Review of: "NP on Logarithmic Space"

Szabolcs Iván[1]

1 University of Szeged

Potential competing interests: No potential competing interests to declare.

The submission claims to prove, among other things, L = NL, with its key points being Hypothesis 1 (more on this below), Theorem 2 which states “if hypothesis 1 is true, then L = NL” (and NP is a subseteq of L^<L>) and Theorems 7 & 8 which are together said to imply Hypothesis 1. Hypothesis 1 is a conjunction of two statements, the first being proved in Theorem 7, the second being proved in Theorem 8.

Unfortunately, the proof of Theorem 7 is flawed irreparably, the proposed reduction does not preserve membership. For a counterexample, consider the graph 1 -→ 3, 2-→3, 2-→4 with topological sorted representation 1,2,3,4, source s = 1, target t = 4. In this graph t is *not* reachable from s. However, the generated formula is unsatisfiable, as setting x1 to true would force x3 being true due to (-x1+x3), that would force x2 being true due to (-x2+x3) which would force x4 being true due to (-x2+x4) which would contradict to -x4. As Theorem 7 would be the key component of showing the existence of an 1NL-complete language being in L, this ruins the whole chain. (Of course if s is the only source in the directed acyclic graph, then all the other nodes are reachable from s, making that version of the problem trivial instead of 1NL-complete so that possibility cannot be excluded.) This construction works for *undirected* graphs actually, but for those, reachability is known to be in L.

I did not check the other parts of the paper (in particular, whether hypothesis 1 would imply L=NL or the other statement, and Thm 8 with Hypothesis 1's second part) but the proof of Thm 7 is certainly wrong.

(Another note: it is well-known that NL = coNL so I believe literally no-one in the scientific community says things as “a language belongs to coNL” but “to NL” instead, apart from some early parts of textbooks where the Immerman-Szelepcsényi theorem is not yet shown at that point.)