v1: 11 March 2024

Research Article

# Continuum Models and Singularities for Heat Distributions From Light

Peer-approved: 11 March 2024



Qeios, Vol. 6 (2024)
ISSN: 2632-3834

**Lena Strömberg**[1]

1. Independent researcher

**Air flow and quasi-static heat around heat sources and shields are exemplified and analysed. The purpose is to (improve thermal efficiency, i.e.,) obtain much heat adjacent to the device and its surroundings. Knowledge from single devices and sources in a row is used and interpreted into comparisons with, e.g., heat waves. Navier-Stokes equations, other balance equations, and rules from continuum mechanics are scrutinized and combined with proposals for the buoyancy of heated air. Results for singularities are derived and visualised with the aim of describing heat power potentials in room layers.**

**Corresponding author:** Lena Strömberg, lena_str@hotmail.com

## 1. Introduction

Oscillations and traveling waves co-existing with a streamline flow have been cast into modeling in various applications, e.g., river flows, heat, EM, and viscous flows [1][2].

In the present paper, heat distributions from an electric lamp will be modelled. The main factor giving the distribution is buoyancy [3][4], giving an upward flow of lighter heated air. With a smart wide shield, some heat can be gathered at suitable low locations in the room. In modeling, superimposed waves [2] may give static steady heat. Analytical quasi-static solutions are related to results in FEM for the symmetric case. The purpose of the study is

- to understand the development of heat layers and matter
- to obtain heat in a room, i.e., develop low-power heaters with the function and size of a conventional heater (preferably with no fans).

The content is organised into sections: First, general models for heated particles are revisited. Then, calculus with differentials and couplings to a solid shield will be proposed. Finally, the real macro-scale heat distribution with buoyancy is considered. From that, conditions for singularities in a power representation are derived.

## 2. Fluid Dynamic Models and Quasistatic Heat

In fluid mechanics, the Bernoulli law (BL) is a governing balance when following the streamline of a Navier-Stokes flow. The equation reads

$$du^2/2 - dgh + p = const \tag{1}$$

where d is density, u is velocity, g is gravity, h is a relative distance, and p is the pressure.

With the preliminaries in [1], e.g., assuming that density d depends on an internal variable proportional to velocity u, we obtain traveling waves. The superposition of exponential traveling waves gives solutions in terms of cosh() and sinh(); i.e., hyperbolics.

The energy equation in continuum dynamics gives Poisson's equation for a scalar temperature field (aka the heat equation). With generalisations into nonlinear parameters, internal friction, and input due to mechanical work, the balance equation changes. The additional terms are e.g. p(d, t/d) and n(u, x)$^2$ where t denotes time differentiation and n is viscosity.

Depending on the sign of parameters (corresponding to linearisation of input/output), the equation is hyperbolic or elliptic, and the solutions are e.g. hyperbolic cosine and cosine.

For the solid elastic case, an FEM formulation [5] of the linear equation with boundary conditions is solved numerically on a domain c.f. [6]. From the results in Figure 1, the temperature level might be traced as the sum of cosh() + const for each direction (vertical and horizontal). Since it is lower in the centre, partly, the remote behaviour may be sufficiently described with a radial coordinate, r.

*Remark.* With a weak formulation, a singularity at the r-dependency in polar coordinates is not always regained, which means that a symmetric solution could be smooth also for that case.


doi.org/10.32388/UXITTP.2

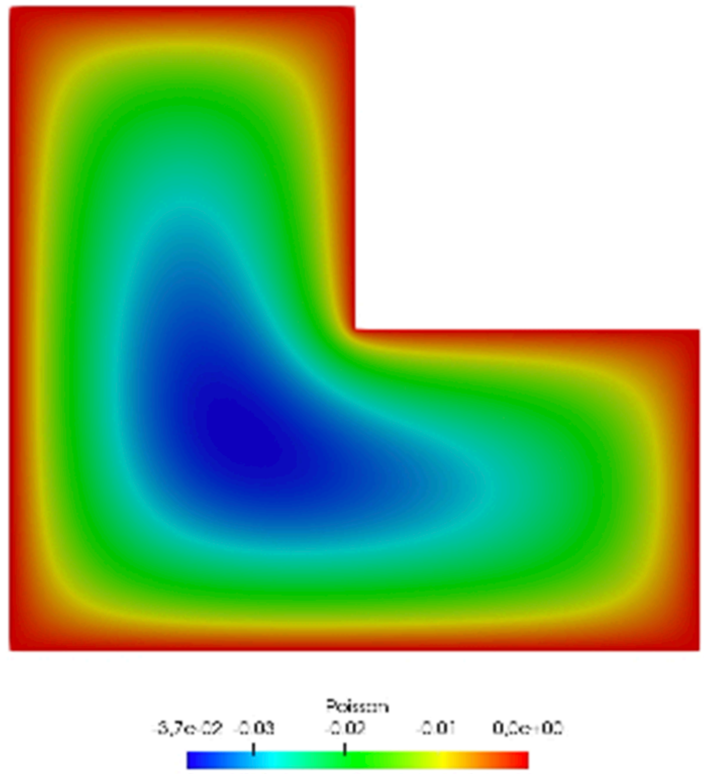


**Figure 1.** Temperature level for the heat equation (Poisson's equation) with boundary conditions on input heat, [6]. *Remark.* In FEM, the Lagrangian element has an exact Gaussian integration, and other formats are e.g. serendipity elements.

## 3. Models for coupling to layers

In order to couple with material adjacent to air, we shall assume additional densities, parameters, and pressure (e.g., as sums with the air matter).

This admits:

- options for modeling layers without knowing the perpendicular radial velocity
- inputs to models with differential geometry (; convenient when matter appears as several d.o.fs)
- coupling between temperature and e.g. electricity in the static heat equation

Adjacent to a lamp shield, a proposal for the gas law is: $p = (d + d_L + d_E)(r + r_L)\ H$

where d is density, H is temperature, $r_L$ is a material parameter that changes with material layers, and $d_L$, $d_E$ are additional densities at a layer or electricity. Here, we will consider a non-electric air layer close to matter such that

$$p = dr_L H \tag{2}$$

With (2), a differentiation of BL (1) gives $u\,du + dr_L H + r_L dH = 0$..... (3)

where $du$, $dr_L$ and $dH$ denote differentials of the fields. This admits several kinds of solutions for temperature, e.g., $H \sim u$ and $H \sim u^2$. The latter is similar to that obtained in the original version (1) and has a certain symmetry.

*Remark 1.* Theoretically, from the format (3), and assuming boundaries, it is seen that layer parts may propeller themselves by decreasing the temperature.

*Remark 2.* In reality, heated air is lighter and moves upwards due to buoyancy. This will be further discussed in Section 5, below.


doi.org/10.32388/UXITTP.2

# 4. Horizontal solutions on a flat wide shield


**Figure 2.** Realisations with a flat wide shield are found to give a warm surface.

Consider a shield a distance from a lamp, Figure 2. With the velocity u proportional to a coordinate y from the center and parallel with the matter, H=H(y) in (3) gives

$$(H'(y)r_L + Cu)dy = -dr_L H \qquad (3)$$

where C is a constant. Next, we will tacitly include time to obtain a semi-analytical solution. With $dr_L$ having support at the layer; $dr_L$~dy and H initially being the Dirac delta distribution at a coordinate, a Heaviside function is obtained for the temperature, H. This is a promising solution for heat to spread sideways on a wide shield. With the symmetric solution H~u$^2$, derived in Section 3, the entire shield will be heated. However, it may extend (violate) the preliminaries for the scale of a material point in a Lagrangian frame with local coordinates. More realistic solutions may be arrived at with approximations:


doi.org/10.32388/UXITTP.2

$H'(y) r_L$

$<< Cu$ and $dr_L$ proportional to $dy$ gives $H(y)$ proportional to $Cu$

$H'(y) r_L$

$>> Cu$ and $dr_L$ prop. to $dy$ gives $H'$

$(y)$ proportional to $H$ i.e., exponential solutions.

## 5. Spatial Buoyancy Model with Pressure Domination

Heated air gets lighter and moves upwards due to buoyancy. These boundary actions with relative motions are not included in the pointwise BL (1); however, the pressure function may rule and balance into such streaming expressions. The upward velocity of the light, hot air may increase and decrease with a distance y, c.f. Figure 3.

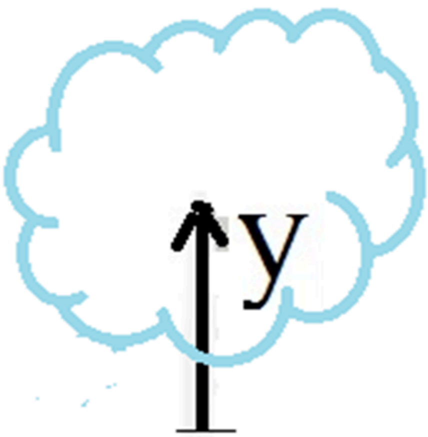


**Figure 3.** Vertical coordinate y for a parcel of hot air moving upwards due to buoyancy.

With a balance for kinetic energy and pressure, this will be proposed with functional relations:

Density is decreasing when lighter air with a higher temperature moves upwards. A pressure potential for this uplift is proposed to read: p(y)= $-Py^m{}_*\exp(-s*y)$...... (4)

where P, m, and s are parameters, and y is a vertical coordinate, as shown in Figure 3.

With d=1, the velocity from an energy balance similar to (1); $du^2/2+p=C$, is given by

$$u = 2 * sqrt(C + Py_*^m exp(-s*y)) \qquad (5)$$

For some values of the parameters (C, m, s), u=u(y) is shown in Figure 4, with the code [7].


doi.org/10.32388/UXITTP.2

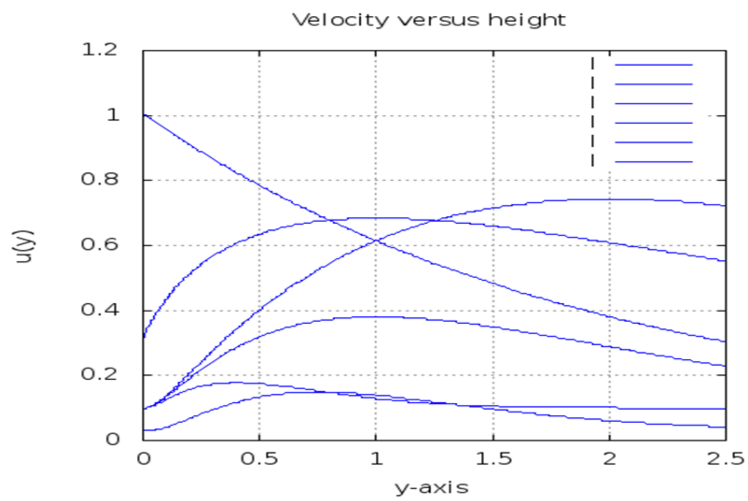


**Figure 4.** Draw2d(/* global options */ title = "Velocity versus height", xlabel = "y-axis", ylabel = "u(y)", grid = true, dimensions = [500,500], /* implicit function */ key = "I", implicit(sqrt(0.01+exp(-x))-y, x, 0,2.5, y, 0,1.2), implicit(sqrt(0.1+x*exp(-x))-y, x, 0,2.5, y, 0,1.2), implicit(sqrt(0.01+x*x*exp(-x))-y, x, 0,2.5, y, 0,1.2), implicit(sqrt(0.01+x*x*exp(-2*x))-y, x, 0,2.5, y, 0,1.2), implicit(sqrt(0.01+x*x*exp(-5*x))-y, x, 0,2.5, y, 0,1.2)), implicit(sqrt(0.001+x*x*x*exp(-4*x))-y, x, 0,2.5, y, 0,1.2);

### *5.1. Continuum fluid analysis*

The pressure function (4) is a constitutive equation dependent on an inverted length-scale and a non-dimensional parameter m. Since it approaches zero at boundaries, it may serve to model pressure both vertically (at the buoyancy) and horizontally.

A differentiation of p(y) or, for brevity, ln(p/P), gives a maximum at y=m/s.

A known distribution related to temperature is the pressure of an acoustic medium. For an array of hot candle spots, it is found to produce a significant heat wave in reflecting semi-thick steel plates. Maybe also with only one hot spot, a more efficient surrounding shield can be created with a certain geometry. Another possibility is to induce a galvanic stress on a material, leading to a spreading of heat.

*Remark.* The pressure function proposed in (4) in a periodic pattern may trigger acoustic behaviour, which on occasions (subjected to small perturbations, or due to rules for sound waves being dual) appears in other distributions, thus giving inputs to heat.

### *5.2. Derivation into functional formats*

In continuum mechanics, the constitutive equations do not include coordinates. With the assumptions proposed above, a relation without coordinates for specific mechanical power will be derived. Subsequently, this will be referred to as specific power or power.

*Theorem 1.* Neglecting the constant C in the velocity u in equation (5), the (mechanical power)/(unit volume) p*u, y for a material point reads...... ½ sqrt(p) p′(y)..... (6)

*Proof:* Differentiation of (5) and (4) and substitution of p from (4), assuming C=0.

This specific power enters into a balance with the rate of energy. Energy E is assumed proportional to temperature as E=dcH, where c is a specific heat parameter.

*Remark.* Geometrically, p may be naturally composed such that the sqrt gives a format of coordinates with 'integer-degree'; i.e., $x^k$, where k is an integer, which means an area measure or composition of such.


doi.org/10.32388/UXITTP.2

### *5.3. Decomposition into energy balance*

*Example.* Consider a horizontal sequence of p(y) from (4), on lengths of magnitude 2m/s, i.e., the pressure function repeated periodically. This constitutes the spatial pattern of a stationary wave. The insertion of the expression p′(y)=p(y)(m/y-s) in (6) gives two parts. The first part is more concentrated close to y=0, and the other part is the function ½(-s) $p^{(3/2)}$, i.e., similar to the pressure p(y), but with other parameters.

In order to describe temperature, we will compare with the balance equation for energy E in continuum mechanics when neglecting heat flux:

$$E_{,t} + u^*grad(E) = pu,y \tag{9}$$

*Remark.* Neglecting heat flux is motivated when pressure dominates and describes more rapid and local behaviour. Heat conduction by diffusion can occur afterwards and in surrounding areas.

Next, we will discuss how the two terms on the left side of (9) could be balanced by each term in (6): With E=dcH, as suggested in section 5.2, the time differential of E may be negative when the temperature rises and the density decreases. That is known to happen generally for heated fluids (and is known as buoyancy [3][4]). Here, the first term may balance the negative term, (-s) $p^{(3/2)}$/2, on the right side.

The other term, ½(m/y) $p^{(3/2)}$/2, is more concentrated and may have a singularity. That could be a distribution for the light ('one hot spot'), and the gradient of E may still spread in other directions than the velocity. In general, the direction of the velocity could also be horizontal, and then generate a temperature gradient along a shield. (A sub-structure-localised velocity may emanate from local buoyancy or be a particle velocity in a stationary sound wave.)

### *5.4. Singularity for specific power (6), in terms of bound for m.*

For smaller values of the parameter m than in Figure 4, the distribution for specific power can be singular.

*Preliminaries*. A singularity at y=0 is defined as $y^k$ where k<0.
*Exercise*. Derive the bound for m at which the first term in (6) has a singularity.
*Solution.* Insertion of (4) and p′(y) in (6) gives the exponent for y, reading 3m/2-1... (10).

From the preliminaries, 3m/2-1<0, i.e., m<2/3.

In Figure 5, the pressure p(y) from (4) is shown for m=0.5 and s=0.9, 0.5. The strength of the singularity for specific power (6) is given by (10), i.e., 3/4-1=-1/4.

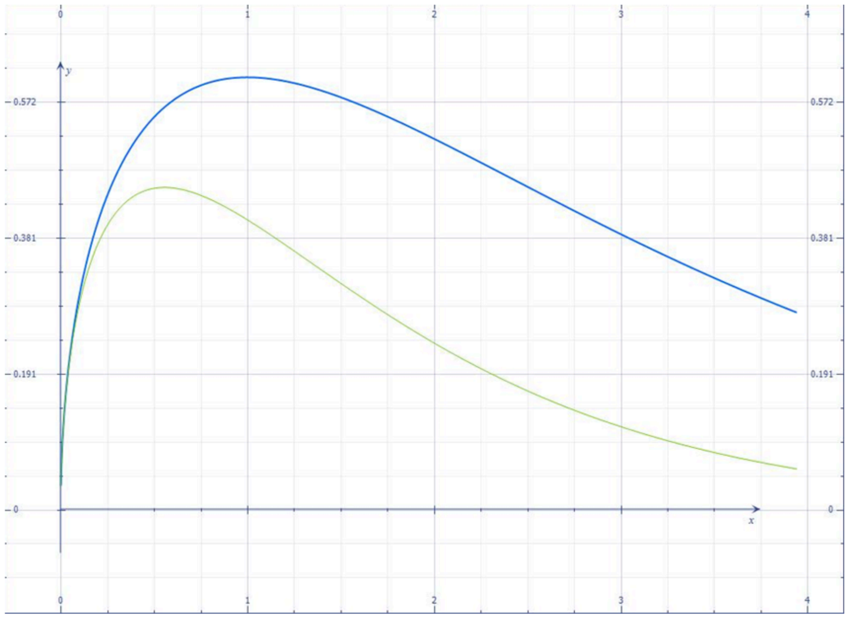


**Figure 5.** Pressure function (4), p(y)/-P= $y^{m}*\exp(-s*y)$ with parameter m<2/3, such that the power (6) is singular at the origin. Upper curve m=0.5, s=0.5. Lower curve m=0.5, s=0.9.

*Remark.* Consider equation (9), when heat conduction is included and grad E is perpendicular to u such that convection vanishes. When the singular part of power balances the Laplacian of temperature, a Poisson's equation similar to that of gravity for a 'density'-distribution is obtained.

*Exercise.* For m<0, i.e., when the entire pressure is singular at the boundary y=0, a stronger singularity is obtained for the specific power, expression (6). Derive the exact expression.

## 6. Concluding summary of the analysis

Traditionally, electric heat generators are high power, i.e., costly. In the present paper, potential and prospects for heat from low-power light were gathered.

- Solutions on shields were outlined, such that temperature fields and velocity materialise in certain fashions.
- After buoyancy & reaching the limiting shield, heat may spread sideways in accordance with heat waves, Poisson's equation, and BL.
- In addition, with differential geometry and multi dimensions at layers, several options for temperature are given. These ideas were gathered in Sections 3 and 4.
- At buoyancy, to balance the kinematics, an input potential energy provided by pressure p was proposed. For certain assumed spatial distributions p(y), the velocity u(y) was visualised.
- Singularities for the specific power; (6) balancing the rate of energy c.f. (9) were derived.

## 7. Future Realisations

*Semi-dynamic solution for maximised located quasi-static heat.*

The solution proposed in Section 5 has a non-uniform distribution vertically. This induces motion sideways, after which it may spread in the material of a shield. Hereby, combining the results, the shape of a shield with a larger heated surface could be made wider. Experiments show that:

1. circulation from below is needed, e.g., with holes for colder air to enter
2. a suitable material is a thin flexible responsive metal (or plastic-metal composite). This is because it probably requires that the layer materials on the solid surface can move on a micro-scale. Two


doi.org/10.32388/UXITTP.2

realisations with thick glass and thick plastic were non-responsive in spreading heat.

3. The ability to spread depends on material properties. In a FEM-solution scheme [5][6], which can be implemented into details, as well as possible coupling with galvanic stress between regions, to foresee and simulate temperature distributions. When flows and other fields occur [1][2][3][4][8][9][10], semi-analytic approaches are present to manage the interactions.


## Acknowledgements

The author would like to thank the reviewers for valuable advice concerning structure and clarity.


## References


1. a, b, c *Strömberg L (2023). Response to curvature, gradients and temperature in a PPB-fuel cell, with a LED and a Li-B. Advances in Environment and Energies, Sanderman PH. Volume 2, Issue 1, No. 3.*
2. a, b, c *Strömberg L (2023). Viscous and thermal Eulerian velocity models for electric heat generation. Security and Management in Modern Civil Engineering, Sanderman PH. Volume 2, Issue 1, No. 4.*
3. a, b, c *Robertson, Glen. (2023). VEM Drive as a Buoyancy Effect. 10.13140/RG.2.2.13814.98886.*
4. a, b, c *Li, Scott & Woods, Andrew. (2023). Boundary mixing. Part 1. Transport of buoyancy and passive tracer. Journal of Fluid Mechanics. 976. 10.1017/jfm.2023.796.*
5. a, b *https://doc.freefem.org/tutorials/thermalConduction.html https://doc.freefem.org/tutorials/poisson.html*
6. a, b, c *Liu, Wing & Li, Shaofan & Park, Harold. (2022). Eighty Years of the Finite Element Method: Birth, Evolution, and Future. Archives of Computational Methods in Engineering. 29. 10.1007/s11831-022-09740-9.*
7. ^ *Maxima online http://maxima.cesga.es/*
8. ^ *Brzenski, Jared & Castillo, Jose. (2023). Solving Navier-Stokes with Mimetic Operators. Computers & Fluids. 10.1016/j.compfluid.2023.105817.*
9. ^ *Truesdell, Clifford & Noll, Walter (2004). The Non-Linear Field Theories of Mechanics. Springer. ISBN 978-3-662-10388-3.*
10. ^ *Riesen P, Hutter K, Funk M (2010). A viscoelastic Rivlin-Ericksen material model applicable to glacier ice, Nonlin. ProcessesGeophys., 17, 673–684, https://doi.org/10.5194/npg-17-673-2010*



### Declarations

**Funding:** No specific funding was received for this work.

**Potential competing interests:** No potential competing interests to declare.