Research Article

A Hubbert Curve from Six Wooden Sticks: A Mechanical Test of Single-Cycle Lotka-Volterra Dynamics

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M. King Hubbert introduced in 1956 the concept of a bell-shaped production curve to describe the extraction of a finite oil resource. The curve is often represented phenomenologically as the derivative of a logistic function, but it can also emerge from a feedback-based interaction between a resource stock and the capital stock that exploits it. This interaction was previously described as the "Single-Cycle Lotka-Volterra" (SCLV) model. One mechanistic way to generate the Hubbert curve in the laboratory was found to be an array of spring-loaded mousetraps and flying balls. Here, a much smaller mechanical system was tested: a compact "stick bomb" made from six interlocked wooden sticks storing elastic potential energy. The experiment supports the interpretation of Hubbert-type curves as the macroscopic signature of a finite stock of available potential being accessed by an active stock and ultimately dissipated by a combination of enhancing and dampening feedbacks. The result it shows that the same minimal dynamics can connect interactions in a variety of fields: fissile nuclei, petroleum extraction, mousetrap chain reactions, elastic stick release, and more.

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1. Introduction

The bell-shaped Hubbert curve is one of the best-known graphical representations of the exploitation of a finite resource. Hubbert used it to describe the rise and decline of fossil-fuel production, most famously in his 1956 analysis of United States petroleum production [1]. In its simplest form, cumulative extraction is represented by a logistic function and annual production by its first derivative. Production begins

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slowly, accelerates as extraction capacity expands, reaches a maximum, and then declines as the remaining resource becomes progressively less able to sustain the productive system.

The logistic derivative is useful as an empirical curve, but it does not by itself identify the physical mechanism that generates the curve. A mechanistic interpretation was proposed by Bardi and Lavacchi [2] and later developed in several applications [3][4]. The basic idea is that production requires the autocatalytic interaction of two coupled stocks: a resource stock and a capital stock capable of exploiting it. These two stocks must be in a reciprocal interaction of feedback that governs their dissipation (autocatalysis). Oil remaining underground is the resource; wells, drilling equipment, refineries, infrastructure, and the knowledge embodied in the productive system form the capital stock. Resource extraction creates or maintains capital, while capital depreciates and dissipates. When resource abundance declines sufficiently, capital can no longer replace its losses, and both production and capital eventually decline.

This interpretation is closely related to predator-prey dynamics [5][6]. It differs from the classical Lotka-Volterra cycle because a nonrenewable resource does not reproduce on the timescale of exploitation. The resulting system performs only one rise-and-fall cycle and was therefore termed the Single-Cycle Lotka-Volterra model, or SCLV [3][4][7]. In thermodynamic language, the system contains an initial stock of available potential or free energy, an active structure that accesses and transforms that potential, and an irreversible pathway toward dissipation.

This paper originates from a previous mechanical test carried out by Perissi and the present author [7], who used an array of loaded mousetraps carrying wooden balls. This experiment was initially proposed by Sutton [8] as an illustration of the chain-reaction that powers nuclear warheads. In Sutton's interpretation. cocked traps represented U-235 nuclei, while flying balls represented the neutrons generated by the fission reaction. The present study asks whether the same dynamics can be observed in a system that is mechanically different and far smaller than the mousetrap one: six interlocked wooden sticks. The purpose is not to claim that every collapse must be Hubbert-shaped, but to test whether a finite elastic system undergoing autocatalytic release produces the same family of curves.

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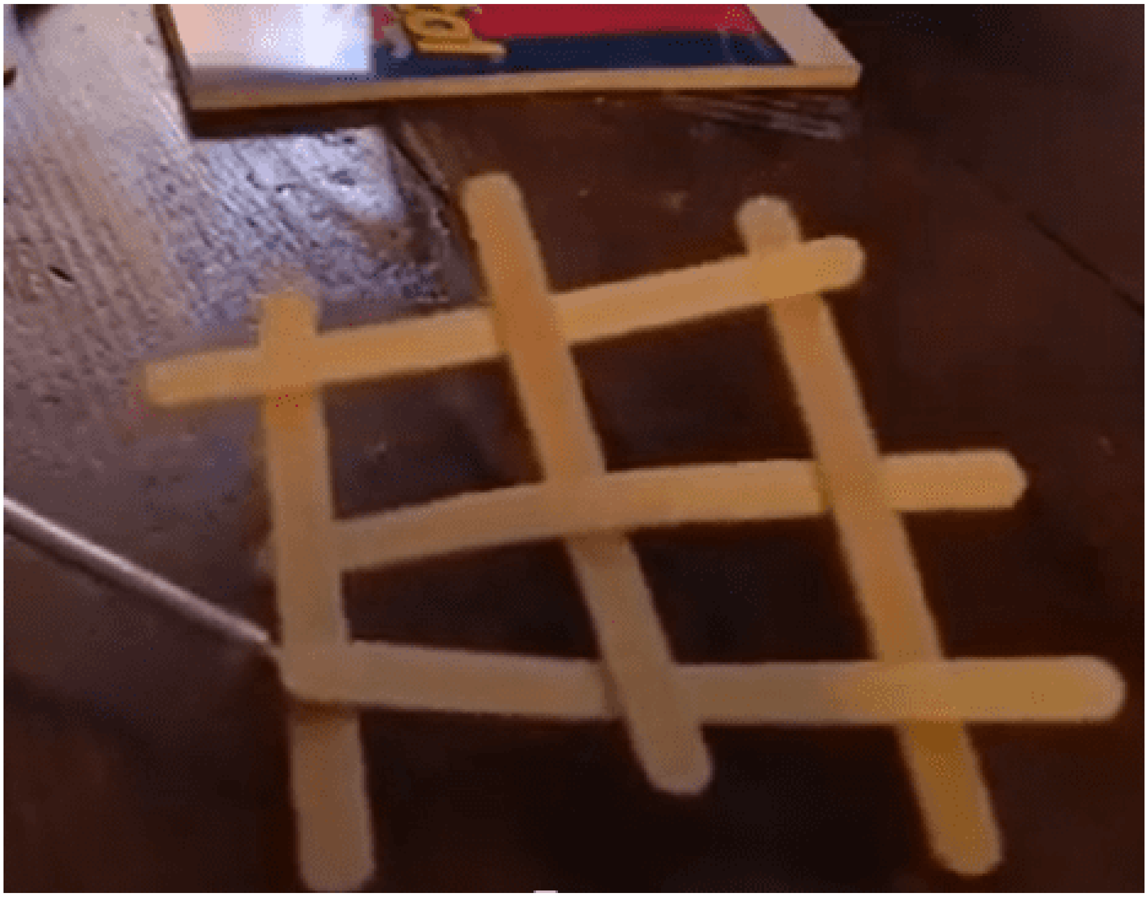

Figure 1. One of the “stick bombs” used in the present study shown as a single frame from a movie in which it was “detonated” using a screwdriver..

2. The Single-Cycle Lotka-Volterra model

Let R(t) be the resource (or prey) stock, in this case, the number of sticks still bound in the loaded structure. Let C(t) be the Capital (or predator) stock, in this case, the number of sticks in the active, flying state. The minimal model used here is

$$dR/dt = -aRC,$$

$$dC/dt = aRC - bC,$$

where a is the interaction coefficient and b is the loss rate of the active stock. Because one bound stick becomes one flying stick, the same coefficient *a* appears in the depletion and capital-creation terms. A third stock, G(t), contains grounded sticks and closes the balance:

$$R(t) + C(t) + G(t) = 6,$$

$$dG/dt = bC.$$

The production flow is the rate at which the bound stock is released:

$$P(t) = -dR/dt = aRC.$$

P(t) is the variable that's expected to show a Hubbert shape.

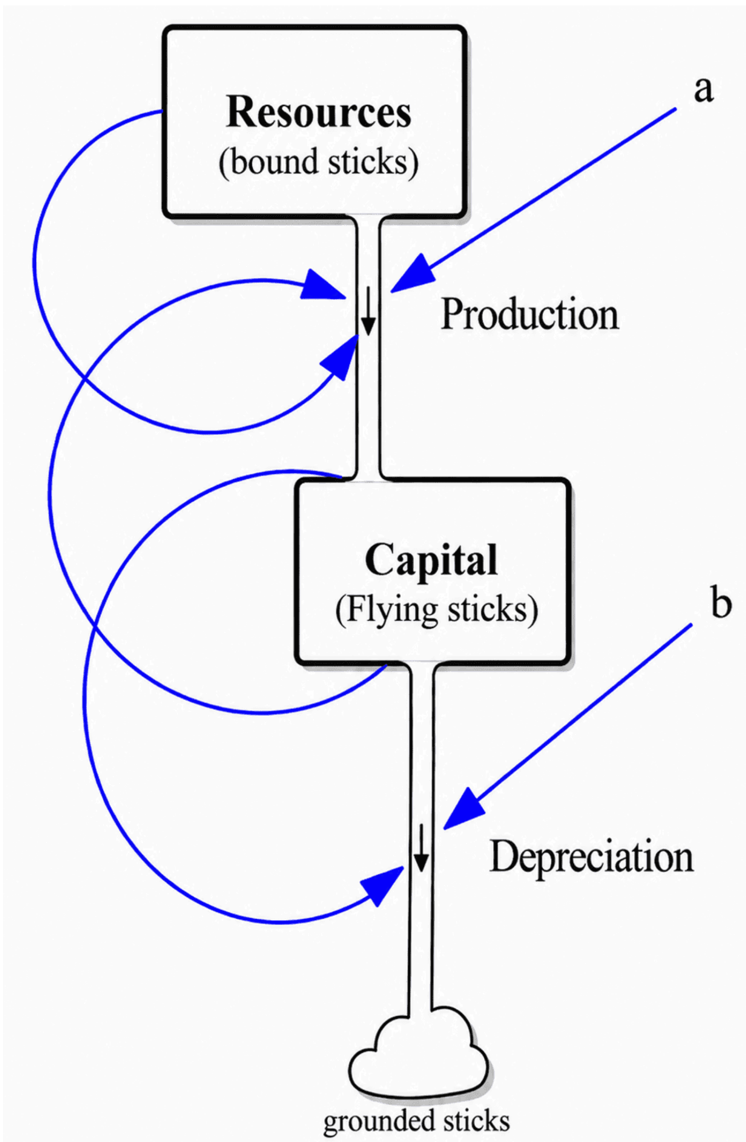

Figure 2. Stock-and-flow representation of the normalized SCLV model used for the six-stick experiment. Flying sticks act as the active stock that promotes release while losing activity as they land.

3. Materials and methods

3.1. Mechanical system

The experimental system was a compact square frame made from six commercial wooden craft sticks – the kind normally defined as a “stick bomb”. The sticks were interlocked under bending stress so that each element constrained other elements. The structure therefore stored elastic potential energy and remained metastable until a small mechanical disturbance started the release.

Once the constraint failed, the remaining sticks separated rapidly and were projected outward. This compact frame differs from the long “cobra weave” commonly studied as a traveling elastic wave [9]. Here, the system was treated as a lumped six-element system.

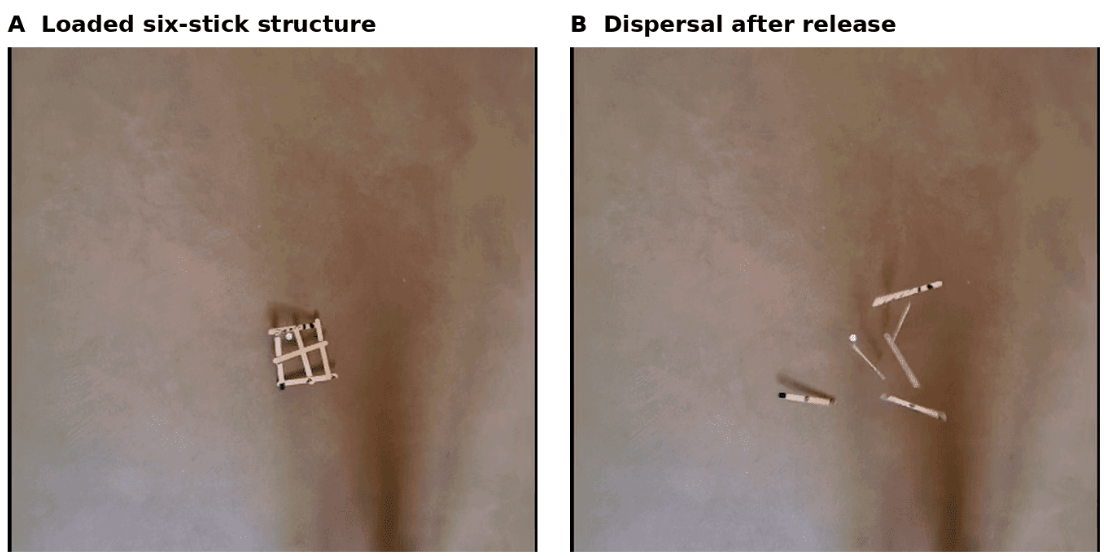

Figure 3. Video frames from a representative run. (A) The loaded six-stick structure immediately before release. (B) The sticks during dispersal. The photographs are unaltered frames cropped from the experimental video.

3.2. Recording and selection of runs

Eight releases were recorded with a Samsung Galaxy S22 smartphone. Five files were recorded in the ordinary slow-motion mode at 1920 × 1080 pixels. Their metadata explicitly reported a capture rate of 240 frames s^{-1} and they contained audio. Three additional files were recorded in the phone’s super-slow-motion mode at 1280 × 720 pixels. Because the exact effective capture rate of the latter mode is less transparent and the files contained no audio, these three runs were used as qualitative controls but excluded from the primary parameter estimation. This conservative choice avoids introducing a timing conversion that depends on the phone's interpolation procedure.

3.3. Event scoring and construction of the production curve

Each of the five primary videos was inspected frame by frame. A stick was scored as released at the first frame in which it was no longer mechanically constrained by the original structure. Six release events were therefore obtained from each run, for a total of 30 events. The temporal resolution was 1/240 s, or 4.17 ms. The complete-release interval was measured from the first to the sixth release.

Runs were aligned at the midpoint between the third and fourth releases, corresponding to half depletion of the six-stick resource. Alignment at the first release was avoided because it would force one event from every run to the same time and create an artificial spike. Release events were grouped in 20 ms bins centered at half depletion. The estimated rate in each bin was $n/(N\Delta t)$, where n is the number of release events, $N = 5$ is the number of runs, and $\Delta t = 0.020$ s. Error bars were estimated as $\sqrt{n}/(N\Delta t)$, using Poisson counting uncertainty.

3.4. Model fitting

The SCLV equations were solved numerically with the initial bound stock fixed at $R_0 = 6$ and a very small active stock used to initiate the trajectory. The resulting curve was shifted in time so that $R = 3$ at $t = 0$. The two free parameters were a and b. Expected event counts in each 20 ms bin were obtained from the modeled decline of R(t), and the parameters were estimated by maximizing the Poisson likelihood of the observed counts. Fit quality was evaluated with the Pearson chi-square statistic. A logistic-derivative fit was also examined as a phenomenological comparison, but it was not used to infer mechanism.

4. Results

All five 240 frame s^{-1} recordings showed the same qualitative sequence: a short initial release, rapid multiplication of release events, and a final tail involving the last one or two sticks. The interval between the first and sixth release averaged 72.5 ± 19.5 ms (mean ± sample standard deviation). Thus, the depletion of the six-element resource stock was complete in less than one tenth of a second in a typical run.

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Figure 4 shows the experimental production-rate estimates and the SCLV fit. Despite the small number of elements, the image displays the expected single pulse. The fit gives a = 21.9 $stick^{-1}$ s^{-1} and b = 39.5 s^{-1}. The fitted peak production rate is 117 sticks s^{-1} and occurs 3.3 ms before the chosen half-depletion origin. The full width at half maximum is 41 ms. The production maximum occurs while the modeled bound stock is approximately 3.38 sticks. The active stock reaches its own maximum later, when the bound stock falls to b/a = 1.81 sticks.

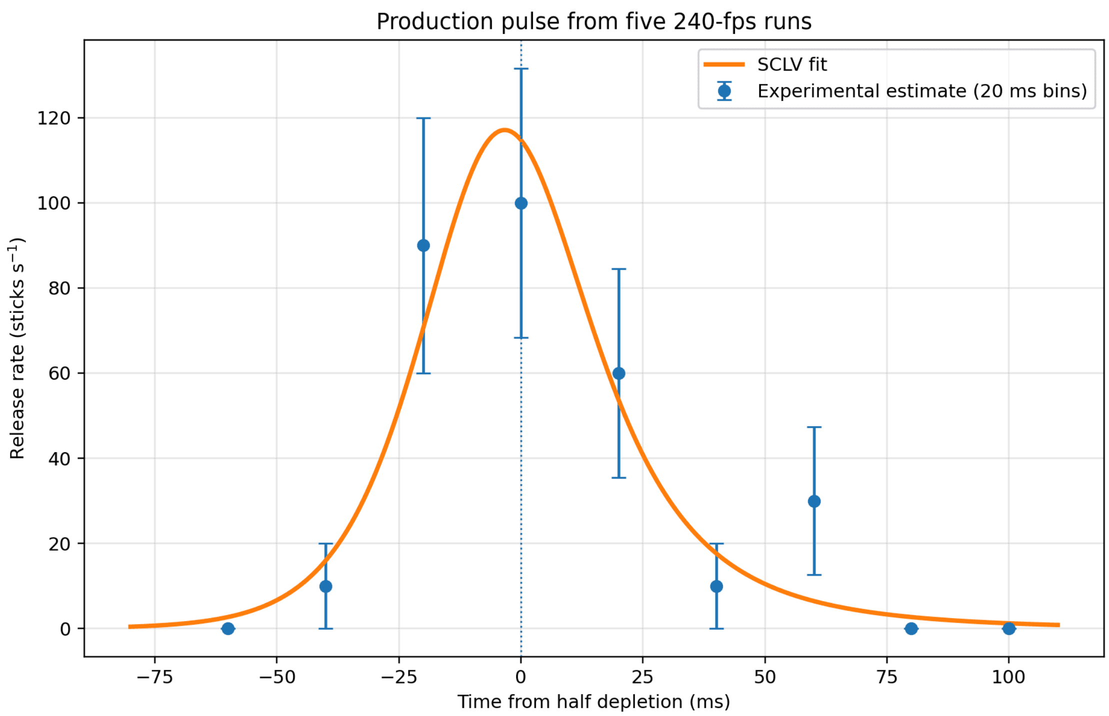

Figure 4. Experimental release-rate estimates from five ordinary slow-motion runs and fit with the Single-Cycle Lotka-Volterra model. Points are 20 ms bins; error bars are Poisson counting uncertainties. Time zero is the midpoint between the third and fourth releases in each run.

The Pearson statistic is χ^2 = 10.1 for 7 degrees of freedom, corresponding to p = 0.18. The data therefore provide no evidence of an inconsistency with the SCLV curve at this level of precision.

The fitted model leaves a residual bound stock of 0.25 stick, which is physically indistinguishable from complete depletion in a six-stick experiment. A logistic derivative also fits the binned data acceptably (χ^2 = 9.56, 7 degrees of freedom, p = 0.22). The two curves cannot be statistically distinguished with 30

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events. The significance of the SCLV description is consequently not a superior numerical fit, but the fact that the curve emerges from an explicit resource-capital interaction.

4.1. Fitted parameters

Quantity	**Estimate**	**Interpretation**
A	21.9 $\text{stick}^{-1}\ \text{s}^{-1}$	Interaction between bound and active sticks
B	39.5 s^{-1}	Loss rate of the active/flying stock
b/a	1.81 sticks	Bound-stock level at the capital maximum
Pmax	117 sticks s^{-1}	Peak modeled production flow
FWHM	41 ms	Width of the production pulse at half maximum
χ^2	10.1 (7 df; $p = 0.18$)	Goodness of fit to binned event counts

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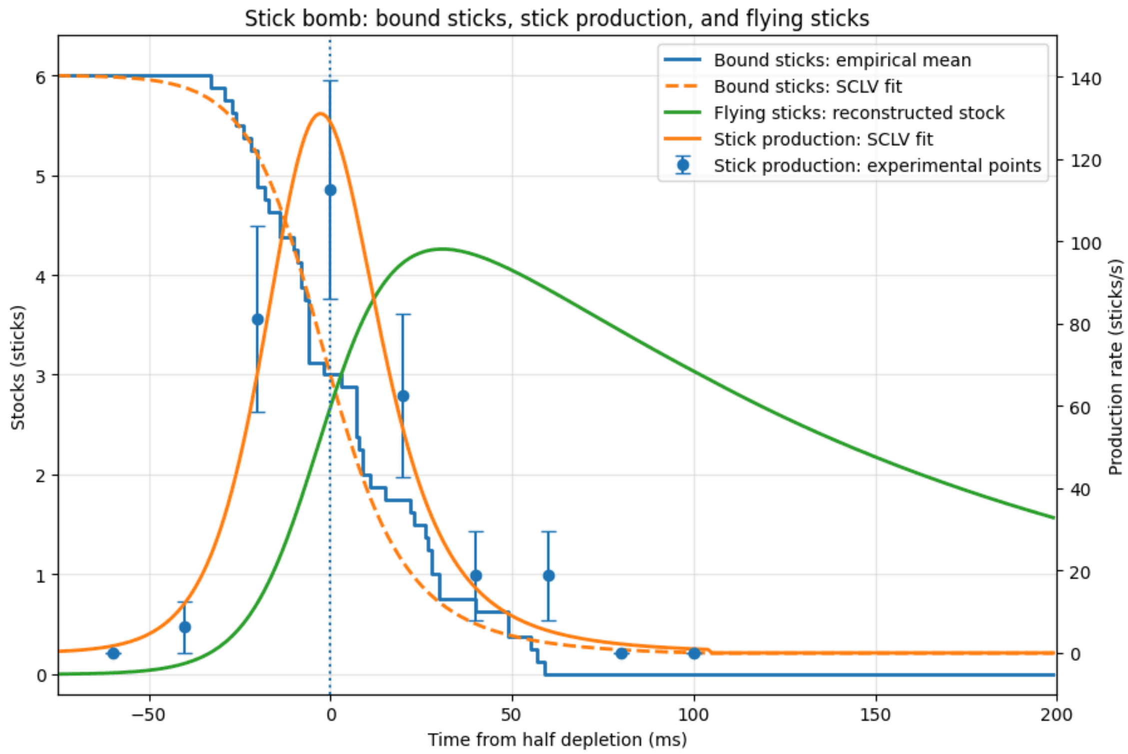

Figure 5. Experimental data for the three measured parameters from five ordinary slow-motion runs and fit with the Single-Cycle Lotka-Volterra model. Points are 20 ms bins; error bars are Poisson counting uncertainties. Time zero is the midpoint between the third and fourth releases in each run. The staircase shape of the empirical bound-stick curve reflects the discrete nature of the experiment rather than the time bins used to estimate the production rate. In each run, the bound-stock variable can take only the integer values 6, 5, 4, 3, 2, 1, and 0, decreasing by one whenever a stick is released. Since the figure reports the mean of eight runs, each individual release changes the ensemble-average stock by $1/8$, or 0.125 stick, producing the smaller visible steps. By contrast, the production-rate points were obtained by counting release events within 20 ms intervals and dividing by the bin width. Thus, the stepped curve is a direct representation of the finite-element experimental observations, while the smooth SCLV curve represents their continuum approximation.

5. Discussion

The mousetrap and stick experiments differ in geometry and mechanism. A mousetrap array is a spatial network of spring-loaded devices activated by mobile balls. The six-stick frame is a compact prestressed structure in which the moving elements and the stored-energy elements are the same physical objects. Nevertheless, both systems contain three similar stages: a stored potential, a transient active stock, and a

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dissipated final state. In both cases, release is autocatalytic because active motion promotes further release while the remaining bound stock is progressively depleted. For the mousetrap array, it is the elastic energy of cocked springs. For the stick structure, it is the elastic bending energy maintained by the interlocking constraints

The term "resource" can be generalized as a stock of thermodynamic potential that can be detected in many kinds of systems, not just mechanical ones. For nuclear explosions, it is the energy potential of the fissile U-235 nuclei. For petroleum, the relevant potential includes chemical exergy and the geological concentration that makes extraction possible. In every case, the active stock channels the potential into motion and useful or observable work, after which the energy is dissipated as heat, sound, deformation, and friction.

This common structure explains why the production flow can be Hubbert-like even when the mechanisms of the dissipation processes are different. The general concept of the curve is not dependent on the physical characteristics of the system, but it is determined by the interplay of the feedbacks occurring during the dissipation process. That explains why the results are similar for so many different systems: mechanical systems, as in this paper and [7], economic systems such as oil production [1] and fisheries [10], socioeconomic systems and many others [11].

The experiment does not prove that every dissipative process follows the SCLV equations. It does show that a mechanical minimal resource-capital model survives a change of scale and mechanism: from regional petroleum production, to dozens of mousetraps, to only six wooden sticks. The Hubbert pulse may therefore be viewed as a generic signature of a broad class of finite-potential dissipation processes characterized by autocatalytic access and loss of the active stock.

6. Limitations and further work

This is a pilot experiment with several obvious limitations. First, the resource contains only six discrete elements, so the production curve is necessarily granular and sensitive to individual release events. Second, release times were identified manually and carry an uncertainty of approximately one video frame. Third, the fitting used the production flow derived from release events; the active flying-stick stock was not independently fitted across all runs. Fourth, no direct measurement of elastic energy was made. Finally, all experiments used one geometry, one stick type, and one operator.

A more complete test would use larger structures and multiple geometries, record both release and landing times, and measure stick dimensions and bending stiffness. This would permit simultaneous fitting of R(t), C(t), and P(t), and would test whether the fitted coefficients scale with the stored elastic energy and the geometry of the constraints. Automated image tracking could also reduce the uncertainty associated with manual scoring.

7. Conclusions

A compact structure of six interlocked wooden sticks produces a reproducible rise-and-fall pulse in the release rate. The pulse is described adequately by the Single-Cycle Lotka-Volterra model previously used for nonrenewable-resource exploitation [2][3][4] and for the mousetrap chain-reaction experiment [7]. The result supports a mechanistic interpretation of the Hubbert curve: production peaks because it requires the simultaneous presence of an available resource stock and an active stock capable of accessing it. The experiment is small and inexpensive, but its simplicity is precisely its value. It demonstrates that Hubbert dynamics can emerge from the dissipation of stored elastic potential without fissile nuclei, petroleum, markets, or even mousetraps.

Statements and Declarations

Funding

No specific funding was received for this work.

Potential Competing Interests

No potential competing interests to declare.

Ethics

The study involved no human or animal subjects, and no personal data.

Data Availability

The processed event table, binned production data, fitted curves, and analysis code are provided as Supplementary Data. The eight original MP4 recordings are available from the corresponding author on request.

Author Contributions

U.B.: Conceptualization, methodology, investigation, data curation, formal analysis, visualization, writing—original draft, writing—review and editing.

Use of Generative AI

OpenAI ChatGPT (GPT-5.6 Thinking) was used to assist with frame bookkeeping, numerical fitting, figure preparation, and language editing. The author inspected the video records and derived data, reviewed the calculations and manuscript, and accepts full responsibility for the accuracy and integrity of the work. No generative AI was used to create or alter the experimental photographs.

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Supplementary data: available at https://doi.org/10.32388/SOCIJ4

Declarations

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