Review of: "Does the Time Dimension has to be Perpendicular to the Space-Dimensions?"

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The author states the following question “Does the time dimension has to be perpendicular to the Space-Dimensions?”. The article is well-written for an opinion. It is clearly stated and it provides simple graphs and mathematics to argue the author´s opinion.

However, in my opinion the article does an incomplete work.

First, it does not consider the problem fully from a mathematical point of view. The article confuses the concept of dimensions with the concept of coordinate system. The article considers Minkowski space as the basis for our current understanding of reality and observation, when it is not. Our current understanding is General Relativity. Minkowski space is just a special case. Part of the problem that I see with the article is that it does not approach the problem using the appropriate mathematics for such problems.

To avoid confusion and wrong interpretations, we need to approach the problem with tensor calculus, so we can avoid any confusion with the specific coordinate system that we chose. Therefore, for a given general space we can describe the quadratic line element as:

$$ds^2 = g_{\alpha\beta} x^\alpha x^\beta$$

The quadratic line element defines the space in which we are operating, and it is given by the contraction of the metric tensor with the corresponding coordinates. Dimensionality is given by the values taken by the covariant and contravariant indexes.

The metric tensor is the mathematical structure that provides the relationships between the covariant basis vectors that define any dimensions of our space without the need of specifying the coordinate system. As these are basis vectors, these mathematical structures are invariant, unlike the coordinate systems. This is expressed as follows

$$\vec{u}_\alpha \vec{u}_\beta = g_{\alpha\beta}$$

Thus, we can have combinations of dot products between unitary basis vectors that do not fulfil orthogonality, so we cannot conclude that Relativity assumes orthogonality because in **general terms it does not:**

$$g_{\alpha\beta} \neq \delta_{\alpha\beta}$$

In the specific case of Minkowski space, it is true the following:

$$g_{\alpha\beta} = \eta_{\alpha\beta}$$

Hence, in the general form of the metric tensor,**no orthogonality is assumed**. Therefore, this is not truly a problem, at least in a way interpreted by the author. In general relativity we can have pseudoRiemannian space-times in which the “time coordinate” is not orthogonal to the spatial coordinates. No problems with that. It is not an issue.

The real issue is on the linearity of the time dimension. Here we have an issue. Minkowski and General Relativity spaces are constructed assigning a non-zero value for the null vector. The null vector is identified to the speed of light. So that the speed of light is the “zero distance” in this particular space:

$$0 = g_{\alpha\beta} x^{\alpha} x^{\beta}$$

Applying the conditions for Minkowski space with the speed of light representing the zero line element, from the equation above we conclude that

$$\frac{x^2 + y^2 + z^2}{t^2} = -c$$

This identification is where the problem with the author may lie. This identification is made to ensure Lorentz co-variance. If we do not do that, our space-time will not be Lorentz co-variant. But this statement also makes the time coordinate DEPENDENT on the spatial coordinates. Hence, as the author said, we can indeed describe a model of our reality with a time coordinate that is not orthogonal to the spatial coordinates. But if we consider the time coordinate linearly independent (and we can do it), then it does not fulfil Lorentz covariance.

Therefore, the author needs to prove that in his space, he can explain the phenomena associated to Lorentz covariance, which is characteristic of relativity. He can indeed construct his space-time as he is describing, but my advice is to use proper tensor mathematics to avoid confusing dimensions with coordinates.