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Review Article

Early Renaissance Concepts of Time and the Invention of Mechanical Clocks

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Mechanical clocks appeared in Europe at the end of the thirteenth century and became widespread during the fourteenth century. Clocks utilized a periodic process – the oscillation of a verge – for accurate time measurement, which is consistent with the Aristotelian concept of time defined by motion. The concept was at the center of debates among scholastics and theologians, which culminated in the Church's Condemnation of Aristotelian teaching in 1277. The debate also led to the discovery of Impetus (inertial motion) by Buridan and Oresme. Sand clocks became a popular device at about the same time. The pendulum clocks invented by Huygens in the 1650s utilized linear oscillations, which, unlike the earlier verge mechanism, had their natural frequency. Both mechanisms are analyzed to explain why the introduction of the linear oscillator led to a 30-times accuracy improvement of the clock.

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Introduction

The origins of the Scientific Revolution of the seventeenth century are a subject of long-lasting discussions. One of the central issues in these discussions is the role of the fourteenth-century scholastics in the development of the ideas about time, space, and motion, which culminated in the discovery of the Galilean Principle of Inertia almost three hundred years later[1]. It has been widely recognized that the attitude of these medieval scholars towards the concepts of Aristotle's physics was influenced by the so-called "Condemnation of 1277," which was a prohibition by the Catholic Church in Paris to teach certain radical Aristotelian ideas[2].

The impact of the Condemnation on the development of the physical concepts of time, space, and motion, has been studied by historians of science since the early twentieth century and, in particular, after the 700th anniversary of the prohibition in 1977. The influences of the prohibition on theology, physics, metaphysics, and logic have been widely discussed in literature[3]. However, one aspect of this influence has not been investigated in detail: the development of the practical technology of measuring time, and, in particular, the invention of mechanical clocks[4].

The objective of the present article is twofold. First, I would like to bring the attention of researchers to almost simultaneous parallel developments in time-measuring technology, namely, the invention of mechanical tower clocks at the end of the thirteenth century and the emergence of sand clocks at about the same time. Both inventions became widespread in Europe in the fourteenth century, at about the same time when the new attitudes towards time and motion had crystalized and were actively debated. Second, I would like to investigate, from the viewpoint of a mechanician, the emergence of the pendulum-less mechanical clock in the thirteenth century and its relation to the invention of a much more accurate pendulum clock in the 1650-1670s by Huygens. According to horological literature, after the introduction of the pendulum, the accuracy of clocks had increased by about 30 times[5]. This increase requires an explanation from the dynamics point of view.

The Condemnation of 1277 and the concepts of space, time, and motion

Already in 1906, Pierre Duhem (1861–1916), a renowned physicist and historian of science, paid attention to certain aspects of the Scientific Revolution, whose origin can be traced in the Middle Ages. According to Duhem, the so-called "Condemnation of 1277[6]" encouraged scholars to question the tenets of Aristotelian science. One particular Aristotelian postulate, which was criticized, was the idea that there is no absolute time and space. Another idea was that motion always has a cause, which is acting instantaneously. The abandonment of these concepts, according to Duhem, was very productive, because it resulted in the invention of the theory of impetus (the motion by inertia or without a permanently acting force) by Jean Buridan (1301–1359/62) and Nicole Oresme (1320/5–1382), which eventually led to the genesis of the modern, empirical scientific method almost four hundred years later, in the seventeenth century. This included the Newtonian concepts of absolute space and time as opposed to Aristotle's notion of time as defined by motion. According to Duhem, 1277 was the birth date of modern science.

Many historians disagreed with Duhem's thesis. Thus, Alexandre Koyré suggested that the condemnation was insignificant to scientific developments because it concerned theoretical logical possibilities, rather than the actual state of the world[7]. Other scholars stressed that the Condemnation of 1277 was never fully enforced, and many of its articles had been revoked and canceled in the first half of the fourteenth century[8]. However, there is no doubt that the thirteenth and fourteenth century discussions about the nature of time and motion influenced the emergence of the theory of impetus and affected the attitudes towards time as a measurable property.

There are two main philosophical concepts of time: time as defined by motion (or the Aristotelian Relationism concerning Time) and absolute time (sometimes called Platonism or Absolutism with Respect to Time)[9]. The Aristotelian concept of time prevailed in the Middle Ages, and it became especially influential in Europe in the thirteenth century following the translation of several of Aristotle's works into Latin[10], and due to the penetration of ideas of Averroes from the Muslim world and, perhaps more importantly, due to the influence of Thomas Aquinas (1225–1274) who combined Catholic theology with Aristotelian ideas[11].

Among the Aristotelian theses condemned in 1277, several were relevant to the concepts of space, time, and motion. Thus, it was forbidden to profess:

"49. That God could not move the heavens with rectilinear motion; and the reason is that a vacuum would remain[12].

87. That the world is eternal as to all the species contained in it; and that time is eternal, as are motion, matter, agent, and recipient; and because the world is from the infinite power of God, it is impossible that there be novelty in an effect without novelty in the cause[13].

139. That an accident existing without a subject is not an accident, except, equivocally; and that it is impossible that a quantity or dimension exists per se, since it would make it a substance[14].

140. That to make an accident exist without a subject is an impossible argument implying a contradiction.

141. That God cannot make an accident exist without a subject, no make several dimensions exist simultaneously at the same place."

Some of these articles were intended against the Aristotelian concept that nature does not tolerate emptiness and no vacuum is possible. The Condemnation affirmed the position that while nature might not tolerate a vacuum; this does not imply that God cannot create a vacuum. Other prohibitions were directed against the idea that the world always existed, rather than it was created. Another important consideration was whether accidents (meaning attributes or properties) can exist without a subject (meaning without substance). This is particularly relevant to the notions of absolute space and time, which were viewed by some scholastics as attributes without substance.

The general trend of the Condemnation was to affirm and emphasize the omnipotence of God when it comes to certain issues questioned by scholars. Many of the condemned articles forced scholars to concede that God could do something that had been previously denied him, without being subject to any limitations, even highly theoretical limitations including those imposed by God's own laws.

The argument between Aristotelian scholars and more conservative theologians about Aristotle's heritage had

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far-reaching consequences for the history of natural sciences. Thomas was influenced by the Aristotelian concept of an acting force required to maintain motion, and he employed it as an argument for his proof of the existence of God. Buridan and Oresme intended to fight Thomas when they suggested the idea of impetus, or motion by inertia (i.e., without an acting force), which eventually led to the discovery of the principle of inertia.

Perhaps the most important for the concept of time was the article of the Condemnation prohibiting teaching:

200. “That eternity and time have no existence in reality but only in the mind” (“*Quod evum et tempus nichil sunt in re, sed solum in apprehensione*”)[15].

Here the Aristotelian concept of time as an illusion defined by motion was explicitly refuted. Obviously, the necessity to refute something indicates that the concept had become quite popular. And indeed, the idea of measuring linear time by a periodically oscillating process led to the invention of the mechanical clock.

The invention of the mechanical clock

Accurate time measurement was an important technological problem from the early history of humankind. Earlier devices designed for this purpose included the sun clock (sundial), **Fig. 1**[16], water clock (clepsydra), **Fig. 2**, fire clock, and sand clock (sand glasses). These relatively primitive time-keeping devices relied on a process having more or less a constant rate, such as the sun’s motion and candle burning, or on a uniform flow of material, such as water or sand[17].

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Fig. 1. A first century Nabatean sundial from Mada’in Salih (Hejaz, Saudi Arabia), Istanbul Archeological Museum, Inv. 7664. Photo by the author.

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Fig. 2. Remnants of the *Dar al Magana* water clock (Fez, Morocco), built in 1357. Photo by the author.

Mechanical clocks, which appeared in Europe by the end of the thirteenth century, employed a completely different principle[18]. For accurate time measurement, mechanical clocks relied on a periodic oscillatory motion of a mechanical shaft called the *verge*, and they used an escapement mechanism called the *verge-and-foliot* mechanism. Although the verge oscillated periodically, achieving an isochronic periodic motion, i.e., motion with more or less constant frequency, was a challenging task until the pendulum was invented by Galileo in the late sixteenth century.

Neither the name of the inventor of the verge-and-foliot mechanism is known, nor even the country in which it was first invented. Some historians suggest that Eastern, and in particular, Muslim influences could be involved, since the Muslim civilization was quite advanced in astronomical knowledge, for example, using the astrolabe[19]. However, mechanical clocks are not known in Muslim countries of the period. So, if the Eastern influence existed at all, it was likely indirect, through the proliferation of Aristotelian teaching.

According to some historians, a vague description of the mechanical clock mechanism is contained in the 1271 note by an English astronomer Robertus Anglicus (Robert the Englishman). He wrote a commentary passage for an astronomical textbook *De Sphera* by Sacrobosco: “Conantur tamen artifices horologiorum facere circulum qui omnino moveretur secundem motum circuli equinoctialis” (“[They] are trying to build a wheel moving exactly according to the motion of the equinoctial circle”)[20]. However, this description turns out to be a mere statement by the author that a mechanical time-measuring device would be desirable, although it had not been successfully built.

Another hypothesis attributes the first escapement mechanism to the French architect Wilars de Honecourt. His notes composed between 1240 and 1251 contain drawings of various mechanisms. One of these mechanisms might be a type of escapement, although alternative interpretations are also possible.

As far as the actual clocks, the first known example of a mechanical tower clock, which employed the verge-and-foliot escapement mechanism, was built in the town of Dunstable in England in 1283. Other famous early mechanical tower clocks include London’s St. Paul Cathedral (1286), Westminster (1288), Canterbury (1292),

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Strasbourg (1352/4), Paris (1362), Padua (1364), and Salisbury (1386)[21].

The first clear drawing of an escapement was found only in a 1364 manuscript of a treatise by father and son, Jacopo (1290–1359) and Giovanni (1318-1389) de' Dondi "Dondi dall 'Orologio" (**Fig. 3**). Jacopo designed a clock (hence "dall'Orologio" was added to the family name), which was installed in 1344 in the Torre dei Signori of the Palazzo del Capitanio at Padua. His son Giovanni Dondi was also a clockmaker, who built an elaborate astronomical clock, which he completed in 1364 following sixteen years constructing it. Francesco Petrarch (1304-1374) mentioned Dondi's work, referring to it as a "planetarium.[22]"

A clockmaker of similar caliber to the Dondis was English Benedictine abbot of St. Albans, Richard of Wallingford, (c. 1291–1336), who wrote *Tractatus horologii* with the description of complicated automata for astronomical simulations[23].

During the fourteenth century, tower clocks became a common element of urban life in Europe and they appeared in contemporary literature. Dante mentions clocks in his *Divina Commedia* at least twice: the "wheels in the movements of a clock" ("*cerchi in tempra d'orïuoli*" in *Paradiso* 10:139) and "a clock that calls us at the hour" ("*orologio che ne chiami ne l'ora*" in *Paradiso* 14:13). This part of the *Commedia* was written between 1315 and 1321. Amazingly, Dante compares the action of a clock with love: "Then, like a clock that calls us at the hour when the bride of God gets up to sing matins to her bridegroom, that he should love her still, when a cog pulls one wheel and drives another, chiming its ting-ting with notes so sweet that the willing spirit swells with love, thus I saw that glorious wheel in motion, matching voice to voice in harmony and with sweetness that cannot be known except where joy becomes eternal." (Par. 10.139-148)

The English poet Geoffrey Chaucer also mentioned the clock in his "Canterbury Tales" written between 1387 and 1400. He compared a rooster, a bird that wakes up early in the morning at approximately the same time, with the clock: "Well sikerer was his crowing in his lodge / Than is a clock of any abbey orloge". Here "sikerer" means "more accurate," "clock" means a "bell," and "orloge" means a "clock").

The symbolism of the mechanical clock

The spreading of tower clocks affected various aspects of urban life, as their regular chime replaced the ringing of the tower bells. The concept of the seasonal hour (dividing day time by 12 hours and nighttime by 12 hours) was gradually replaced by equinoctial (equal) 24 hours as measured by mechanical clocks; the difference was particularly significant in northern countries. While in cities mechanical clocks were more symbolic than practical; in monasteries they were used for the establishment of a daily routine, including the sevenfold division of the day into the canonical hours (*matins*, *prime*, *terce*, *sext*, *none*, *vespers*, and *compline*) by the Benedictine Rule[24]. Monastery regulations implied that more psalms be recited during winter nights than summer nights. The clocks affected even non-Christians. Thus, Rabbis in northern France and Germany adjusted some rabbinic regulations for winter or summer months[25].

Clocks also obtained symbolic significance as a metaphor of Nature representing a microcosm. Moreover, moving mechanical dolls of people or animals were often attached to clocks. These were known as "Jacks o'clock" (also *jacquemarts*, *batticampana*) and they symbolized animate objects and often underwent ceremonies of baptism and naming stressing their anthropomorphic qualities[26].

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Fig. 3. King Solomon repairing a mechanical clock, from *Horloge de Sapience*, 1461-65, Bibliotheque Nationale, MS fr. 455, fol 9.

The clocks were also viewed as a symbol of "temperance," the quality which implied moderation or measure. The same quality was also associated with "wisdom", **Fig. 3**[27]. While automated dolls attached to the clock mechanism symbolized living figures, the human actions and emotions could be compared to clocks. French author Jean Froissart (c. 1337 – c. 1405) compared mechanical clock with the emotions of a lover's heart in his allegorical poem *Orloge amoureus* ("The Clock of Love", around 1380) inspired by automatic striking clock in the Royal Palace of the Paris built in 1370 by Henry de Vick. This is how Froissart poetically describes the verge-and-foliot mechanism, whose mechanics will be considered in more detail in the next section, as an example of balancing and avoiding extremes, which is a quality of Temperance:

"And, because [this first wheel] would go without governance / And all too swiftly and without restraint / Had it not something that from its unruliness / Should hinder it and bring it back to rule, / And by its due control should regulate it, / For this, by proper art arranged, / There was a second wheel adjusted, / Which slows it down and makes it move / By governance and with restraint, to wit / By virtue of the foliot as well, / The which without cessation thus is moved: / One stroke to the right and then one to the left, / Nor may it nor it cannot be at rest; / Because by it this wheel is checked / And by true moderation stayed."[28]

Amazingly, the symbolism of the clock was used by both Thomas Aquinas in support of the idea of instantaneously acting force and by his opponent Nicole Oresme in support of the opposite idea of impetus. Thomas listed *horologium* and other man-made engines as evidence that motion originates not in a movable object itself, but in the mover. This is the concept of the First Mover by Aristotle[29]. In contrast,

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Oresme attributed the permanent motion of the heavenly spheres to the inertial motion or to the operation of the mechanical clock, which did not require human intervention: “When God created the heavens, He put into them motive qualities and powers just as He put weight and resistance against these motive powers in earthly things... the situation is much like that of a man making a clock and letting it run and continue its own motion by itself. In this manner did God allow the heavens to be moved continually[30]”.

Mechanics of the verge-and-foliot mechanism

Let us now pay more attention to the operation of the early clock from the mechanician’s point of view. The technical details (equations and calculations) are presented in our earlier work[31], where we demonstrated how the introduction of the pendulum resulted in the 30-fold improvement of the clock accuracy (more exactly, the improvement is by μ/π times, which is approximately $\mu/\pi \approx 31$ for the coefficient of friction $\mu = 0.1$). Here we will build upon the qualitative results of that work. The mechanism of the *verge-and-foliot escapement* in early mechanical clocks consisted of a shaft (the *verge*) connected to a crossbar called a *foliot* with weights attached to each end (**Fig. 2**)[32]. Moving the weights to different positions on the crossbar adjusted its moment of inertia, so that the period of oscillations was dependent on the distance of the weights from the center. The verge escapement mechanism also included a crown wheel with saw-like teeth driven by a weight. The crown wheel could alternately hit the two pallets fastened to the verge shaft with about 100° of angular separation. The revolving motion of the crown wheel caused an alternating circular movement of the foliot. A push on the first pallet resulted in a rotational movement in one direction (clockwise), whereas a push on the second pallet resulted in a rotational movement in the opposite direction (counterclockwise) (**Fig. 4**).

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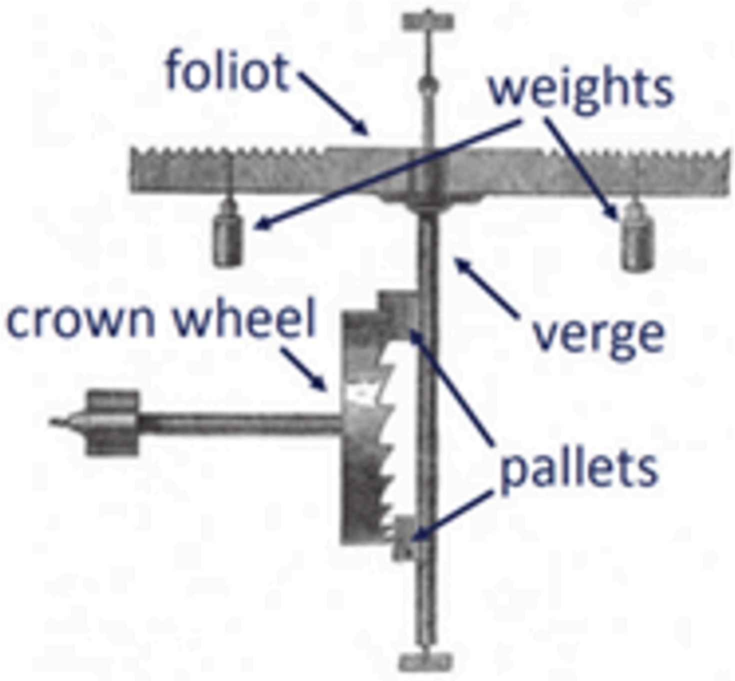

Fig. 4. The verge-and-foliot escapement mechanism from the 1379 Henri De Vick tower clock in Paris

After a tooth of the crown wheel escaped, the crown wheel rotated freely by the small drop angle of about 2° for an instant until another tooth stroked the other pallet. At that moment, the other pallet hit the tooth, and the foliot changed its direction of rotation under the influence of the force exerted by the crown wheel upon the pallet. The rotation continued for about 100° until the pallet allowed the tooth to escape again. This periodic process continued indefinitely.

In the horological literature, the introduction of the pendulum is often viewed as a revolutionary invention, whereas the emergence of the verge-and-foliot escapement mechanism is often treated as an evolutionary development of the clepsydra (water clock) used in Europe, Asia, and the Middle East. Indeed, sophisticated water clocks had their type of escapement mechanisms[33]. However, there is also a significant difference between the water clock and mechanical clocks. The former employed continuous flow to measure time intervals, while the latter used an oscillatory periodic process as a time-measuring device. The escapement in the clepsydra is essentially a water flow meter. While the transition of technical and astronomical knowledge from the Muslim world played a significant role on many aspects of the European Renaissance, such as the emergence of the Copernican heliocentric theory[34], there is no evidence of such influence with the verge escapement mechanism, which appeared at first in Europe. Therefore, early mechanical clocks with the verge-and-foliot escapement mechanism constitute a technological leap, which requires a thorough investigation.

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Fig. 5. Wall Clock with Automation, Southern Germany 1550/1600. Iron, bell metal, brass, gilt copper, and polychrome decoration 14 × 6 × 6 1/2 in (35.56 × 15.24 × 16.51 cm). Milwaukee Art Museum, Purchase, with funds in memory of Betty Croasdaile and John E. Julien M2002.182. Photographer credit: John Nienhuis (Reproduced with permission).

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Besides public tower clocks, the verge escapement mechanism was also used for private wall clocks. An amazing example of an animated wall clock mechanism is found in the Milwaukee Art Museum (**Fig. 5**)[35].

The verge mechanism is not unknown in literature on mechanical vibrations[36]. However, the question of why the accuracy of the verge escapement mechanism remains so low in comparison with the pendulum-based clock has not been addressed in most of these studies. Headrick states that "the greatest problems were caused by changes in temperature and levels of friction," however, he suggests no quantitative analysis[37]. Our earlier analysis showed that friction force was the dominant underlying reason in this effect[38].

From the mechanics (or physics) point of view, the verge-and-foliot mechanism is a non-linear oscillator, which is a very important observation for the objective of the present article. The property of non-linear oscillators is that they do not have a natural frequency (i.e., the frequency independent of the amplitude of oscillations). Instead, their frequency may depend on the balance of the driving force and friction force, which makes the period of vibration very sensitive to friction. We will return to this concept when we will compare the verge-and-foliot mechanism with the more accurate pendulum mechanism.

The sand clock

It is remarkable that the tower clocks with the verge and foliot escapement mechanism were invented and became widespread in Europe almost simultaneously with the sand clock. The sand clock or hourglass seems to be a relatively simple device that could have been invented in antiquity simultaneously with water clocks. However, sand clocks were not known or at least not wide spread until the late Middle Ages[39].

The earliest medieval evidence of a sand clock appears in the 1338 fresco "Allegory of Good Government" by Ambrogio Lorenzetti, where it serves as an allegory of Temperance. A figure of a lady holding a large sand glass filled with a granular material appears in the fresco, and the clock seems to symbolize such qualities as patience, measure, and temperance (**Fig. 6**). The fact that an audience could recognize this artist's symbol indicates that the sand clocks had become quite widespread by the year 1338[40].

There were some attempts to relate the emergence of the sand glasses with the improvement of the naval technology typical to the thirteenth century. Indeed, by the beginning of the fourteenth century sand clocks were commonly used on ships. Thus the "Document d'Amore" by Francesco da Barberino, composed between 1306 and 1313 and devoted to the seamen technology and to the dangers of sea travel, mentions the sand clock (*orologio*) among other instruments needed on a ship, such as the lodestone and chart (*al Compasso steino*)[41].

Balmer investigated a possible link between sand clocks and the emergence of the naval technology. Indeed, knowing the exact time is crucial for navigation, namely for determining a ship's longitude (while the latitude can be easily found from the observation of the Polar star's altitude). However, sand glasses are impractical for measurements of large periods of time (dozens of hours and days) needed for navigation, because of their insufficient accuracy. The task of keeping the exact time for the needs of ship navigation was solved only after the invention of the pendulum-based chronometer by the end of the seventeenth century.

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Fig. 6. Temperance bearing an hourglass; detail of Lorenzetti's "Allegory of Good Government," 1338 (located at Sienna's *Palazzo Pubblico*, image credit: Wikipedia).

Another possibility is the use of the sand clock for determining a ship's speed by measuring a distance ran by the ship within a certain time with the so-called "log". This method implied throwing a piece of wood overboard and measuring a ship's velocity relative to that piece, for which measuring a time period is required. This method was used in the sixteenth century; however, there is no recorded evidence of it in the thirteenth or fourteenth century.

According to Balmer, although sandglasses could hardly be used for marine navigation due to their limited accuracy, their invention and spreading was still stimulated by their use on ships for regulating sailors

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activity. The sand clocks had an advantage over water clocks that they could be used in the wet, rough, constantly moving environment of a ship[42].

Balmer notes that "it is possible that the societal concept of time was evolving from a nebulous continuum to a quantifiable organizable duration... there are two other well-documented uses of [sand clocks] during the late Middle Ages: by scholars, apparently for regulating their routines of study, and by the clergy for regulating their sermons and meditations"[43].

The sand clock has a significant advantage over the water clock. In the former, the rate of granular material flow is almost constant, while in the latter the rate of liquid flow depends on pressure and on the level of liquid in the vessel[44]. However, both verge-and-foliot mechanical clocks and sand clocks cannot compare in accuracy with pendulum clocks.

The pendulum clock

While early mechanical clocks relied upon the verge-and-foliot mechanism, whose frequency of oscillations depended on various factors such as friction, much more accurate time measurement can be achieved with the pendulum clock. The isochronicity of small oscillations of the pendulum, or the independence of the oscillation frequency upon the amplitude, was investigated by Galileo starting in 1588 and published in 1602[45]. Galileo suggested an idea of an original escapement mechanism in about 1637; however, he never built this mechanism.

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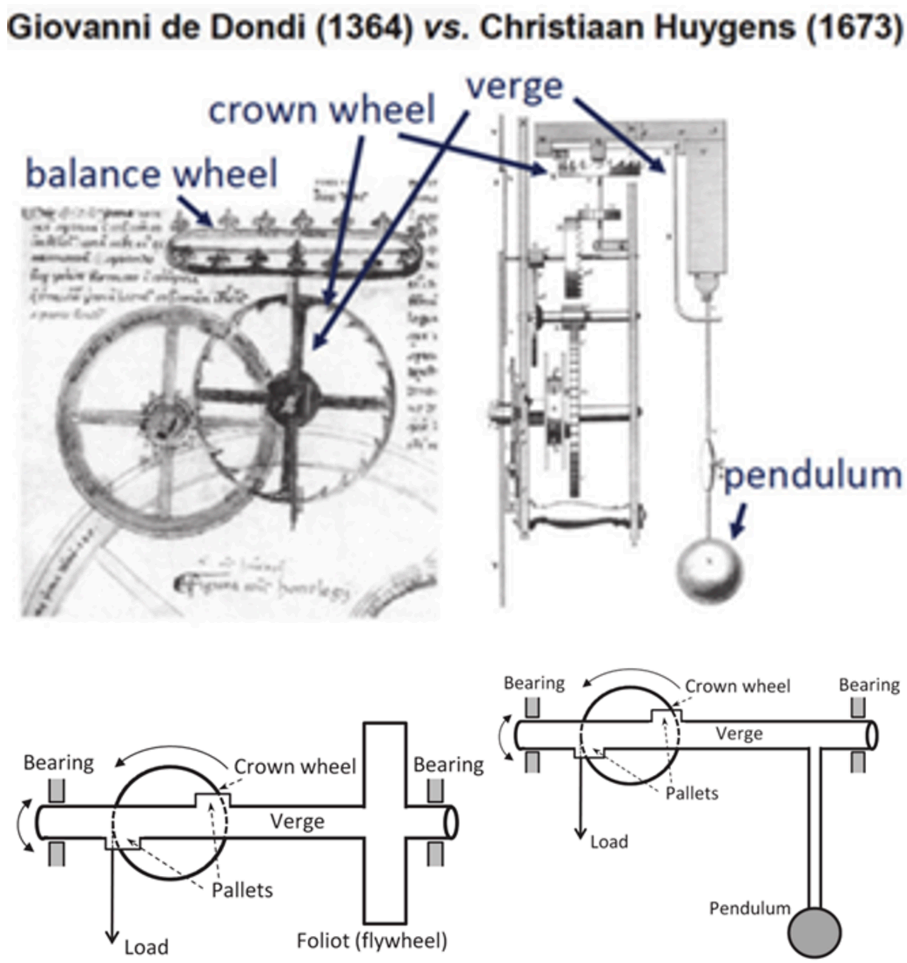

Fig. 7. Comparison of a verge escapement in Giovanni de Dondi's clock (1364, Padua), drawing from his Il Tractatus Astrarii, and anchor escarpment the second pendulum clock built by Christiaan Huygens (1673); drawing from his Horologium Oscillatorium, and kinematic diagrams of both mechanisms.

The first pendulum clock was built in 1658 by Christian Huygens, who used Galileo's discovery of the isochronicity of a pendulum. Huygens' pendulum, combined with an anchor escapement, could swing for about 6° and provided very high accuracy. In 1673, Huygens built a second, improved version of the pendulum clock. Its accuracy was about 10 seconds per day (**Fig. 7**).

As a consequence of the introduction of the pendulum in 1658, the accuracy of the clocks increased by almost 30 times. Verge-and-folio escapement clocks had an error of approximately 300 seconds per day, while the pendulum and anchor-escapement clocks had an error of about only 10 seconds per day[46].

From the mechanics point of view, a pendulum subject to small oscillations is a linear oscillator (in the sense that the restoring torque is linearly proportional to the deviation of the pendulum from equilibrium, angle φ). Unlike non-linear systems, linear oscillators have a natural frequency. The motion of such a pendulum is well approximated by a linear differential equation of the second order (also known as the harmonic equation), $\ddot{\varphi} + \omega_0^2 \varphi = 0$, where ω_0 is the natural frequency (so that $T_p = 2\pi/\omega_0$ is the period of oscillation).

When dry or lubricated friction is applied, the frequency of oscillations will change; however, this change is small, and it is proportional to the magnitude of friction $T_p = (2\pi - C_p f)/\omega_0$, where f is the coefficient of friction and $C_p \approx 1$ is a constant[47]. This is very different from the system without a pendulum, for which the period of oscillation is given by $T_v = C_v/\sqrt{f}$, where C_v is a constant. The sensitivity of the period of vibration to the variation of friction can be calculated as a ratio of the derivative of the period by the coefficient of friction by the corresponding period

$$\alpha_v = \left|\frac{dT_v}{df}\frac{1}{T_v}\right| = \frac{C_v}{2f\sqrt{f}}\frac{\sqrt{f}}{C_v} = \frac{1}{2f},$$
$$\alpha_p = \left|\frac{dT_p}{df}\frac{1}{T_p}\right| = \frac{C_p}{\omega_0}\frac{\omega_0}{2\pi - C_p f} \approx \frac{1}{2\pi}.$$

The ratio of the two sensitivities can be estimated for f=0.1 (which is a reasonable value for lubricated friction) as $\alpha_v/\alpha_p \approx \pi/f \approx 31$. This value is consistent with the literature statements that the introduction of the pendulum between 1658 and 1673 resulted in the improvement of the accuracy of the clock by about 30 times (**Fig. 8**)[48].

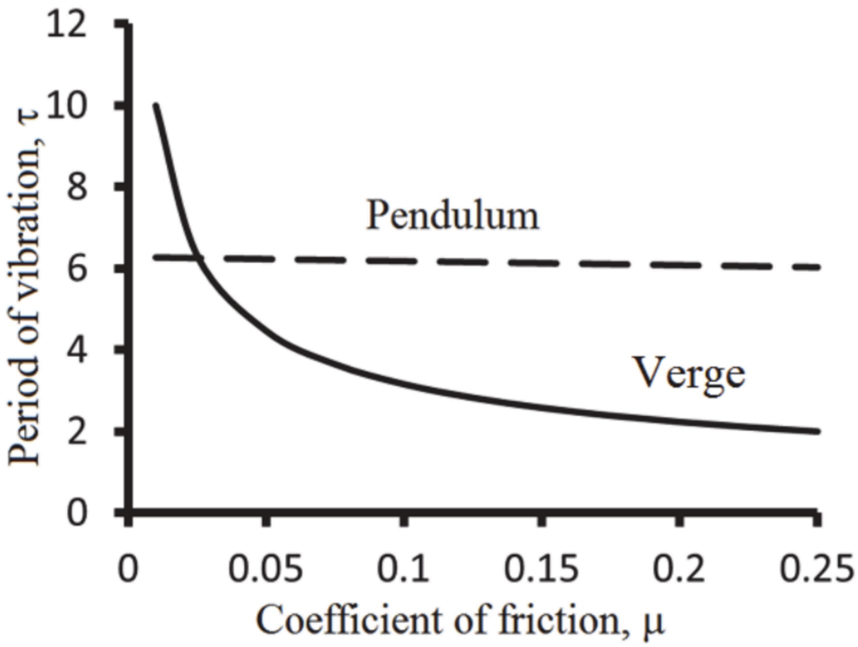

Fig. 8. Typical dependencies of the period of oscillation on the coefficient of friction for the verge (solid) and pendulum (dashed) mechanism.

A system with a pendulum is much less dependent on the variation of friction. Thus, the variation of the coefficient of friction by one percent from f=0.1 to f =0.101 would result in a corresponding change of the period of vibration by 0.5% (or 432 seconds per day). The same variation in the system with a pendulum would result in the change of the period of vibration by only 0.016% or 14 seconds per day.

The principal property of the pendulum clock which provided its enhanced accuracy is its linearity, which resulted in isochronicity or independence of the frequency of the amplitude of oscillations. A linear system can often be decoupled into parts, and many properties of the system is a sum of its parts' properties, thus, vibration with twice the original amplitude would have twice the original restoring force, hence the same frequency.

Galileo's discovery of linear systems, along with the astronomical observations by Galileo, Kepler, and Newton that the motion of planets in the Solar System can be decoupled as a combination of two-body problems, had far-fetching consequences for the history of physics in the early modern period. These discoveries strengthened the reductionist method of establishing the laws of nature from the observations of parts of a system, whose behavior is independent of their context. An early example of successful application of the reductionist method would be the laws of motion of a body or of a particle studied in isolation from the surrounding objects, whose action is substituted by a sum of forces exerted upon the body. Galileo and Newton are credited for the discovery of such laws. Pre-Galilean mechanics studied natural occurrences as opposed to the study of phenomena introduced by Galileo[49]. The study of phenomena ("the invariant forms that allegedly underline natural occurrences") should systematically exclude causal accidents as impediments, such as friction, which should be eliminated in the search of refined and purified phenomena. The discovery of inertia as a phenomenon required, at first, to eliminate friction.

The clock as a metaphor of Nature is a common motif of many scholars. Kepler wrote that his goal was "to show that the celestial machine is not like a divine being, but like a clock."[50] William Paley argued for the existence of God invoking the Watchmaker God as proof of a rational and orderly universe in his "Natural Theology."[51]

Concluding remarks

The invention of the oscillating verge and foliot escapement mechanism at the end of the thirteenth

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century was a breakthrough in the development of time-measuring technology. It resulted in the emergence and rapid proliferation of mechanical tower clocks and, later, wall clocks in Europe. While the exact circumstances of this invention are unknown, it happened almost simultaneously with the emergence of sandglasses, which was likely stimulated by changing attitudes towards the organization of time during the early Renaissance.

The Condemnation of 1277 was a reaction against the growing influence of Aristotelian philosophy including the idea that time is defined by motion, such as periodic oscillation. The discovery of impetus (motion by inertia) by Buridan and Oresme was directed against the Aristotelian concept of a permanently acting force, shared by Thomas Aquinas. The rigorous formulation of this concept was achieved only by Galileo more than two hundred years later, which was a success of his reductionist approach to natural phenomena. In a similar way, only the pendulum clock mechanism could provide a high accuracy method for measuring time.

The pendulum mechanism introduced in 1658 by Huygens was a linear oscillator, which had its own natural frequency, so that friction had only a minor effect on the period of oscillations. Scaling arguments suggest that the clocks' accuracy improved by about 30 times, which is consistent with actual historical data. The invention of the pendulum can be viewed in the broader context of the Scientific Revolution of the seventeenth century as a success of the reductionist paradigm of the natural philosophy associated with Galileo and Newton, when simple linear phenomena (such as linear oscillations and two-body gravity problems) were identified to explain the behavior of complex systems.

Footnotes

[1] Two particularly significant groups of fourteenth century scholars were the followers of William Occam (1287–1347) in Paris – Jean Buridan (1301–1359/62) and Nicole Oresme (1320/5–1382) – as well as the Oxford group known as the Oxford Calculators.

[2] There is rich literature regarding the Condemnation of 1277 and its role in the emergence of modern science. The original claim by Pierre Duhem (*Etudes sur Leonard de Vinci*. Paris: Hermann, 1906-1913), Vol. I, p. 412.) that 1277 marks the birth of modern science, was to some extent, disputed by various historians. For example: Edward Grant, "The Condemnation of 1277. God's Absolute Power, and Physical Thought in the Late Middle Ages," *Viator* 10 (1979): 211–244; J. E. Murdoch (1998). 1277 and late medieval natural philosophy. In J. A. Aertsen & A. Speer (Eds.), *Was ist philosophie im mittelalter?* Berlin: Walter de Gruyter; Thijssen, J. M. M. H. (2003). "Condemnation of 1277." In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*.

[3] Sara Uckelman, Logic and the Condemnations of 1277 *J. Philos. Logic*, Vol. 39, No. 2 (April 2010), pp. 201-227.

[4] One significant exception is the work by Christopher Brown, *Writing Time: Dante, Petrarch, and Temporality*. Doctoral dissertation, Harvard University, 2015). However, Brown considers the attitude towards time during early Renaissance from the viewpoint of literary history, rather than the history of science. See also Ricardo J. Quinones,. *The Renaissance Discovery of Time* (Harvard University Press, 1972).

[5] Carlo M. Cipola. *Clocks and culture* 1370-1700 (Walker and Co., NY, 1967).

[6] The Condemnation issued on March 7, 1277 by the Bishop of Paris, Étienne Tempier, prohibited the teaching of 219 theological and philosophical concepts of Aristotle at the Faculty of Arts of the University of Paris, which was under the bishop's jurisdiction. The condemnation was issued by instruction of Pope John XXI, and it was a result of a long-standing polemic between conservative Catholic theologians and peripatetic scholars, who supported the views of Aristotle, Thomas, and Averroes. Almost simultaneously, a similar prohibition to teach 30 theses at the University of Oxford was issued by the Archbishop of Canterbury Robert Kilwardby (Uckelman, 2010, p. 202).

[7] Duhem's attention towards the medieval roots of physics was a reaction to an earlier position that nothing important was done in physics and mechanics during the "Dark Ages" between ancient Greek authors and Galileo. For example, Lagrange wrote in his famous *Mécanique Analytique* (1788), one of the most important treatises of mechanics, that between Archimedes and Galileo, science has experienced eighteen centuries of darkness (Horia-Roman Patapievici. The 'Pierre Duhem Thesis.' A Reappraisal of Duhem's discovery of the Physics of the Middle Ages. Logos & Episteme, VI, 2 (2015): 201–218, p. 203). See also Grant, 1974, p. 43.

[8] Thijssen (2003) notes that already in 1297/98, Godfrey of Fontaines, a member of the Theology Faculty at Paris, wrote "at the University of Paris, the Paris Condemnations of 1277 were ignored completely or interpreted in a way entirely contrary to the intentions of their framer." He suggested "the Bishop of Paris should at least suspend the condemnation of those

propositions which appeared to have been taught by Thomas." This was done in 1325, when Tempier's successor as bishop of Paris, Stephen de Bourret, proclaimed that the 1277 condemnation "had no canonical value" with respect to any censured Thomistic proposition. Scholastics of the fourteenth century, including Buridan, frequently referenced the Condemnation of 1277 and propositions condemned by it (Grant, p. 239).

[9] Ned Markosian, "Time" *The Stanford Encyclopedia of Philosophy* (Fall 2016 edition), Edward N. Zalta (ed.).

[10] Until the twelfth century, few works of Aristotle were available in the Latin west, translated by Boëthius. Throughout the twelfth century, several translations by Boëthius of other texts of Aristotle were rediscovered, and new translations of his otherwise unknown works were made. By the beginning of the thirteenth century, a wealth of new secular material became available to Latin scholars (Uckelman, 2010, p. 209).

[11] Wippel, J. F. (1995). Thomas Aquinas and the condemnation of 1277. *Modern Schoolman*, 72, 233-272.

[12] For the text, Grant, 1974, p. 48. According to Aristotle, space was defined by the relationships of objects, hence empty space or vacuum was impossible.

[13] The idea that time is not eternal led to the question whether time was created when the world was created. However, the concept of the eternal world was seen by many theologians as contradicting the Biblical concept of creation. Moreover, the claim that there is no novelty in an effect without novelty in the cause prohibited motion by inertia.

[14] Articles 139-141 implied that no accident (property) exists without a subject (a material substance) thus making problematic absolute time and space, which are properties of a vacuum.

[15] Uckelman, p. 215.

[16] A first century Nabatean sundial from *Mada'in Salih* (Hejaz, Saudi Arabia), with the name of the owner *mnš br ntn šlm* ("Menashe son of Nathan, peace"), apparently, Jewish (Healey, J. F. A Nabatean Sundial from Mada'in Salih. In: *Syria: Revue d'art oriental et d'archéologie* LXVI. (Paris, 1989). Pp. 331–336); stone sundials (*'avne ša'ot*) are mentioned in early Rabbinical literature.

[17] H. C. Brearley. Time telling through the Ages (NY, Doubleday, Page & Co., 1919); F. J. Britten. Old clocks and watches and their makers (NY Bonanza Books, 7th ed.); E. Burton. The history of clocks and watches (Rizzoli, NY, 1979).

[18] C. Cipola. *Clocks and culture* 1370-1700 (Walker and Co., NY, 1967).

[19] Muslim astronomical knowledge penetrated into the West, and the most famous example is the Tusi Couple, a theorem suggested by the Persian astronomer Nasir al Din al Tusi in the 1240s, which became crucial for the development of the Copernican system. Apparently, Oresme was aware about the Tusi Couple (Claudia Kren (1971). "The Rolling Device of Naṣir al-Dīn al-Ṭūsī in the De spera of Nicole Oresme". *Isis*. 62 (4): 490–498). Famous water clocks, as well as some other automated devices, existed in the Muslim world, such as *Jayrun* in Damascus (1257/1277) and *Dar al Magana* in Fez (1357). However, there is absolutely no evidence of mechanical clocks in the Muslim world of that period. On the origin of earliest clocks, see also North, John D. "Monasticism and the First Mechanical Clocks." In *The Study of Time II: Proceedings of the Second Conference of the International Society for the Study of Time*, edited by J.T. Fraser, N. Lawrence, 381-98. Berlin: Springer, 1975; Landes, David S. *Revolution in Time: Clocks and the Making of the Modern World*. Cambridge, MA: The Belknap Press of Harvard University Press, 1983,. 53-66.

[20] Lynn Thorndike, "Invention of the Mechanical Clock about 1271 A.D.," *Speculum* 16, no. 2 (1941), 243; also Brown p. 35. Robert's note was that it is possible to build a machine, which would mimic the daily celestial motion. Remarkably, a similar idea had been expressed by Roger Bacon in 1248, who mentioned a self-moving astronomical sphere, which would mimic the daily celestial motions. He mentions it among other machines that could be constructed in the future. For Bacon, this shows a connection between human mechanical art and divine art (North, *God's Clockmaker: Richard of Wallingford and the Invention of Time*. London and New York: Hambledon and London, 2005, p. 158; Brown, p. 45).

[21] Brearley, p. 77

[22] Maddison, Francis. "Dondi, Giovanni". Encyclopedia.com

[23] North, 2005, p. 158

[24] The Benedictine Rule divided the twenty-four hours into eight periods of three hours each. The day began at sunrise (*hora prima*). The length of these hours varied in accordance with the season depending on sunlight hours.

[25] Kalman, David Zvi *Turning Clockwise: Jews and Timekeeping from Antiquity to Modernity* (PhD Dissertation, U Penn, 2019). The earliest Jewish

acknowledgments of clocks appear around 1400 in Italy, particularly, in the colophons of manuscripts, where copyists indicated not only the date of finishing their work, but also the hour, and book owners recorded the hours at which births, deaths, and other events took place. These colophons employed the "Italian" (equinoctial) hours beginning at sunset (for example, "22 hours" means two hours before sunset).

[26] Brown, p. 61. On the role of automata on mechanistic philosophy see de Solla Price, Derek J.. Automata and the Origins of Mechanism and Mechanistic Philosophy. *Technology and Culture*, Vol. 5, No. 1 (Winter, 1964), pp. 9-23

[27] Wisdom (*Sapience*) could also be associated with the image of the Biblical King Solomon. White, Lynn "The iconography of Temperantia and the Virtuousness of Technology", In: *Action and Conviction in Early Modern Europe: essays in Memory of E. H. Harbison*, ed. by Th. K. Rabb and J. E. Seigel, Princeton University Press, 1969.

[28] Brown, p. 63.

[29] Apparently, water clocks are implied by the *Horologium,* since mechanical clocks have not yet been invented. See also Brown, p. 44.

[30] Nicole Oresme, *Le Livre du ciel et du monde*, ed. Albert D. Menut and Alexander J. Denomy, C.S.B., trans. Menut (Madison: The University of Wisconsin Press, 1968), p. 288; see also Brown, p. 49

[31] Blumenthal, A.S.; Nosonovsky, M. Friction and Dynamics of Verge and Foliot: How the Invention of the Pendulum Made Clocks Much More Accurate. *Appl. Mech.* 2020, 1, 111-122. https://www.mdpi.com/2673-3161/1/2/8/htm.

[32] P. Dubois. "De Vick tower clock, built Paris, 1379, by Henri de Vick." In.: *Historie de l' Horlogeri* (Paris: Editorial Maxator), 1849. P. 221

[33] A. M. Lepschy, G.A.Mian, and U.Viaro, "Feedback Control in Ancient Water and Mechanical Clocks," IEEE Trans. Education, Vol. 35, pp. 3-10, 1992

[34] Nosonovsky M. Abner of Burgos: the missing link between Nasir al-Din al-Tusi and Nicolaus Copernicus? *Zutot* 2018, 15:25-30; Barker, Peter and Heidarzadeh, Tofigh. "Copernicus, the Tūsī Couple and East-West Exchange in the Fifteenth Century.

[35] L. Winters and J. Bliss. *A Renaissance Treasury: The Flagg Collection of European Decorative Arts and Sculpture*. (NY: Hudson Hills Press, 1999), p. 26. About German automated clocks see also Otto Mayr "A Mechanical Symbol for an Authoritarian World," in *The Clockwork Universe: German Clocks and Automata 1550-1650*, ed. Klaus Maurice and Otto Mayr (New York: Neale Watson Academic Publications; Smithsonian Institution, 1980).

[36] A model of a verge escapement mechanism was presented in the vibrations textbook by A. A. Andronov, A. A. Vitt, and S. E. Khaikin, Theory of Oscillators, (Dover Publications, Inc, New York 1966). The verge mechanical clock was also studied by control engineers as an example of an early feedback mechanism. Thus, Lepschy et al. compared the feedback loop of the verge escapement mechanism with that of the Ktesibios' water clock (c. 230 AD). A. Roup and D.S. Bernstein, "On the dynamics of the escapement mechanism of a mechanical clock" In: Proc. Conf. Decision and Control (Phoenix, AZ, Dec. 1999), pp. 2599-2604 investigated limit cycles of the verge escapement mechanism. See also Blumenthal, A.S.; Nosonovsky, M. Friction and Dynamics of Verge and Foliot: How the Invention of the Pendulum Made Clocks Much More Accurate. *Appl. Mech*. 2020, 1, 111-122. https://www.mdpi.com/2673-3161/1/2/8/htm.

[37] M. V. Headrick (2002). Origin and evolution of the anchor clock escapement. *Control Systems*, IEEE. 22. 41 - 52. 10.1109/37.993314

[38] Nosonovsky and Blumenthal, p. 7-8.

[39] There is evidence that sand clocks could be used in the eighth century in Europe; however, they were extremely uncommon until the late Middle Ages (Britten p. 16).

[40] Robert T. Balmer, 1978. The operation of Sand Clocks and their medieval development, *Technology and Culture* 19:615-632.

[41] Balmer, 1978, p 616.

[42] Balmer, p. 621.

[43] Balmer, p. 618. Similarly to the origin of the mechanical clocks, a possibility of transition from the Muslim world has been investigated; however, no evidence was found. While many concepts have been transmitted to the West in the thirteenth century, including the use of Hindu decimal numerals introduced by Fibonacci, it appears that clocks were a local European invention. Note that international transition of knowledge, including the astronomical and mathematical knowledge accelerated in the thirteenth century. In the 1240s, almost all of Asia and parts of Eastern Europe became parts of the same state, the Mongol Empire, which facilitated international exchange. In Europe, early Renaissance cultural

developments started to emerge, such as the *Pecia* system of copying manuscripts.

[44] From the physics point of view, as it was discovered in the 1980s, granular flow in a cone of sand is an example of a so-called Self-Organized Criticality system, which tunes itself into a so-called "critical state" of more or less constant flow rate. The sand pile is a classic example of a self-organized critical system, as was shown by (Bak, P. *How Nature works: the science of Self-Organized Criticality*, NY, Copernicus, 1996). For physics of sand glasses see also Mills, A.A. et al. Mechanics of the Sandglass, *Europ. J. Phys*. 17:97-109 (1996).

[45] S. Drake, *Galileo at Work: His Scientific Biography*. (Chicago: University of Chicago Press, 1978), p. 419.

[46] Cipola, C. *Clocks and culture* 1370-1700 (Walker and Co., NY, 1967)

[47] Nosonovsky and Blumenthal, p. 7.

[48] Nosonovsky and Blumenthal, p 8.

[49] Wiltsche H. A. Mechanics lost: Husserl's Galileo and Ihde's telescope, Husserl Stud. 2017, 33(2):149-173.

[50] Johannes Kepler, *Opera omnia*, ed. Christian Frisch, vol. 2 (Frankfurt: 1858-1871),

[51] Brown, p. 66.

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