Review of: "Quantum mechanics and symplectic topology"

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The author of this article essentially proposes a sort of derivation of non-relativistic quantum mechanics from the uncertainty principle assumed as a postulate. He is seduced by the fact that the limit on the information, or definiteness, of quantum states is fixed by the finite value of h and that this seems to be the real difference with respect to the classical case. Therefore, the author reasons, the uncertainty principle must play a more fundamental role in the construction of the theory than that of a simple theorem on the magnitude of the fluctuations of conjugate variables.

However, the point that must be kept in mind is that the conjugate variables involved in the various formulations of this principle are represented by non-commuting operators. Notoriously, it is the non-commutability of these operators that gives rise to the uncertainty relations.

We must also take into account that our understanding of the theory has greatly evolved in the long decades since its original formulation as it is explained, for example, in the handbooks of Von Neumann or Dirac.

In particular, since the eighties of the last century it has become clear that non-commuting operators can be associated with macroscopic properties of classical systems, when the experimental tests of these properties are not commutable. Hence, uncertainty relations can also be deduced in relation to classical systems governed by classical mechanics, and are not unique to the quantum world. In other words: the uncertainty principle is not a salient trait of the quantum domain, which detracts much from the motivations that the author places at the basis of his project. Quite simply, for an accidental fact certain probability structures were encountered for the first time in the study of quantum mechanics, but they are actually more general and indeed ubiquitous.*

An absolutely analogous discourse could be carried out on quantum states and the superposition principle. Classical systems described by fuzzy variables support a description in terms of probability amplitudes and their superpositions; complex probability amplitudes and their linear superpositions are not unique to the quantum domain. Quantum like modeling of non-quantum phenomena are now widespread in sectors ranging from quantum cognition to artificial intelligence and linguistic analysis of texts.

The use of complex numbers to describe interferences in these domains does not constitute any mystery, as it does not in quantum mechanics, which is nothing more than a particular application.

In summary, the real peculiarity of the quantum domain does not lie in its probabilistic structure, but in the relationship that quantum entities maintain with space. This relationship, which remains the true misunderstood aspect of the ontology of these entities, has no classical analogue. It is the true origin both of the spatial delocalization of these entities and of their correlation in the phenomenon of entanglement. With respect to all this, postulates i-iv that the author summarizes on the last page neither add nor subtract anything.

Another obvious limitation of the proposal is that starting from a real variable as the minimal radius of the neighborhood in the phase space, it is practically impossible to arrive at a surrogate for complex amplitudes. A drawback that the author, with great honesty, acknowledges. Roughly speaking, complex numbers enter the history through quantum discontinuity, which causes the Psi state to jump into the Phi state or viceversa. Since the processes Psi --> Phi, Phi --> Psi are generally distinct, we have two transition amplitudes < Psi | Phi >, < Phi | Psi > distinct. It can then be seen that the relationship between them is that of a complex conjugation.

Along the paper, the author's position on the meaning of the connective "-->" appears to oscillate between two interpretations between which he does not make any choice: that according to which Psi --> Phi is a transition between states and that according to which it represents "all the Phis that are Psi", Psi being the temporal evolute of the initial state. I think this point should be clarified. Consider for example Psi(x) = < x | psi >. It is obvious that | x > is contained in | Psi >, and that the probability of observing x is | Psi(x) |^2. But this is to be understood as the probability of the transition Psi --> x, i.e. as the probability that, having observed x, one was in Psi, rather than the probability that being x we are (still) in Psi. I fear that, by eliminating the time step constituted by the transition and with it its direction, the basis for the introduction of complex amplitudes will also be abolished.

The article is overall well written, but some points seem obscure to me and should perhaps be better explained:

1) The section "state overlap" describes the superposition of "balls" on the same phase space in relation to the superposition of states of different systems. We are actually talking about superposition of different states of the same system, otherwise we would have distinct phase spaces without superposition.

2) Note 3. "the possibilities are that c(k,csi) = c(k,eta) or that one of the symplectic capacities are enclosed by the other". Why?

3) Page 5, left column. "Furthermore, due to the indeterminacy relation, the Hamiltonian cannot quantify changes in the overlap with infinite precision. The Hamiltonian can therefore only be defined in units of the greatest possible resolution". What exactly does this sentence mean? Is there a limited definition of the Hamiltonian?

4) Page 6, section "Ensemble of similar states". It begins with the sentence: "Consider an ensemble of closed systems". Or are they different states of the same system?

5) Page 7, section "Superposition of overlaps". We find the sentence: "The quantum ensemble is for this reason physically constrained by the requirement that there exist no mutual overlaps between the state preparation and two, or more, members of the quantum ensemble". This requirement is unnecessary if we interpret Omega in terms of transition. A transition connects two states: csi and (say) eta1. It cannot connect three states as csi, eta1 and eta2.

6) Equation (29). In the case of Psi(x) and the various values of x, this is quite evident. But the interpretation in terms of

fidelity should perhaps be explained better in relation to the meaning of the connective “-->”. AFTER the transition Psi --> x we have x, which is a new preparation, and the old Psi therefore cannot be "mistaken" for x. BEFORE the transition Psi is also x, and this is not "accidental".

*See the link: https://www.youtube.com/watch?v=9C3vtVADL1o&list=PLRHyCKf119nMl8GCUNNNV_Bw9VKLPgLgd&index=1

for an amusing, elementary introduction to quantum structures.