, when the e_i's are iid $N(0,\sigma_e^2)$, this result becomes

$$I(\delta) = -\sigma_e^2\delta^2/2$$

In short, the usual limit theorems of statistics, namely the law of large numbers and the central limit theorem, are distribution independent, and they give us results only about the limiting behaviour of the measurement errors. On the other hand, in order to obtain results about the rate at which the errors converge when we use a large but finite number of samples, we must make use of large deviation theory, and these asymptotic results are error-distribution dependent because they do not merely give us the limiting distribution, they give us the rate at which the limiting distribution is approached.

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