## Review of: "From General Equilibrium to Algorithmic Equilibrium"

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Potential competing interests: No potential competing interests to declare.

(Five stars - meaning: deserving publication, perhaps with some modifications to help interdisciplinary reading.)

Review by Manuel Tarrazo, PhD, and Professor of Finance, School of Management, University of San Francisco, January 2024.

Prof. Li's contribution, "From General Equilibrium to Algorithmic Equilibrium," is a very difficult and meritorious effort to clarify an overwhelmingly complex and complicated domain—the general area of economics.

It suggests that we all—economists and non-economists, but especially theoretical and academic economists—should embrace a perception of a situation that can be characterized as equilibrium and disequilibrium at the same time, and the best to hope for is to be able to progress by employing the best problem-solving we can, as the middle-age algebraists, as doctors were known, would do when setting broken bones. Professor Li's subject matter is likely to present several challenges even to dedicated, interdisciplinary, committed readers. I felt that the biggest problem for the reader was being aware that some perfectly accepted terms have no empirical content whatsoever and cannot keep being used as if they were useful.

"Economic equilibrium" (EE) implies that both concepts are good descriptors of what they imply, the economy and equilibrium, and that both allow for a clear and logical happy marriage.

In reality, beyond basic language, "economic" may refer to "things related to the exchange of goods and services both for profit and not-for-profit, ways for agents to earn a living, and the participation of the public sector in those pursuits." In turn, "equilibrium" may refer to various conceptualizations, each having different degrees of abstraction, theoretical and practical implications, and purposes. However, many recently educated economists may not even concern themselves with the EE materials found in textbooks written in the 1900s, where the traditional approach to EE can be found.

Walras's multimarket equilibrium may work in some products, industries, and services, but it is not applicable at the level of an economy or region of the world. It posits that a given "n" demand and supply functions (not necessarily linear) and a single n-point dimensional solution can be found. The economy is usually understood as a collection of n markets for n products and services, each having a price,  $p\neg i$  (i = 1, ..., n) (see Samuelson (1947)). Most of the time, it is very difficult to find suitable supply and demand functions for many goods and services; worse yet, some of them do not have markets (e.g., public goods, administration of justice, defense, and organs for transplants). Furthermore, the solution to a set of

linear, simultaneous equations representing the problem may contain negative prices! The theory of inequalities, game theory, and mathematical programming provided some hope during the 1930s and facilitated some implementations of Leontief's input-output theory, which goes as follows: An economy of n markets could comprise firms selecting some inputs and producing n outputs to satisfy resource and budget constraints and represent cost and profit schedules. On the demand side, consumers could simply choose various bundles from all the goods and services available, given their budgets. These developments helped a great deal to optimize resource allocation. They also helped not only to consider interactions between consumers and producers at a given point in time (i.e., a static model) but also to visualize how things could change over time (see, for example, Von Neuman's model (1945) and Champernowne (1946)). The story became more difficult to summarize during the last half of the 1900s. Welfare economics and economic growth became specific topics in economics, and computing and modern advanced mathematics fed much theoretical and practical work (see Takayama (1974) and Judge et alia (1991)).

One cannot think about an equilibrium without thinking about optimality (a word not mentioned in Li's note). A cemetery, for example, may appear to be in a rather stable form of equilibrium—not optimal for the living. Rationality is not possible without perfect information or omniscient decision making. Moreover, with respect to optimization, we do not optimize as we do in algebra or calculus. Agents (households, firms, the government) do what they can with what we have and deal with whatever comes to them the best they can. In Simon's words, instead of "optimizing," we employ our lives mostly "satisficing."

Readers should be aware that the discussion in part three of the paper (equilibria versus disequilibria) builds on mathematical abstractions. For example, Nash's equilibria in markets, when agents act and react to other agents' plays, is an application of game theory. The discussion can only be understood in terms of regions of a supposed algebraic model constructed in a given space of computation, in which we can operate dynamic formulas (linear algebra, first- and second-degree dynamic equations). Yes, Nash's model and other theoretical models from economics help us think about alternatives, but compared to the complexity of what is going on in the real world, they may resemble the plastic shovel and bucket children use to build sand castles by the sea.

To my professional knowledge, there cannot be optimal solutions at the aggregate production or consumption levels because such functions cannot be constructed. There is not yet a solution to old essential questions, such as the relationship between interest rates and aggregate investment by productive units. New complexities have annihilated old categories of thought. Thin, for instance, is the concept of how human capital has superseded the Marxian labor versus capital binary concept so useful for politics. Macroeconomic policies seem to have very little or no theoretical support. The monetary area seems rudderless without guidance on the relationship among monetary aggregates, effective amounts of money, interest rates, and private investing, a shortcoming that is evident when the inflation hydra monster rears its head, causing havoc time and time again. The fiscal areas—government budgets, revenues, and expenses—seem to have taken on lives of their own. Governments spend and spend and tax and tax, and no other dynamic is in sight. A summary of a recent conference organized by the IMF mentioned the lack of theoretical and quantitative support to guide policy (see Blanchard et alia (2016)).

Curiously, it is at this point of apparently disarming complexity that Professor Li's contribution comes in very handy indeed because it invites us to think of two paths. One is the quant/algorithmic/step-wise, qualitative process in what resembles a new and improved embodiment of Walras' tâtonnement, systematically orienting us toward some objective and marching toward it step by step. The other is a unifying process that can only succeed if it is understood as going beyond the traditional domain of what started as unassuming household management ("oikos nomos") and became a way to disguise arbitrary decisions, political and otherwise, with imposing concepts and baroque, heavy, and manifold academic regalia.

It is in this framework conceptualized by Prof. Lee that we can acceptably use intellectual constructs, such as those models containing the very expectations of their outcomes (rational expectations), with qualitative insight distilled mostly from historical analysis (e.g., Milton Friedman's work on monetary theory), and add comprehensive and encompassing thoughts on system complexity (see Hayek (1988, 1974, and 1964)), in some cases even questioning whether decision making can be simply reduced to a mathematical model.

Our quantitative limitations and the fact that we are in a dynamic process, in which the future is only known (if at all) imperfectly, push us toward the qualitative, interdisciplinary area. In other areas, we find other intellectual constructs and insights that may cross-fertilize our work in economics and other areas. For example, we may study those models from biology and ecology that focus on species and growth. May (1976) shows that even simple model specifications can include very sophisticated dynamics. That is the case with logistic growth, in which each period's growth is not necessarily determined by stationary carrying capacities. The biologist Volterra and the chemist Lotka provided models in which the carrying capacity may depend on species interaction (predator-prey) or on support characteristics of the respective ecosystem (e.g., plant food) (see Bernstein (2003)). Chaos theory models (e.g., Lorentz's three-equation weather model) provide insights into formerly unknown states (e.g., strange attractors) and alternative pathways (see Gleick (1985), Strogatz (1994)).

Once we start visiting those worlds that we economists once considered not our own, things seem to get a bit easier and we can find the synergy we needed. This situation has been called consilience: "The concurrence of evidence that occurs when data from very dissimilar fields provide support for the same scientific hypothesis" (Mautner (1997, p. 109)). For example, from ecology and biology, we may enhance our understanding of ways economic agents interact beyond naked competition. We may consider cooperation behaviors that result in mutual benefit because they are good for us, our partners, and the ecosystems. The tools for decision making and our thinking are also likely to go to the next plateau. For example, the logistic equation, with its built-in sustainability, may replace our exponential functions, limited only by the artificial imposition of constraints. For example, mortgage loan calculations use exponential formulas to calculate payments and interest, but this only works because we have a set time constraint: the length of the loan.

Professor Li's study adds yet another very welcome bonus that will help scientists from various fields learn from each other: communicability.

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