Review of: "On the statistical arrow of time"

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The physical origin and conditions of the arrow of time, manifest in a plethora of phenomena in physics and elsewhere, are the subject of a vast body of literature, from popular science to highly sophisticated theory. Any further contribution to the subject must therefore be measured by one crucial question: whether it really contains new material or arguments that enhance the debate. I have doubts that the present manuscript meets this criterion.

The author pretends bringing forward reasons why an information-theoretic interpretation of time asymmetry is incompatible with its physical embodiment, the Second Law of Thermodynamics. He avoids taking a personal standpoint in favor or against this interpretation. Instead his reasoning culminates in the seemingly paradox conclusion that by using increasingly powerful equipment for measuring and computing, the arrow of time will be less evident, and for infinitely precise measurement of initial conditions and infinitely accurate calculation of the time evolution of a dynamical system, it would disappear altogether.

My impression is that this reasoning suffers from a number of critical flaws. Above all, it misses elements considered as central in the recent debate on the origins of the time arrow:

– The author mentions and discusses a decisive feature of many-body systems approaching equilibrium, ergodicity, but leaves out another property that has proven at least as important: mixing. It implies ergodicity, yet it is stronger in that it requires the system to cover its accessible phase space not only on average, but at each moment in the long-time limit. Mixing is equivalent to a complete and irreversible decay of auto-correlations between the final distribution and initial conditions and thus already comes close to equilibration.

While many mechanical systems are ergodic, only very few (e.g., the Sinai billiard) could be proven to be mixing.

– In terms of an underlying deterministic dynamics, chaotic behaviour is a necessary and almost sufficient condition for mixing. Deterministic chaos, in turn, allows for a clear information-theoretic interpretation: It conveys information on the initial state from small (microscopic) to large (macroscopic) scales.

– These two features have to be complemented by a third element the author addresses only indirectly, the presence of a large number of degrees of freedom that interact so strongly that the information on the initial state of a single freedom is rapidly scrambled among all the others. This provides the coarse-graining that prevents retrieving the information from the final conditions that would allow reconstructing the initial state.

Together, these ingredients imply an objective asymmetry in the global information flows in dynamical systems, manifest in the dominating role of top-down (from macro- to microscales) currents. With the assumption of an initial state of the universe (at the big bang) far from equilibrium, i.e., exhibiting large-scale structure, they combine to provide an objective interpretation of the arrow of time.

To be sure, there does exist a condition depending on the nature of the observer, thus an element of subjectivity, which the author however does not even address: An observer, who himself follows a time-reversal symmetric time evolution, would not be aable to detect an aymmetry in the observed data, since for him, the time-reversed dynamics would not look different. It needs an internal time arrow of his own, for example memory, to be compared with the external time arrow to find coincidence. Therefore, a quantum computer with its unitary time evolution would be blind for the time asymmetry in its environment. Only the macroscopic nature of us as observers therefore allows us to see and to ponder the time arrow. However, that does not challenge its objective existence.

Finally, the persuasive power of this manuscript is affected by the sloppy handling of some of the technical arguments. I would like to mention just a few weak points:

– Equation (1): A phase-space region occupied by a statistical ensemble can have any shape. Its volume is not necessarily given by a simple product of uncertainties.

– Equation (4): A dimensionless probability is given by an integral of a probability density, of dimension 1/(phase-space volume) over the phase-space region in question. This should be taken into account by chosing a notation for probability densities, different from that for probabilities.

– Equation (5): Invariance of the local probability and information is only guaranteed for the Lagrangian (comoving) time derivative. That ought to be mentioned.

– Why should "coarse-grained regions" in phase space not move around and change their shape, e.g., from circular (as in Fig. 1 in the manuscript) to any kind of complex ramified outline?

– Pure states: The author works consistently in the formalism of classical mechanics. In that framework, pure states are badly defined. If anything like that existed, the notion could only refer to Dirac delta functions in phase space. However, they are singular, and in particular, derivatives such as the divergence of the velocity density, Eq. (10) are not defined. A "total number of pure states" would diverge as well.

While I agree with the partial conclusion of the technical part of the paper, the derivation leading needs improvement.

The bibliography contains a number of important references concerning technical questions. However, for readers not familiar with the debate, citing in addition the one or other review or monograph on the Second Law, equilibration, irreversibility, the time arrow, etc., would have been helpful.

To summarize, I think the author's reasoning omits essential points that have already been widely recognized and proven indispensable for the discussion. His bottom line is a strange conditional: Assuming a subjective interpretation of probability, the time arrow cannot be explained with information-theoretical arguments. Readers are confronted alone with the decision whether to either deny the fundamental nature of the arrow of time, or else abandon its origin in entropy dynamics. I am afraid most readers will not accept this weird alternative or at best be left puzzled, but hardly be convinced.