Qeios

Update on $a + b = c$

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Abstract.
This numeric model organizes the collection of sentences *a+b=c* for $a,b \leq 16$. The update involves speaking also about such states of the world which are not the case. The etalon collection of 136 pairs of *(a,b)* is placed in a habitat that is subject to periodic changes. We sort, resort, and order the logical primitives according to their diverse aspects. For sorting, we use the concept known as permutations. For grouping, we make use of the cyclic properties of permutations. We find an upper limit for the number of group relations that are concurrently possible on an assembly to be *n? = exp(ln(partition(n))²)*. The relations *n! / n?* are fundamental.

The work is an instruction manual on how to build the databases out of which the results are read out. The pairs of natural numbers organize themselves into paths within self-made geometries. The basic patterns that simple logical elements show when being reordered are archaic building material for logic. The terms of place and material become data in a data depositionary that is an element. Above this mother matrix of facts (which element is where when) there is a matrix of order, relations, and predictions. The accounting image of the inventory and the factual state of the inventory are two slightly deviating sets of data, which maintain a common currency of predictability resp. degree of certainty. The mutual expectations - predictions are an inbuilt feature of the symbols set and picture the rationally explainable world within the Eddington delineation. Its rules can be learnt by means of the data set we suggest the reader builds on their own computer.

Keywords: Natural Philosophy, Information Theory, Theoretical Physics.

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1 Introduction

1.1 Target audience

This is an exercise book, where simple reading probably brings a general understanding of some ideas but will not allow active research based on and using the model presented here. One needs a few hours of programming to have a functioning tautomat.

We show how to construct a numeric model which yields patterns of what being where and when, and how predictably. The system is self-contained and uses deictic definitions. The model is comparable to a hybrid between sudokus and the magic cube.

People who like sudokus, play with Rubik's cube, solve logical riddles, enjoy systematizing, will include such who develop computer games, design and write compilers, database architects, operating systems developers. We speak to those here.

1.2 Content matter of the work

Following the tradition of sentence logic established by Wittgenstein [1], we abstract the particularities of things into such generalities which can be represented by symbols that have a definite meaning, like *1,2,3,…* (elements of **N**, units in the Sumerian tradition).

Departing from the Sumerian tradition, we use logical units that are each an *individual*, by building *pairs* of Sumerian units. We establish an etalon collection of sentences with the content *a+b=c* for the first *136* variants of pairs of *(a,b)*.

We group and sort the *136* elements of the etalon collection. For sorting, we use the apparent numeric properties of the elements. For grouping, **we use an additional numeric property of the elements which has not been discussed in the literature before**. This logical category of the elements is given by a small detail in the procedure of resorting, namely by determining, with which other elements any individual teams up into what is termed a '*cycle'* during reorders. The collection of positions and properties of that subcollection of elements which replace one the next during a reorder is a cycle of that reorder. The task teams, into which logical primitives organize during a reorder exercise, confer to the members of the team specific properties, which remain the property of the member even if the reorder exercise is currently not taking place. Of these properties, which belong to the elements by reason of the elements belonging to groups, we create a system of ***liaisons.***

> Example: Person X knows that they are notified by A and must notify F in the case of a flood alarm, that they are notified by G and have to notify H in the case of a fire alarm, that they are notified by Q and have to notify R in the case of an accident alarm, etc. The chains of notification are each a cycle or a part of a cycle. A and G and Q are potential predecessors for Person X, for whom F and H and R are potential successors.

The commutative symbols elements of a collection of pairs of natural numbers share, based on belonging to cycles which are realized during a reorder, while remaining dormant during different reorders, create a basic dichotomy between realized and potential. The collection is always in a state of being reordered. There is an implicated cloud of potentials around any realization of a reorder.

1.3 Structure of the work

Part Infometry introduces the concepts that will be discussed. Part Ordometry shows how the parts interact if everything goes smoothly, as expected. The usual functioning of the mechanism is the carrier medium of the message, serves as its background. Part Tautometry discusses what are typical deviations before this regular background. Inevitable conflicts are shown to be a feature of the system working in order, as foreseen, according to its own rules. Each segment has three main parts: **Action, Context, Technique**. We discuss what one should do, why it should be done, and how one does it.

1.4 Reliability and validity

The numbers published in the steps of introducing the model are **reliable.** The **validity** of the ideas presented in the sections **Context** is by no means axiomatic.

We formally renounce any claim to validity regarding the interpretations offered in the sections **Context** of the present work. We do not state, e.g., that the plane with axes *x: [a, 2a-b], y: [b-a, a]* is one of two planes that are a valid picture of *electromagnetic fields.* We do not state that the axis *z: {[a+b, a], [a+b, b]}* is a valid picture for *gravity.* Rather, "… makes the impression of, … appears to be useful as a picture of, … could also be seen as representing the idea of", etc. should be used. A philosopher's opinion, like an artist's impression, shows a possibility, how it *could be.*

What we discuss here are relations among numbers. A complex picture of a model emerges, which model could be used to interpret some features of Nature as being based in elementary logical implications.

We use well-known, traditional tools the industry of data processing has evolved, for building nontraditional applications that bring forth nontraditional views and results.

2 Infometry

2.1 Cohort of logical primitives, their properties

Action: .
Set up your computer and create a cohort of logical symbols (pairs of *(a,b))* Set *d=16.* Add properties *c=a+b, k=b-2a, u=b-a, t=2b-3a, q=a-2b, s=d-(a+b), w=2a-3b*. The table is now 9 columns, 136 rows.

Context: .
We discuss the logical sentence *a+b=c.* The pair of *a,b* values is one individual basic logical unit.[1]
We deviate from the tradition established by the Sumerians, where units are defined as multiples of the *Basic Unit* named *1 (one).* Using *two* of the Sumerian logical units to concatenate into *one* infometric unit, the model is based on units being *distinguishable individuals.*

The advantage of using individuals as symbols comes with the drawback of a limited size of cohorts. The model depicts buildup, behavior, dissolution of aggregate units (like atoms, molecules, drops, crystals, functional units, planets, galaxies: these all have a property of the constituents belonging together). There is a limit to the extent of a Whole of which the parts are related to each other. The Whole as such is a collection of the relations among elementary units, which are in the model represented by the pairs (a,b).

> The natural limit for the number of interacting objects observed in Nature is explained by the divisor in the expression for the *fine structure constant,* which was given by Eddington [4] as *136,* which later research corrected to ~ *137.* The model shows both theoretical upper limits *(136, 137)* to be referring to the same threshold, the difference being an artefact of counting *'=' before a background of '≠',* as compared to counting *'≠' before a background of '='.* The fact and extent of the inbuilt incongruence will be discussed in Part Tautometry.

Technique: .

Deictic definition of cohorts

Table 1. Connection *to OEIS:* The number of members of a cohort is driven by *d,* the number of diverse variants of *(a,b):* this agrees to the triangular numbers *oeis.org/A000217. [8]*

:

No of distinct properties *d*	No of distinct elements in the cohort n_d	Elements of the cohort
1	1	*(1,1)*
2	3	*(1,1),(1,2),(2,2)*
3	6	*(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)*
4	10	*(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4)(3,3),(3,4),(4,4)*

[1] The name *"logical primitives"* was given to the collection of *(a,b)* by M. Abundis ˙[2]

2.2 Sorting orders and planar places

Action: .

Establish *72* sorting orders by pairing each of the properties *{a,b,c,k,u,t,q,s,w},* to be used as primary, outer sorting criterium with every other different property, to be used as secondary, inner sorting criterium. Sort the collection *72* times and register for each element its linear rank in the relevant sorting order. This is database SQ.

Create *72*71* planes, pairing two of the sorting sequences, which are here axes with gradations *1..136*. Assign to each element one dot with coordinates *x: rank in sorting order [αβ], y: rank in sorting order [γδ].* This is database DOT.

Context:

The central tautology of the model is that two ranks on two axes are equivalent to one planar pair of coordinates, on a plane of which the axes are the sorting orders. The linear contradiction $pos([ab], (1,3)) = 3 \neq pos([ba], (1,3)) = 4$ is dissolved in $pos([ab,ba], (1,3)) = 3,4.$

$$\textbf{Rank(d, [}\alpha\beta\textbf{], (a,b)) \& Rank(d, [}\gamma\delta\textbf{], (a,b))} \Leftrightarrow \textbf{Place(d, [}\alpha\beta,\gamma\delta\textbf{], (a,b))} \qquad (1)$$

Technique:

Some typical dot patterns:

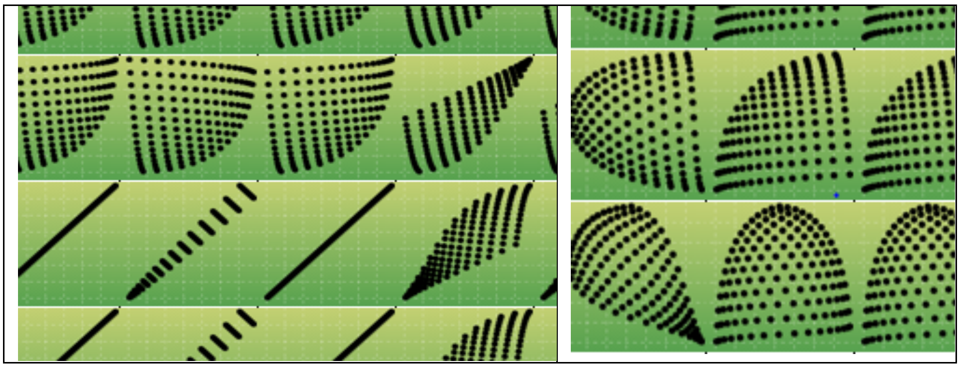


Fig. 1. The dots represent the place each element *(a,b)* occupies on the plane with axes ***αβ,γδ***

2.3 Cycles and their properties

Action:

Create a database T in which the steps of place changes are registered. Its columns are: *a, b, sorting_order_from, sorting order_to, rank_from, rank_to.*

Establish all cycles that exist on the collection *(46.260)*, using the sequences of steps in database T. Create database C which contains the cycles, by adding columns to database T: *cycle_nr, step_nr.* Draw the cycles.

Context:
The procedure of moving each element from its rank in sorting order *αβ,* to its rank according to sorting order *γδ* is called a reorder. The idea of reorders comes from the experiences of **periodic changes**. Periodic changes are a pre-axiomatic fact, at least on the surface of the planet Earth (which is subject to tidal, daily, and yearly periodic changes). The model is based on the axiomatic existence of reorderings, which simulate periodic changes.

Cycles are elementary, fundamental, immanent relations among pairs of natural numbers. **Cycles have an a-priori logical existence.** Nature makes use of the relations that are pictured by the cycles.

Technique:

Either **a.** use the following deictic definition:
We reorder *10* objects with two qualities each. The qualities we denote by *{(1,2,…,10),(a,b,c,…,j)}*.

Presently, the collection is in following order:									
1c	2g	3a	4d	5b	6i	7e	8f	9j	10h
The places themselves are enumerated									
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
We wish to achieve following order:									
3a	5b	1c	4d	7e	8f	2g	10h	6i	9j

No of replace-ment	**Ele-ment moving**	*From rank*	*To rank*	**Pushing away element**	Arrives at empty place	Nr of cy-cle	No of mem-ber in cycle
1	**1c**	*1*	*3*	**3a**		1	1
2	**3a**	*3*	*1*		yes	1	2
3	**2g**	*2*	*7*	**7e**		2	1
4	**7e**	*7*	*5*	**5b**		2	2
5	**5b**	*5*	*2*		yes	2	3
6	**4d**	*4*	*4*		yes	3	1
7	**6i**	*6*	*9*	**9j**		4	1
8	**9j**	*9*	*10*	**10h**		4	2
9	**10h**	*10*	*8*	**8f**		4	3
10	**8f**	*8*	*6*		yes	4	4

Table 2. Deictic definition of the term 'cycle' by giving a step-by-step demonstration of the properties of cycles.

Or **b**. use the procedure defined in *oeis.org/A235647* [9]

2.4 Definition of information

Action: .

By sorting and ordering Database C, establish measures of cycles like *number of members in cycle, Σa, Σb, Σdist*$(x_1,y_1 - x_2,y_2)$ and more you choose freely.

Observe reorder [ab] ↔ [ba] pars pro toto. Draw its *12* cycles and tabulate the meta-properties of cycles.

Context: .

Humans can distinguish and name diverse forms (realizations) of information. We can exchange logical sentences about the form and extent of specific realizations of the idea of information (in the sense of Wittgenstein, in whose system every word in a logical sentence refers to mental concepts of which the meaning is agreed on).

If the content of our logical discussion is understandable to all, then the content must be included in the implications of the symbol set we use. We uncover the flexibility inbuilt in the symbol set, which flexibility allows us to point out that something is *otherwise* than expected. **On this point, we deviate to the traditional interpretation of the symbol set**, according to which in the symbol set developed by the Sumerians, *nothing ever can be otherwise*, lacking the background of *relative to an expected value*. Here, we introduce an immanent property of the symbol set, namely that there are relations among its members; that the relations have their own set of relations; that there are mutual pairs of *{expected, observed}* values.

> Example: We imagine 136 pairs of youngsters, each one containing one girl and one boy. We say we expect a boy of *x* kg mass to be in a pair with a girl with *y* kg mass, based on the properties of the whole collection of pairs. There exist values for Δ $(x_{obs} - x_{exp})$, Δ $(y_{obs} - y_{exp})$, *etc.* (Think of *mésalliance.)*

The extent of mismatch is descriptive of the individual pair and of the cycles as an aggregation of individual pairs. The relative oddity of each pair (relative to other pairs) is a summand in an extent for the relative oddity of the cycle the pairs are members of (relative to other cycles). In the course of (during, as the consequence of) periodic changes, the collection becomes reordered.

> E.g. the extent of being a mismatch (deviating to expectations), will be measured by Δ $(q_{obs} - q_{exp})$, Δ $(r_{obs} - r_{exp})$, where *q, r* stand not for kg, but for cm.

Each individual pair has an extent of oddity in some measures, be these *a, b, b-a, 2a-3b, a+b, etc.* relative to its *135* peers. The extent of oddity will be also observable in cycles, relative to a. the other cycles of the same reorder, b. to cycles across reorders, c. reorders across cycles.

The cycles, into which the elements have aggregated, can also be seen as units of a meta-level of counting. These units **create their own set**.

The basic inner deviation within the symbol set's mutual references is a feature of the symbols set. Using it as a **numeric extent of Δ *(observed, expected)*** needs a structure in which there are mutual relationships that can be ranked on a general property. We use the extent of *oddity* on which we can compare diverse forms and appearances of relative oddity. This **is what we call information**. It appears always in *two* variants, which are logically the same but numerically different, namely

q as observed	deviating to	*q as expected by r,* and
r as observed	deviating to	*r as expected* by *q*,

where *q, r* can refer to any aggregational states and units of *a, b.* We have chosen **Cycles 3, 6 of the reorder [ab] ↔ [ba] of Cohort 16** to serve as a **deictic definition** for the term and extent of information, because these two are easy to compare. We define: these two cycles are deviating to each other to the extent of unit. The unit has several measurement dimensions. One is free to improve on the definition offered here, as it is clearly arbitrary. Semantic interpretations of the information extent are, e.g., how heavy, stretched, extended, dense, stable, varied, competitive, sustainable, etc. are the cycles. These extents can relate to each other in the dimension of oddity. We use the extent of the *mésalliance* aggregated in the system, measured by the difference in numeric aggregates along all member pairs of two cycles, related to geometric properties of the two cycles.

Technique:

Generate the general form of the reorder [ab] ↔ [ba].

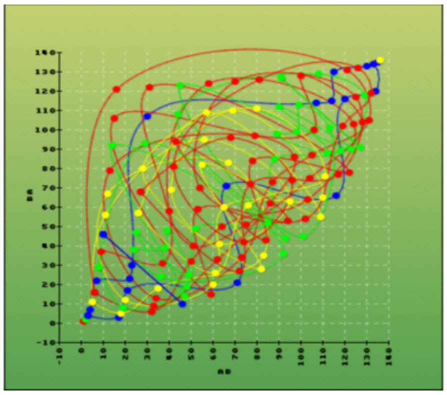
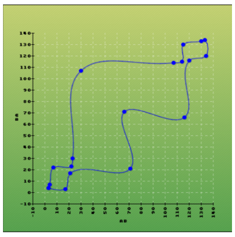
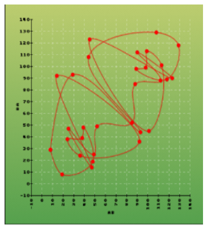
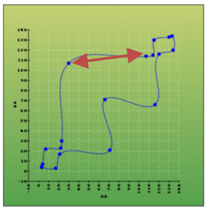
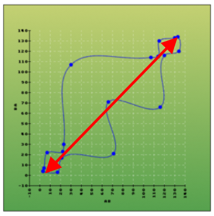


Fig. 2. Cycles of reorder [ab] ↔ [ba]

Extract cycles No 3, No 6, to be used as etalon unit of inner deviations Δ ***(observed, expected),*** which is the definition of information. The cycles have many different properties. The proportion between the aggregates for *{a,b}* members of a cycle and geometric properties of a cycle is descriptive of a cycle.

Table 3. Deictic definition of the term “information”

Name:	*Cycle 3, (d=16, (a,b),[ab],[ba])*		*Cycle 6, (d=16,(a,b), [ab],[ba])*
Picture:			
Basic Pr.:	*members: 18*		*members: 30*
	Carry_a= $\sum a$		*Carry_a=* $\sum a$
	Carry_b = $\sum b$		*Carry_b=* $\sum b$
	Run: $\sum$ *dist i*		*Run:* $\sum$ *dist i*
	Extent: max(dist(j,k))		*Extent: max(dist(j,k))*
Derived:	*run per members*		*run per members*
	members per extent		*members per ex-tent*
	carry per run		*carry per run*
Information:	***(Carry_b – Car-ry_a)/run***		***(Carry_b – Car-ry_a)/run***

If the length of the run is *0* because the element stays in place, one can either:

a. exclude the result, using a strict definition of movement, or

b. change the rule to *(run + 1)*, and have a more general picture of the process.

3 Ordometry

Chapter Context:
We approach the etalon sentence *a+b=c* from the viewpoint of what is true and remains true among its implications, if we visualize all its variants sharing a common logical space, their habitat, in which the collection undergoes periodic changes. We can use this brute force attitude in our days, because since Wittgenstein, 100 years of technological advances have passed. We are now in a position, by using computers, to generate the complete truth table regarding *a+b=c*, at least for the first few values of **N.**

We show that by using identical units, one ends up with a world view, that contains *one, big* deviation between its *two* parts. This chapter, we see the process of *polarizing the same,* next chapter will discuss *unifying the distinctions,* which will show themselves to be *many, small* deviations. Humans are habituated to perceiving the world under the aspect of sameness; our logical language is rooted in concepts of **N,** where order is imagined *au fond* to be based on a grid of equal distances.

3.1 Ranks in sequences, places on planes, Standard cycles and turns

Action:
Create a database S which aggregates the cycles. Build types of cycles.

Filter out such reorders which fulfil following criteria:

1. Number of cycles: *46,*
2. Cycles contain members*: 45 * 3 + 1 * 1*,
3. For the 45 cycles: *Σ a = 18, Σ b = 33,*
4. For the remaining (central) element: *(a,b) = 6, 11.*

These we call the standard reorders.

Context:
The world consists of coincidences[2]. The world is ordered. The sentence that details order is a composite statement about the properties of a thing and its placement among its peers.

> We certainly look an order into the world (because our neurology works by using pre-fabricated patterns). Whether there are specific collections of abstract sentences that state specific forms, types, categories of relations between properties of things and the placement of things among its peers to exist, is what remains to be seen, is what we investigate here. Such a kind of order is not originated in the human brain but exists *as such, eo ipso, a priori, immanent* to the system of symbols, being an artefact of the system of symbols. (Not the brain hallucinates abstract a-priori relations, but the abstract a-priori relations determine the functioning of the brain.)

[2] Die Welt ist alles, was eine Koinzidenz ist. The world is everything that is a coincidence. [3]

Human neurology causes us to look (project) into the data of standard reorders patterns of triangles, enclosing one (centrally located) point. At first, we see in the picture of a standard reorder the concept of *time,* which is inseparable from the concepts of *causality, consistency, continuity, tautology.*

> Example: The term 'year' is defined as the sequence of the seasons S, S, A, W, in Europe. If elsewhere a year consists of seasons P, Q, R, S, T, the idea of parts of a whole that follow each other in a strict sequence during a period remains constitutive to both.

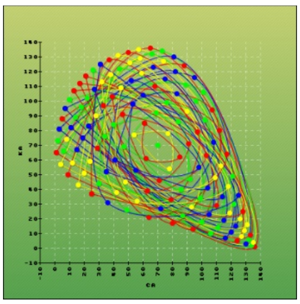
The standard cycles are a Peano unit. (We call a sequenced collection of *3* symbols with no first or last element a Peano unit.) Each of the members is related to its two temporal neighbors. We return to the basic tautology of **2 R = 1 P,** which says that two ranks in linear sequences and one set of two coordinates of a place on a plane are equivalent. The Peano units have a Fibonacci property.

We may declare a moment of *now* in which one specific element of a cycle is the case. The collection of *nows* over all cycles yields the possibilities of coincidences. Which elements *(a,b)* can be concurrently *now* is what we are filtering out.

We assume *synchronization,* leaving the 45 variants' differences regarding the *run* properties to the background. (We believe that the idea of space is easier to mentally compress and dilute than the idea of time.)

Technique:

Unusual coordination of brain areas: It may be helpful to build a path of associations in one's brain connecting the area where the experience of counting happens with more archaic regions that have learnt ordering and sorting. Convince yourself of the truth of the above axiom by picking any *two* describing properties of a few (> *10*) of your things (say, length and weight, color and value, form and frequency of use, etc.). Give ranks to (sort, sequence) the things in both descriptive dimensions. Draw two rectangular axes. Find the place each thing occupies in this map. Reorder the things and watch the place changes and the cycles that are a realization of the idea of reorder. Observe that the cycles are a linearized variant of the contents of the reorder. Write the steps of cycles on long paper stripes. Make a Las Vegas machine by picking a stripe on a wheel.



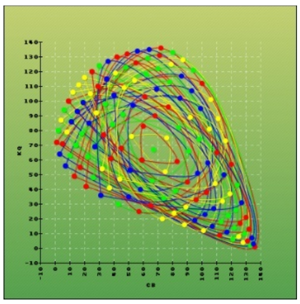


Fig. 3. Two examples of standard reorders ([a+b,a] ↔ [b-2a,a] ; [a+b,b] ↔ [b-2a,a-2b])

3.2 Two Euclid spaces, two central elements

Action:
Filter from the standard reorders such that can be assembled into a rectangular structure with 3 axes.

> Find axes: A.: *z:[a+b, a]; y: [a-2b, b-2a]; x: [a-2b, a]*; B.: *z: [a+b,b], y: [a-2b, a], [b-2a, a-2b*]. Note that two more planes transcend both Euclid spaces. (Electro-magnetic fields.)

Both the left and right Euclid spaces contain *one central element.* Its *a,b = 6, 11.* The two central elements are logically identical, their geometric location in their respective webs is different, if we use one, common **N** to enumerate geometric positions (if we use a concept of **geometric** distances) . The standard cycles connect the planes generated by the axes. The web made up of **triads** surrounds both central elements, each central element having its own swarm of possibilities.

Context:
The standard reorders are **geometric**, **not temporal** distances here**.** The patterns emerging from sorting pairs of natural numbers yield a basic duality. The numeric model is well suited to depict agglomerations of *two* variants of entities, which are logically equivalent parts of a logical whole, yet possess different quantitative properties.

> Examples: proton – neutron, DNA – RNA, female – male, background – foreground, sequenced – commutative [readings of symbols], electric – magnetic [the two transcending planes], etc.

The central elements are a definition of the idea of a **point**, without taking recourse to an axiom. Their spatial relation within their respective webs is well defined in a space that is constructed by the standard cycles. The axiomatic definition of a point can be updated to mean two points that are in a definite geometric relation to each other.

The standard cycles imply an idea of *continuity, consistency, predictability, stability.* This property can be used to maintain continuous **tautology**, which will allow us to see, that it is self-evident, that a *positional* change in a sequence of logical tokens (in the DNA) will (under specific, ideal circumstances) be equivalent to establishing selection criteria regarding properties of logical tokens, which are concurrently present, in a 3+D space. The selection mechanism is based on the repeated application of the basic tautology **2 R = 1 P**, that is: two linear ranks are the same as the coordinates of a point on a plane.

In practice, this means that there are *three turns (phases)* in which the elements that have a place on a plane, have ranks on two axes respectively. To this place corresponds a rank in a third axis. We neglect coordinate *x* of the place description *(x,y)* of an element *(a,b)* on plane *XY,* and gain a coordinate *z* from the rank of *(a,b)* on the axis *Z,* arriving at a place *(y,z)* on plane *YZ,* drop coordinate *y* and gain coordinate of place *(z,x)* from plane *ZX, and so forth*

The idea of continuity etc. is also observable by the fact that the standard cycles are a variant of the **Fibonacci rule of $n_i = n_{i-2} + n_{i-1}$,** here in the specific form of $a_3 = 18 - (a_2 + a_1)$ resp. $b_3 = 33 - (b_2 + b_1)$.

Technique:

Principle: The hypothetical word *(p,q,r), (here e.g. 5,6,7)* stands for *3* consecutive members of a cycle. Its coordinates are: *x: 5, y: 6* on plane **PQ**; *z: 7, y: 6 on plane **QR**; x: 5, z: 7 on plane **RP***. The three planar coordinates yield one 3D position. The word of the cycle is understood here to mean the placement of an element in three dimensions.

3.3 Merging into one Newton space

Action:.

Drop the 2nd, inner sorting criterium of the aspects *(a+b,a), (a+b,b), (b-2a,a), (b-2a,a-2b), (a-2b,a), (a´2b,b-2a),* creating such *3 main axes **Z: a+b, X: b-2a, Y: a-2b.***

Point out the 4 possible exact interpretations of the place of *(a,b)* on each of the planes ZX, XY, YZ.

Context:

The model shows one axis ***Z*** *(of similarity, a+b)* of a Descartes 3D space, constructing it together with two axes ***X, Y*** *(of diversity, a-2b, b-2a).*

What we experience as our common Newton space is a conglomerate of two Euclid spaces, which in parts crowd each other out.

Nature appears to use a 4 fields decision table to point out differing geometric properties of the 4 alternatives that come from simplifying the coordinates of a point *(a,b)* on plane ***ZX, ZY*** into a rank on ***Z,*** and those on planes ***YX, YZ*** into a rank on ***Y,*** and those on plane ***XY, XZ*** into a rank on **X.**

The 3D point with coordinates ***ZYX*** is an inexact approximation. To know its exact location, one needs a sequence of 3 parameters, each parameter having 1 of 4 possible variants of forms. This is what the DNA does: points out one of 4 possible variants in units of 3 sequenced turns.

Technique:

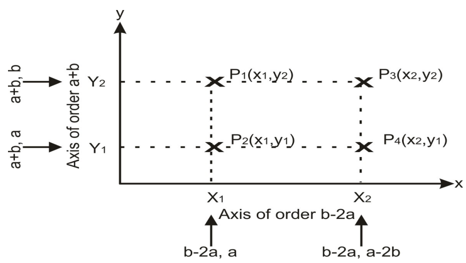


Fig. 4. One 3D Newton coordinate is one of four sequenced variants in two 3D Euclid spaces that build it, being based in 2D planes that root on linear sequences.

Repeat this assignment of 4 potential exact places on planes [a-2b,a] ↔ [a-2b,b-2a], [b-2a,a] ↔ [b-2a,a-2b]. Each plane has 4 variants. There are 3 planes that turn within one temporal-spatial moment, in sequence.

> Remark on the advantages of diploid information transmission: Using the two central elements as reference points in the midst of their respective

swarms, being a picture of *a, b* each in its separate world, allows formulating a hypothesis about the reason resp. advantages of having two sexes. Nature might make use of following reasoning:

> 1. It is possible to describe the world by means of sentences of the form $a = c_a - b$;
>
> 2. It is possible to describe the world by means of sentences of the form $b = c_b - a$;
>
> 3. The identity $a+b=c$ remains true, as long as $diff(c_a - c_b) \leq threshold$;
>
> 4. There exists a method of cross-validation, whether the two (sub)systems are still part of the same system.

In the model presented here, female and male variants of a general concept are like the union of two sets. It is necessary that the two sets contain an intersection, in which the fundamental rules of the general concept are repeated and re-validated.

3.4 Which fits next turn, the Syntax of the DNA

Action:
Draw two Euclid spaces, possibly as two 3D pictures. Observe, which sequence of turns defines the left, as opposed the right, Euclid space.
Run some non-standard cycles in both subspaces. Observe the geometric differences.

Context:
The **triads** of the standard reorders create a web of relations of numeric nature which yields two consistent 3D spaces (and two transcending planes). The inner consistency of the triads is given by their Fibonacci nature $(a_3 = 18 - (a_2 + a_1))$.

> The Sumerians have mastered that method of counting which uses *one* basic unit. Their method is still in use in our days. The present model introduces methods of counting by using *two* different units (which can agree in value and extent). The coexistence of the duality creates a third describing dimension, namely that, how finely the two basic webs are integrated into each other.

The two Euclid spaces are ordered in themselves, but their geometry is different. The planes that constitute the two Euclid spaces are not interchangeable, because the junior part of the axis would not fit with the senior part of the axis in the next turn.
The two Euclid spaces are furnished slightly differently. Their diverse spatial positions are differently densely occupied by the swarms surrounding CEL, CER.

Technique:
There is an additional restriction on the properties of elements that can be on places (turns) 1,2,3 of a word.

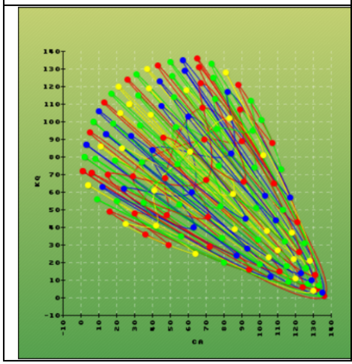
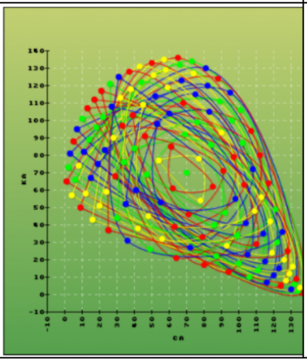
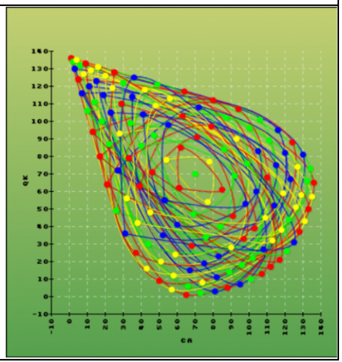
	This vertical plane *z:a+b,a ↔ x: b-2a, a*	
cannot attach to this horizontal plane *z: a+b,a ↔ y: b-2a,a-2b*		**can** attach to this horizontal plane *z: a+b, a ↔ y: a-2b,b-2a*
		

Fig. 5. Left and right Euclid spaces

On the example above: *(a+b)* axis of agglomerative Space C can be on any of the places (turns), originating from either *(a+b,a)* or *(a+b,b)*. It is open, whether the message refers to the left or the right subspace. The axis which creates plane: *up/down-left/right* cannot be any, because only *(2b-a,a)* fits to *(a+b,a)*, and only *(2b-a,2a-b)* fits to *(a+b,b)*. Therefore, although at first glance, on any of the three parts of the logical word, any of *4* symbols can sit, because of the restrictions, that the logical sentence must provide coordinates, only *2* of the arguments can appear together, namely such that belong to the same, left or right subspaces (in the genetic context).

We are very much used to reading *a+b=c* in the context of c, where everything is of unified nature. The sentence is a snapshot in time and shows us an uneasy truce between *a* + *b* on one side and *c* on the other side. These states of the world are both in existence.

> Example: In the historic era of the Warring Kingdoms, there were *two* central elements, and two sets of laws regulating the relations of the subjects to their respective central elements, in geometric terms. The only thing the Kingdoms could agree on, was that gravitation is the same everywhere in case one weighs *1, 32 or 97* units and that it is impossible to use the system above ca. *136*. After unification, with one law for all, the central elements are no more recognized as such *(damnatio memoriae)*. The subjects have no relations to them or among each other, officially, because the Su-

> merian rule of equality by uniformity for all decrees, that the subjects are in no other way different to each other than being differently many of the unit. Notwithstanding the efforts of the Sumerian thought police, the subjects do have relations among each other, like in old days, but it is risky to raise the theme of fundamental differences and what structures these generate. The thought police explain the observed facts of subjects building alliances among each other as Mysteries of Nature, by no means as an artefact of the unifying regime change by the Sumerians.

That what the theoretical DNA of this model does, is pointing out the background for the constellation described. The point in 3D space can connect to a next point, in a geometric version of genetic, if it is a continuation of its predecessor along a line of causation (which we see in the inner consistency of the 3 members of a standard cycle). The real DNA is one of the realizations of the theoretical DNA. For the theoretical DNA it is sufficient to be a working model, if a collection of reorders exists, which are permutations of each other and can follow each other.
We basically say that Nature speaks Shannon's language by using 3 consecutive choices of 0,1 which refer to which of the two possible Euclid 3D spaces is meant to continue to exist in the next turn in Newton 3D space. This gives the furniture of the common 3D space; the coulisses, frames, prerequisites are assembled from two possible storage rooms in each of three axial / planar directions. The furniture of the stage influences the kinds of dramas that can be played on that stage.

The syntax

*{***word1***[{p,q,r,s}, {p,q,r,s}, {p,q,r,s}],*

word2*[{p,q,r,s}, {p,q,r,s}, {p,q,r,s}], ..., etc}*

has great similarities to the syntax assumed to be used by Nature when registering into and reading out from the DNA.

In case the hypothesis is correct, that the message within the DNA refers to properties of subspaces, the syntax would be restricted into

*{***word1***[{{p,q}, {r,s}}, {{p,q}, {r,s}}, {{p,q}, {r,s}}],*

word2*[{{p,q},{r,s}},{{p,q},{r,s}},{{p,q},{r,s}}], ..., etc.},*

which syntax appears to be observed in Nature.

The word of the DNA is concurrently a time stamp on the metronome, because in its folded variety (as one among 128) it points to one of the solutions of 129 into 2 (which point out sentences that relate to $a = 129 - b$, as other sentences relate to *32, 97.)*

4 Tautometry

Chapter Context:
We investigate the relations between *(a,b)* before and during the wedding ceremonies, after which they become a part of *c*.

First, we discuss saturation limits for sentences that can be said about a collection. "*How many objects are needed for x logical relations to be present?"* is answered slightly differently in dependence of whether perception uses similarities or diversities to serve as background. These numeric facts are the fundament for the model.

Second, the material of the discongruence shall be organized. The snippets that are traditionally wished away during the transformation of *a,b* → *c* do not cease to exist. They give cause and form to archetypical multitudes of coincidences.

Last, the idea of the sequence as such is placed in an economic context. The sequence is a realization of order. It appears that a double accounting is practicable. We contrast the data of two data sets: one depicting the reality (of a match between a unit and its place), the other depicting plans partly realized. Units aggregate into cycles, cycles into reorders. We develop a system of deferred and credited rewards based on facts being the case or not.

We mix (wash together) spatial and material properties of symbols and in a later step filter out of the mixture (re-separate) properties that can be interpreted as spatial, and properties that can be interpreted as material, and furthermore properties that can be interpreted as neither-nor but rather relational. The common currency is related to the certitude of the prediction.

The prediction of the properties of the whole based on properties of some of the parts and the prediction of the properties of some of the parts based on properties of the whole are contents of one and the same database matrix of facts and probabilities.

4.1 The numeric facts

Action:
Recreate on your computer the two sequences of *oeis.org/A242615. [5]* These are: *n!, oeis.org/A000142; n? = exp(ln(part(n))²),* where *part(n)* refers to the number of partitions of *n, oeis.org/A000041. [6], [7]*

Context:
A sorting operation creates coincidences by assigning a place to a property and a property to a place. The implication of the members of the symbols set being individuals is, that there will appear specific aggregational locations, temporal segments, and quality types of said agglomerations.

It is everyday experience, that about a collection of a limited number of members only a limited number of distinct sentences can be said. After every possible relation of every member to/with every other member has been pointed out, the subsequent sentences are redundant repetitions (opening a way to learning).

It is irrelevant, whether the sentences are stating an equivalence between or among members, or the sentences state a difference between or among members. **There exists an upper limit to the number of nonredundant sentences that can be said about a collection consisting of a limited number of members.**

Two forms of sentences

The sentences that describe an assembly can come in two forms: such that state a difference between *a,b (<,>)* and such that state an equivalence between *a,b (=).*

Our neurology treats signals of = before a background of ≠ differently to signals of ≠ before a background of =. The former procedure is called *building groups* as we gather all those that share a property and leave to the background those which are different. The application of symbols <, > is called *building sequences,* as we establish a difference in the foreground between two different things and leave to the background all the others with their undefined properties, which are therefore similar in their being one like the other.

We count groups and sequences differently. The algorithm for the maximal number of different sequences that can be built on *n* individuals is well known, as *n!, oeis.org/A000142.* The algorithm for the maximal number of different concurrent groups had to be evolved, as the idea called ***multidimensional partitions*** is not defined in mathematics.

Upper limit for number of variants of coexistent groups on a limited assembly.

Based on test theoretical considerations, we have arrived at a concept of *structural saturation* which uses the identity relation between the number of items that can be validated on a group of *n* probands and the number of groups the *n* probands can build. Both are *f(n),* the quadratic form yields

$$\mathbf{n? = exp(ln(part(n))^2)} \qquad (2)$$

where *part(n)* refers to the number of partitions of *n*, *oeis.org/A000041.*

The graphic in two forms shows two collections of sentences delineated as “information” (which are *otherwise*), which have issues with density, diversity, and space needs. Relative to how many variants of space (which are made up of similar units) are there, the number of variants of quantitative compositions (which are diverse among each other) can or cannot fit easily. There is a region 1..32 in which the type variants can choose among several places, while in the region 33..96 there are relatively few places are available and the variants must fight for the places. The choice is outside of 33..96 *which place is chosen for any given material variety,* inside of 33..96 *what variety of material can come to one of the relatively few places.*

Equivalence points

The equivalence points near *32, 97* give rise to a view of a co-shared reign between *a,b.* Before the Sumerian-led Campaign of Unification, there were two sets of rules in force in the Combined Kingdom of *a,b.* In one, (contra-intuitively, in the Kingdom of the Bigger), everything centered around *2x16,* in the other, around *6x16.* It is a tolerant society, where e.g., metric and imperial measurements systems coexist.

For this hypothesis, that there are *two* linear equivalences, *32, 97,* near which the translation costs between relation and object, = statements and ≠ statements are minimal, because the available sets of symbols are in balance, speaks also that the *metronome cycle,* which is the longest cycle, is *129* units long. A 3D existence can contain such things that possess 2D coordinates in both merged subspaces, which linearize out on **N** at *32, 97.* The wise monarch advances laws that are valid in both partner Kingdoms.

The Bazaar

The *min – max* pair at *11, 66* gives rise to the concept of the **Bazaar.** Information can obviously be compressed and expanded. If *6 x 11* members are sequenced, more different states of the world can be depicted than if *66* members are not sequenced, in terms of spatial variants. On the other hand, *66* non-sequenced members appear to depict more qualitative variants to be potentially possible than there are variants of space to accommodate such. Something is getting compressed and expanded here. The key point is in how far (to what extent) the neighborhood relations support the hypothesis that reorder $\alpha\beta \rightarrow \gamma\delta$ is taking place.

The upper limit on the number of distinct aspects the collection can be described in terms of similarity, the exponent *ln(part(n))* is near *n = 66* is ~ *15.*

The terms of exchange between and among *{similar, diverse, numbering}* are fluid and in a three-way interdependence.

4.2 Eddington delineation

The numbers explain Eddington's determination of the *fine structure constant* to be *1/136,* which later experiments have shown to be *1/137,03.* His view that both numbers have the same meaning is rehabilitated. The value known as fine structure constant is a logical threshold. At this double limit the divergence between two concurrently used variants of **N** arrives at the first time to come above the extent that is attributable to having one more element in the calculation.

> Example: Surveyors A and B use each their own rods. The two rods are equally far away from Zero if both measure *1, 32, 97.* Above *136*, the Δ of the measurement allows $f^1(x)$ to be *136* on $\mathbf{N_1}$, *137* on $\mathbf{N_2}$. The two $\mathbf{N_i}$ variants are $f^1(x)$ for two sets of logical statements in which equivalences resp. differences are expressed. The surveyors arrive at a place where both say: this place is a limitation, delineation. Inside this realm, anything can be an occurrence, with slight restrictions on how the proportions of = and ≠ interact with *n* resp. *x.* It is possible to maintain a symbols system and a logic where there is an inner coherence. The inner coherence is the syntax (grammar) of sentences that describe that what is the case once in terms of stating under the viewpoint of *a,* once under the viewpoint of *b*. (The same applies to =, ≠.) Outside the Eddington delineation, one cannot maintain the idea that such things exist that have at least two references which are comparable to each other and refer to each other in a common grammar. The threshold is supported by two milestones that say that the web of distances read out of

> these milestones begins to err by a whole mile by the next one. There will exist no *138*th milestone, because it will be unclear, whether it designates *138 or 139.* The crisis in inexactitude is observable with *136.* So many highly organized (uniformed individual) members can create that many relations as *137* members can bring forward based on their being diverse among each other (sets of groupies as diverse as they can get, no one has an identification aside their group tattoos).

A system of calculations that relies on the exactitude of $f^1(x) \sim n_!, n_?$ will not work if the calculation errs to the tune of one whole unit. The spectacle Nature provides for us is based on the exactitude of $f^1(x)$ pointing to the same *n,* on which we recognize the variants caused by the similarity of the describing symbols as contrasted to the diversity of the describing symbols. Outside the Eddington delineation, the sentence "*observe, which effects the* Δ *of* $f^1(f_?(n)) \leftrightarrow f^1(f_!(n))$ *causes"* loses its grammatical correctness, because the underlying identity of the numbers of the objects on which to observe Nature's work is no more given. Experiments that investigate the behavior of probands when flooded with similar resp. diverse inputs lose their credibility as soon as it becomes unclear, of how many probands we gain the different outputs.

Technique:

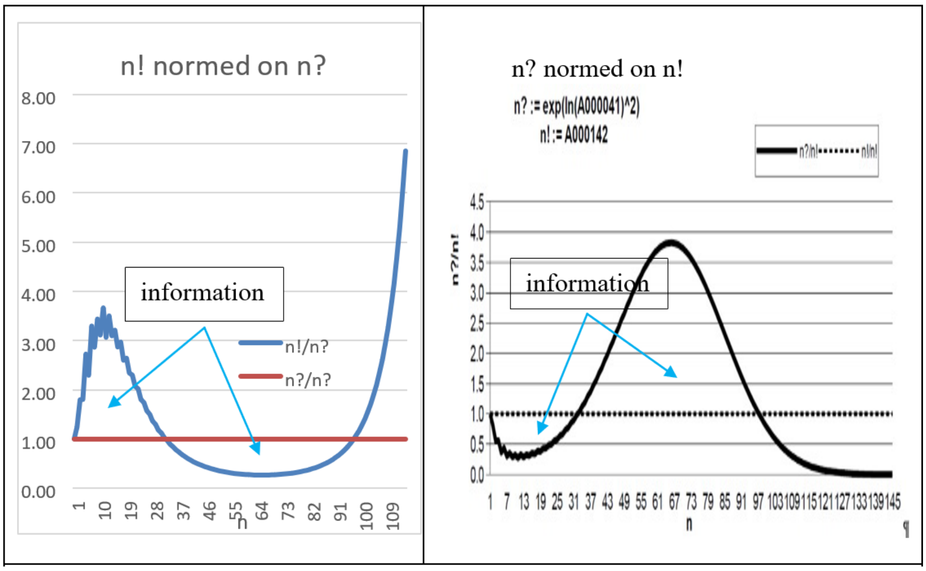


Fig. 6. A242615, in two forms.

Investigate the relative exactitude of $f^1 \sim 137$ of the two functions one to the other.

4.3 Being related means being a part of a whole

Context:
We revert to a Wittgenstein concept of what can be understandably said. Our universe is a part of a collection of units which have *136* individually distinguishable forms.

> There is always a contrasting area of *"not-such"* surrounding a perception, building its background The longest cycle is *129* long (metronome), the next longest (folding) cycle is *128* long (these are inside – outside). The role of those *7 ~ 8* elements that are outside the metronome and folding rules is comparable to that part of the system of logical sentences, about which Wittgenstein advises to remain silent, not knowing how to deal with logical concepts that are not subject to such rules as we know them.

The part we observe are those roughly *60 ~ 63 ~ 66 ~ 70 ~ 73* units that are involved in coincidences that are the case. Together with their counterparts (which are not the case and are the background), these constitute the *120 ~ 128 ~ 136* members that interact in the logical habitat we call rational reality (Eddington delineation).

We shall now investigate, how the members agglomerate into subcollections that possess properties by which properties the subcollections can be categorized and typified, according to aspects: material; spatial; temporal; relational; certain – noncertain.

4.4 Habitat and subjects

Action:
Create the Mother Database, called Matrix M. The Matrix M has axes with value ranges::

Individual	(1..136),
Reorder	(72 * 71),
Cycle	(1..46.260),
total	**3.216.143.232 cells.**

Includes bidirectional counting. Find such bundles of cycles that run parallel to each other. Build bundles and clusters of such *(Task 1)*.

Use the clusters of *Task 1* and typify them. Which and how many cycles are there that are a pairwise connection between established clusters?

Tabulate the clusters according to the quantity and quality of the cycles that interconnect them.

Context:
We have an assembly that contains parts that each are different but share similarity in many aspects. The assembly being ordered and reordered in every conceivable way leads to believing that the assembly is not ordered at all. **We deviate to established tradition** by the assumption, that the seemingly random paths for the diverse elements are in fact not random, because the very fact that the elements are diverse determines that there will be logically possible and logically impossible paths. The catalogued cycles are each a description of a path, providing predecessors and successors

together with planar coordinates for all, under the understanding that a specific reorder is the case.

Taking away the specific reorders, there remain paths which would be followed if reorders were the case. These potential paths are the inner structure of space and matter.

Where there are paths and traffic, there are pileups. Pileups are occurrences, by being an agglomeration of elementary occurrences (both matter *p,q* are in cycles that cross coordinate *x,y*).

Pileups are visual version of the idea of clusters of *Task 1.* These are elements that share (an idealized version of) a spatial coordinate. Like pileups on the number and kinds of vehicles, the clusters can be typified on the number and kinds of parts of cycles of which they are the condensate.

The name **logical archetypes** would be suited for the diverse variants of unavoidable agglomerations observed in an etalon collection while the collection is resorted in any or several ways. The logical archetypes are a mathematical picture of the entries in the periodic table of chemical elements.

Technique:
Work down the 46.260 catalogued cycles.

Investigate if any tendency to self-organization appears: Generate random sequences of *136* natural numbers. Are random sequences R_i, R_j nearer to each other than to a. any of the *72* catalogued sequences, b. in terms of cycles, to so many among the 46.260 catalogued cycles that a whole of a reorder can be assembled of, even if not catalogued?

Dictionary of Basic Concepts: Populate planes with clusters of such cycles that can run concurrently, being disjunct. Add such cycles of which members are in two or more clusters.

4.5 Change perspectives between foreground and background

Action:
Establish a plane AB with axes *a,b (1,16)* and translate the mother matrix M into a form, in which the cycles become **positional attributes of a, b.** The dot on plane AB signifies that one of the *46.260* catalogued cycles passes there (includes this element). Each dot is a data depository.

Context:
We turn the observation around, away from temporal-geometric consequences for different values of *a,b,* (target) given specific reorders are the case (source), into temporal-sequential and planar logical consequences for different reorders (target) given specific values of *a, b* are the case (source).

A blink of dot on plane AB signifies – like a LED on a switchboard – that an element is the case, but does not say explicitly, of which cycle of which reorder.

A combination of dots that blink, and even more so a temporal sequence of such, restricts the number of possible cycles that can take place, and such also the categories of reorders in which such reorders take place. The dot patterns of LED on plane AB are a Shannon-language description of the proceedings. The neighborhood relations of blinks on plane AB are determined by **material** differences between values of *a,b* and contrast with the **geometrical** differences the cycles impose during periodic changes.

Adding planes e.g. *a-2b, b-2a etc.* opens an interesting discussion about identities and synonyms. A blink on plane AB is concurrently a blink on a plane *a-2b, b-2a.* The two blinks are logically identical but have different appearances and neighbors.

We build a numeric concept of the inner incertitude which allows compromising on some apparent contradictions.

The contrast we propose to use is between the existence of a specific form of order - as a theoretical expectation, a plan - and the existence of a specific collection of predecessor – successor relations, observed occurrences.

In other words: we keep two ledgers. In one we register the observations that p_i are on places q_j, and we make predictions of which of the reorders {[αβ] ↔ [γδ]} takes place; in the other ledger we register the observations which of the reorders {[αβ] ↔ [γδ]} takes place and we make predictions of p_i coming to places q_j,

The advantages of using two sets of expectations - observations come from the ability to engage in credit – debit operations within the system itself.

Technique:
Build the system of pairwise tables of *{a,b,c,k,u,t,q,s,w}*. Translate Matrix M.

4.6 An Example of Creative Double Accounting

Action :
Start off an AI process that aims to predict which periodic changes are taking place. Let it learn from messages from mother matrix M that <such and such> coincidences of peg-hole pairs are the case.

With suitable feedback, the mental organization AI can learn the system of place – matter coincidences. If the AI can name an offset as a synonym, one side of the system of mutual expectations and observations is established. Redo.

Context:
. The logical web that the differences among the symbols create is of a geometry that is not Euclidean. We have axes and planes that have less variants than are in mother matrix M. A coordinate made up of *a,b,b-a,2a-b, etc.* is an index into a data heap where the entries are ordered differently than in the geometric web. A point here is a multitude of combinations of cycles, reorders, time ticks which could all be the case.

There is an **element of freedom** in the fact that there exist different sentences that describe the same state of the world.

Seen from above, where periodic changes generate orders and orders realize in the form of reorderings, the cycles are the end result, being parts according to the plans of the whole. Seen from below, the system of facts, that amounts are on places and will gain specific places during a reorder's cycle, the cycles are also an end result.

Two interrelated sets of predictions are to be read out of mother matrix M. In matter relating to wholes, the units are reorders and the subunits are cycles. The ready – not ready distinction is on how far the member cycles of a reorder are all cooperating. In matter relating to parts, the units are the cycles and the subunits are the matter/place coincidences. The ready – not ready distinction is on how far the member elements of the cycle are all cooperating.

Where architects and masons agree on basic principles, the particularities of the observed generalities are like offsets among cycles. Hypothesis: in both systems of accounting, there exists a degree of fulfilment relative to the certainty.

The missing part is present in both systems of predictions. **That is the information content**. (*Relative to a fulfilled reorder, cycles p,q,r are not done; Relative to cycles being fulfilled, t,u,w are not done.)* The range above which a whole is counted as done is below a threshold of certainty. The extent of being not yet completely done relative to being absolutely finished is a basic unit that is subject to barter in the bazaar. Options and derivatives appear as possible bets on how the process will develop.

The freedom in the choice, which relation system to use when describing a state of the world allows generating sentences that are grammatically correct but nevertheless false on the level of facts. The level of facts connects to the level of plans and order via the mediation on the level of cycles.

Technique:
Go through following half-steps of reasoning:

The eligibility of an element to serve as a middleman between predecessor and successor depends on liaison calculations. The Bazaar is the place where everything can be bartered: the value of belonging to a worthy lien may compare favorably to the costs of leaving some others.

Optimal Orders: optimization of one specific aspect results in deterioration in ease of ordering in all the other aspects. If there were an Overall distances summation over all the cells of the mother matrix, one could imagine that to be a part of Parameters of Order, which limit and regulate the interactions of rules applicable to assemblies that consist of distinct parts.

The idealtypical form, Gestalt: human neurology wishes for the best and for the best combination of the best. The whole is built of parts and knowing some parts allows imagining the whole. The collection of imaginations that can be made based on parts is the Gestalt. It is more than the sum of its parts because it includes such properties as coexistent also which are pairwise exclusive.

In the ordered collection, there exist limits for the other two of any of the three properties: similarity, diversity, number. This is the reason why it is not possible that the number of optimal orders reverts to *1, One.* The ideal order is a multitude of optimal orders, none of which constitutes (causes/is the description of) a sustainable ideal.

5 Summary

Seen as a card game, traditional counting uses cards that on the front picture the value of the card and the reverse side is uniform. The update turns over the playing cards and finds 136 different symbols etched into the reverse side. Nature plays a complicated form of a game of patience, finding variants of *a+b=c* that go well together.

Discussing *a+b=c,* by using computers, inevitably turns up some rules about how specific kinds and combinations of *a,b* will behave towards a c which usurps a communality between *a,b,* which these may not wish to be of any relevance.

The left side of the sentence states that it is true that *a,b* are coexisting and possibly different. The two partly independent, partly differing, partly similar part-worlds are very much still in existence. Diversity and differences keep being a fundamental principle of the world.

The duality itself transcends its many particularities. We contrast what we perceive against a background, which is made up differently to the foreground. One counts slightly differently, if one counts diversities before a background of similarities, as opposed to counting similarities before a background of diversities. There is a vivid **bazaar** where symbols have differing values, subject to the market situation dominated by ***{how many, how diverse, how similar}.***

There is a credit – debit relation between the ledgers: facts – relations. The accounting magic appears and disappears with the criterium of being sequenced or not. If *66* objects are not sequenced, the assembly can carry > *3* times as many messages as if the objects were sequenced.

The property of being sequenced means that if contrasting requirements are concurrently the case respective the geometric and the logical successor, the logical argument wins.

We have found the Cycle of Eternity, aka metronome cycle, to be 129 units long. If a true sentence about relations among 129 members of a collection that contains 136 elements can be said, then that collection is well-ordered. Inside the metronome cycle is a folding cycle of length 128.

The numeric picture appears convincing. The relations among the symbols are quite robust, being implications of pairs of natural numbers.

Acknowledgements

Present paper owes much to the fruitful and invigorating discussions in the chat group Fundaments of Information Science. FIS works since 1995 under the chairmanship of Pedro C Marijuán. Fis Info Page: https://listas.unizar.es/cgi-bin/mailman/listinfo/fis:

References

1. Wittgenstein, L.: Tractatus logico-philosophicus. Kegan Paul, London, (1922). also: https://people.umass.edu/klement/tlp/, last accessed 2024/01/10
2. Abundis, M.: Informatic Primitives: Logical vs. Material. In: http://listas.unizar.es/pipermail/fis/2020-July/002595.html, last accessed 2024/01/10
3. Wittgenstein, op. cit., § 1.
4. Whittaker, Edmund. "Eddington's Theory of the Constants of Nature." https://doi.org/10.2307/3609461
5. www.oeis.org/A242615, last accessed 2024/01/10
6. www.oeis.org/A000142, last accessed 2024/01/10
7. www.oeis.org/A000041, last accessed 2024/01/10
8. www.oeis.org/A000217, last accessed 2024/01/10
9. www.oeis.org/A235647, last accessed 2024/01/10