

Commentary

What if Calculations of the Economic Effects on Germany of a Halt to Energy Imports from Russia Are Misled by the Neoclassical Cost-Share Theorem?

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Mainstream economics underestimates energy's role in economic growth. Energy is either not considered as a factor of production, or it is weighted by its cost share. According to the cost-share theorem, the economic weights (output elasticities) of the production factors capital, labor, and energy are equal to these factors' shares in total factor costs. In industrial countries, the shares have been roughly 0.25 for capital, 0.70 for labor, and 0.05 for energy. We look into two publications of leading economists who argued that an immediate German stop of Russian gas imports, in response to the Russian-Ukrainian war, would have only a small impact on German economic growth. Another example could be energy policies aiming at a significant reduction of energy use and related emissions to mitigate climate change. In a model scenario, based on suggested drastic energy reductions, we compute (hypothetical) losses of industrial output with production functions a) that forgo the cost-share theorem, and b) neoclassical ones, which use it. The losses in case a) are by an order of magnitude larger than in case b). Suggestions for an improvement of the real-case analyses are made. The technological constraints, whose neglect in the maximization of profit or overall welfare has led to the misleading cost-share theorem, are recalled.

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Most of the time only the loss of things teaches us their real value.

Arthur Schopenhauer, 1851.

1. Introduction

“The future will be cold and dark if we don’t have a radical rethink,” says the title of an editorial of the *London Sunday Times*, dated August 7, 2022. Referring to Charles Hall, it points out: “Economists have long regarded energy as a commodity like any other, and relatively minor one, given that the energy sector constitutes only 5 percent of a modern economy. Hall and his colleagues retort that without the 5 percent the rest of the economy would grind to a halt.”^[1]

A little earlier, on April 4, 2022, *The Los Angeles Times* published the article “Germany gets fresh criticism over its purchase of Russian natural gas” ^[2]. It reports the Ukrainian ambassador’s demand for an immediate embargo on oil, natural gas, coal, and metals and the German chancellor’s response: “Scholz had bristled at a suggestion last week during a German TV interview by leading independent economists that an immediate embargo on Russian gas would only slow, but not cripple, the German economy. ‘They’ve got that all wrong,’ Scholz said sharply. ‘It’s irresponsible to calculate some kind of mathematic model like that. ...If we were to suddenly stop importing natural gas [from Russia], that would mean entire industrial sectors would have to shut down.’” A suggestion like the one to which the German chancellor had referred has been published via the policy report outlet *EconPol* in the study “What if? The Economic Effect for Germany of a Stop of Energy Imports From Russia” by Bachmann et al. ^[3], see also ^[4]. It reports losses in German gross domestic product (GDP) because of a sudden stop of Russian gas imports that are between 0.2 percent of GDP for the most complex model calculations and up to 2.2 percent for simplified models. Schularick, one of the authors of ^{[3][4]}, confirmed the low estimates of GDP losses in the ARD TV program *Monitor* on April 21, 2022. Other economists added that losses should not exceed 4 percent. The moderator of *Monitor* concluded that in view of the terrible pictures from Ukraine, Germany is morally obliged to tolerate these losses and stop energy imports from Russia right away.

The low estimates of the economic losses due to a Russian gas embargo have raised considerable discussion in Germany about the country’s appropriate response to the Russian-Ukrainian war. Quite a few scholars question the small economic weight attributed to energy by neoclassical economists. They suggested looking into the methodological foundation of the study ^{[3][4]}, which uses standard neoclassical mathematics. We join the critics, because we believe in “The need to reintegrate the natural

sciences with economics” ^[5], especially in the first two laws of thermodynamics—the most powerful laws of nature. They can be summarized by “Nothing happens in the world without energy conversion and entropy production.”

More specifically, in the context of production and growth, the laws of thermodynamics imply two important roles of energy: i) in the short term, it is in fact energy conversion that activates the capital stock, giving rise to varying capacity utilization in industrial production: in this sense, energy and capital are complementary. And, ii) in the longer term, energy and capital can replace human labor in the course of increasing automation: in this sense, energy and capital on the one hand, and labor on the other, are substitutes. Of course, energy can be substituted by capital in various ways by introducing technologies with higher energy conversion efficiencies, but those efficiency improvements are subject to thermodynamic limits. These two roles of energy, being pivotal in both capacity utilization and automation in production, support the intuition that energy has a relatively high output elasticity (in particular, higher than its typically very low factor cost share). This will be explored in more detail by the present paper.

The remainder of the paper is structured as follows. Section 2 briefly reviews energy-dependent production functions, with a focus on the Cobb-Douglas and LinEx functions. Irrespective of the functional form chosen, our emphasis is on the economic weight of the production factor energy, i.e., its output elasticity. We point out the role of the cost-share theorem (CST), according to which the output elasticities of capital, labor, and energy should—but empirically do not turn out to be—equal the respective factors’ shares in total factor cost. In Section 3, output growth in Germany’s industrial sector is calculated with the Cobb-Douglas function and the LinEx function. Their econometrically determined output elasticities are much larger for energy and much smaller for labor than the respective cost shares. In Section 4, German industry is considered in an imagined scenario of sudden energy scarcity, similar to a Russian gas embargo. We compute how this would affect growth if a) neoclassical output elasticities based on the CST or b) econometrically estimated output elasticities were used. This leads to Section 5 and a proposal regarding the studies ^{[3][4]}. Why the cost-share theorem is invalid is sketched and interpreted in Section 6, based on a more rigorous treatment including the formal optimization of profit or overall welfare, where relevant technological constraints regarding capacity utilization and automation are taken into account, whose shadow prices invalidate the CST ^[6]. Section 7 concludes on the importance of adequately taking into account the impact of energy on industrial production and growth in view of the implications for energy policy.

2. Energy-dependent production functions, Cost-share theorem, and Solow Residual

Bachmann et al. [3][4] use macroeconomic energy-dependent production functions for analyzing the generation of a country's gross domestic product (GDP), or parts thereof. So do we. The production of output (value added) at time t is described by the twice differentiable function $Y[K(t), L(t), E(t); t]$ of the inputs capital $K(t)$, labor $L(t)$, and energy $E(t)$. The production function Y is a state function of the economic system (just as internal energy or entropy is a state function of thermodynamic systems). Thus, the infinitesimal change of output, dY , is a total differential, and the growth of output is given by

$$\frac{dY}{Y} = \alpha \frac{dK}{K} + \beta \frac{dL}{L} + \gamma \frac{dE}{E} + \frac{\partial Y}{\partial t} dt. \quad (1)$$

Here

$$\alpha \equiv \frac{K}{Y} \frac{\partial Y}{\partial K}, \beta \equiv \frac{L}{Y} \frac{\partial Y}{\partial L}, \gamma \equiv \frac{E}{Y} \frac{\partial Y}{\partial E}, \quad (2)$$

are the *output elasticities* of capital, labor, and energy. They are the economic weights (productive powers) of the production factors. Roughly speaking, the output elasticity of a factor gives the percentage of output change when this factor changes by 1%, while the other factors stay constant.

Bachmann et al. assume that output elasticities are equal to factor cost shares. In highly industrial economies, the cost share of labor is much larger than the cost share of energy. Contrary to Bachmann et al., we use production functions and output elasticities that ignore cost shares. Their calculation is outlined in the following.

At any *fixed* time t , an increase of *all* inputs by the same factor κ must increase output by κ , because at the fixed state of technology that exists at the given time t a, say, doubling of the production system doubles output; in other words: two identical factories with identical inputs of capital, labor, and energy produce twice as much output as one factory. Thus, the production function must be linearly homogeneous in K, L, E . As a consequence, the Euler relation, in combination with eq. (2), yields the so-called “constant returns to scale” relation:

$$\alpha + \beta + \gamma = 1. \quad (3)$$

References [6], pp. 11-15, and [7], pp. 199-215, show in detail how to calculate production functions and output elasticities by integrating eq. (1), observing eqs. (2), (3), and certain boundary conditions; the law of diminishing returns guides the choice of equations for output elasticities. Free parameters have been

determined econometrically, observing the restriction that output elasticities must not be negative. Statistical quality measures, such as the adjusted coefficient of determination and the Durbin-Watson coefficient d_W , turned out to be acceptable. Cointegration analysis [8] confirmed the magnitude of computed output elasticities. Two of the production functions calculated this way are shown (in dimensionless variables) by eqs. (5) and (6).

So far, we have looked into the economic growth of Germany, Japan, and the USA. Sources of the empirical data for output and capital, labor, and primary energy are (inflation-corrected) national accounts with appropriately chained time periods, labor statistics, and energy balances. Research labs in the USA, Japan, and Germany kindly helped to get high-quality data.

It is convenient to work with dimensionless variables that are normalized to their magnitudes in a base year t_0 :

$$k(t) \equiv \frac{K(t)}{K_0}, l(t) \equiv \frac{L(t)}{L_0}, e(t) \equiv \frac{E(t)}{E_0}, y[k, l, e; t] \equiv \frac{Y[kK_0, lL_0, eE_0; t]}{Y_0}. \quad (4)$$

For these variables, equations (1), (2), and (3) hold likewise.

In Figure 1, economic growth in the industrial sector of Germany is reproduced by the energy-dependent Cobb-Douglas function

$$y_{CDE} = y_{CDE}^0 k^{\alpha_0} l^{\beta_0} e^{1-\alpha_0-\beta_0} = y_{CDE}^0 e \left(\frac{k}{e} \right)^{\alpha_0} \left(\frac{l}{e} \right)^{\beta_0} \quad (5)$$

and the (first) LinEx function

$$y_{L1} = y_{L1}^0 e \exp \left[a \left(2 - \frac{l+e}{k} \right) + ac \left(\frac{l}{e} - 1 \right) \right]. \quad (6)$$

The latter depends *linearly* on energy and *exponentially* on factor quotients. It is a production function of *varying* output elasticities. The constant Cobb-Douglas output elasticities α_0 , β_0 , and the parameters y_{CDE}^0 , y_{L1}^0 , a , and c have been determined by appropriate fitting procedures.

For the German system “Industries,” whose output growth is shown in Figure 1, the time-averaged LinEx output elasticities computed from (6) are for capital $\bar{\alpha} = 0.284$, labor $\bar{\beta} = 0.076$, energy $\bar{\gamma} = 0.640$ [9]. Thus, they are for energy much larger and for labor much smaller than the cost shares of these factors. (There is also a small contribution to output changes from temporal variations of the LinEx efficiency parameters a and c .) The magnitudes of the constant Cobb-Douglas output elasticities α_0 , β_0 , $\gamma_0 \equiv 1 - \alpha_0 - \beta_0$ are similar to those of the time-averaged LinEx elasticities.

Ayres and Warr analyzed economic growth in the USA during the 20th century, using “useful work” as the energy variable in the LinEx function. They defined “useful work” as *exergy* (i.e., the part of energy that can perform physical work), multiplied by appropriate conversion efficiencies, plus physical work performed by animals. The “useful work” data already include most of the efficiency improvements that have occurred in energy-converting systems during the 20th century. In this case, two constant efficiency parameters suffice for a good reproduction of the gross domestic product of the US economy between 1900 and 1998 ^{[10][11]}. The time averages of the corresponding output elasticities of capital, labor, and “useful work” are for “useful work” much larger and for labor much smaller than these factors’ cost shares.

As said above, mainstream economists like the authors of ^{[3][4]} assume that the output elasticity of a production factor is equal to its share in total factor costs. Until recently, in highly industrialized countries the cost shares of capital, labor, and energy have been roughly 25%, 70%, and 5%, respectively. Thus, neoclassically, the corresponding Cobb–Douglas output elasticities would be $\alpha_o = 0.25$ for capital, $\beta_o = 0.70$ for labor, and a meager $\gamma_o = 0.05$ for energy.

The cost-share theorem of neoclassical economics states that the output elasticity of a production factor is equal to the factor’s share in total factor costs. It results from maximizing profit or general welfare *without observing technological constraints*.

There are two such constraints. One is due to the fact that energy activates the machines of the capital stock, and that the flow of energy into these machines is limited by the designs of how much they can take; if an energy flow exceeds a machine’s specification, the machine breaks down. The other constraint is that the degree of automation of the capital stock is limited by the state of technology; trivially, the degree of automation cannot exceed 100 percent.

Robert M. Solow computed the output (per capita) of the US economy during the first half of the 20th century. He used a Cobb–Douglas production function of capital and labor with output elasticities equal to the factors’ cost shares. He found a theoretical output curve far below the observed output curve ^[12]. The big discrepancy between observed and computed growth of output became known as the Solow Residual. To account for it formally, the concept of “technical progress” was introduced. “Technical progress” is *the* driving force in neoclassical growth theory. In 1994 Solow commented on that: “... the neoclassical model: it is a theory of growth that leaves the main factor in economic growth unexplained” ^[13].

In the following, we will consider the data underlying the computation of the German economic losses by Bachmann et al. and look into the relevant mathematical appendices of [3][4]. Criticizing them, we will essentially call upon passages from two publications of ours [6][9]. Wondering about an appropriate way of improving things, without being able to do it ourselves right away, we will suggest modifications to the analyses of Bachmann et al. by using economic weights (output elasticities) of energy and labor that are for energy much larger and for labor much smaller than these factors' shares in total factor cost. They result from production functions that are computed econometrically and well reproduce economic growth in Germany and elsewhere since 1960, thereby mostly resolving the Solow Residual of neoclassical growth theory. The (time-averaged) economic weights obtained from them are for energy about 0.44 (and for labor about 0.19) in Germany's total economy (GTE) [9], see also [6][7], whereas the cost share of energy, and thus energy's economic weight in the simplified models of Bachmann et al., is a meager 0.04.

3. The economic weights of capital, labor, and energy in Germany's industrial growth

Table 1 of [3] (or [4]) shows the total amount of German primary energy usage in 2021 to be 3387 TWh (= 12.193 PJ). Gas contributes 905 TWh to this amount, and about 55% of this gas, i.e., 498 TWh, is of Russian origin. Bachmann et al. assume that substituting coal and liquid natural gas (LNG) for part of the 905 TWh of gas could reduce the energy loss by a total embargo of Russian gas to $30\% \times 905 \text{ TWh} = 271 \text{ TWh}$. This is 8% of the 3387 TWh primary energy used in Germany in 2021.

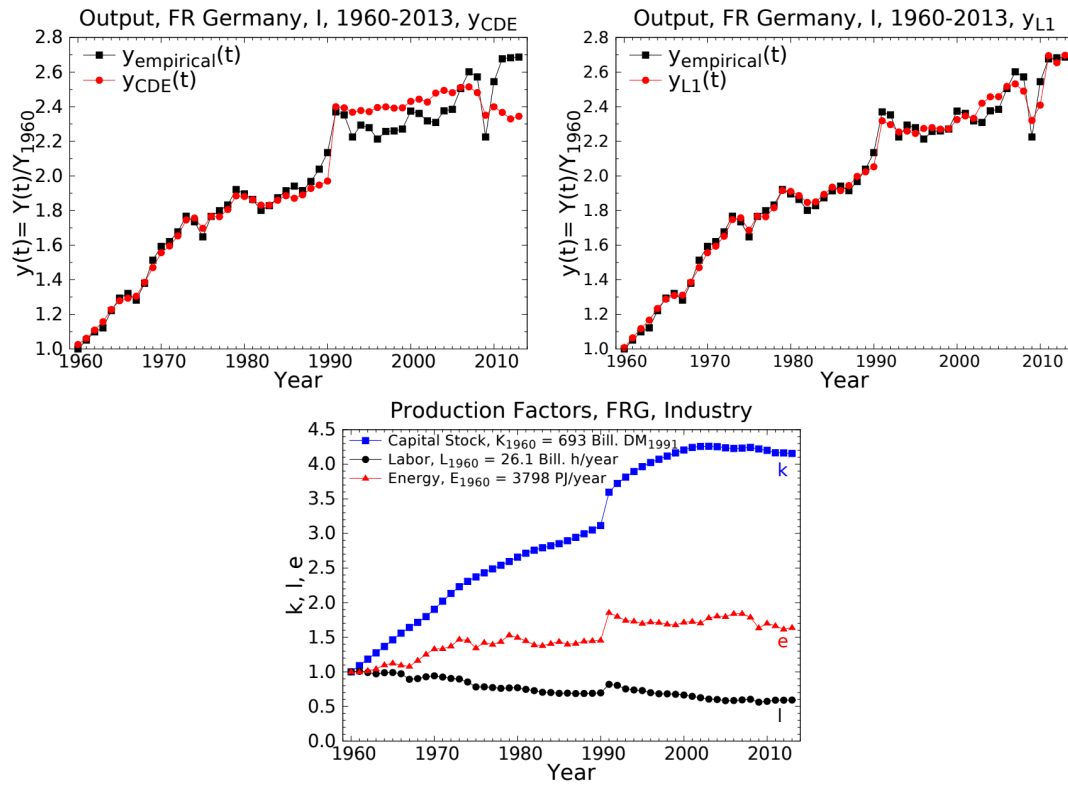


Figure 1. Growth from 1960 to 2013 of the empirical (inflation-corrected) output $y = Y/Y_{1960}$ in the industrial sector of the Federal Republic of Germany (FRG I), black squares, and theoretical growth computed with the energy-dependent Cobb-Douglas function, red circles (top, left), and the LinEx function, red circles (top, right). Empirical growth of (inflation-corrected) capital $k = K/K_{1960}$, labor $l = L/L_{1960}$, and energy $e = E/E_{1960}$ (bottom). 1960 is the base year to which output and inputs are normalized.

$$Y_{1960} = 453.5 \cdot 10^9 \text{ DM}_{1991}.^{[9]}$$

To get an idea of economic losses in Germany caused by a sudden stop of Russian gas, if one calculates them by *not* using the cost-share theorem but rather the method of [\[6\]\[9\]](#), we look into the impact of gas-supply shortages on the output of value added in the German industrial sector “Produzierendes Gewerbe” (defined in the German national accounts by the classification system WZ 2008). This sector, FRG I, is the pillar of the German economy. If it crumbles, the other sectors will follow. Nonlinear interactions between the sectors may enhance the damage.

Figure 1 shows the empirical evolution of output and of the production factors capital, labor, and energy in FRG I from 1960 to 2013, and the theoretical reproductions of output by the energy-dependent Cobb-Douglas function and the (first) LinEx function as well [\[9\]](#). The output elasticities (economic weights) of

capital, α , labor, β , and energy, γ , turn out to be a) for the Cobb-Douglas function: $\alpha = 0.41$, $\beta = 0.06$, $\gamma = 0.53$, and b) for the LinEx function's time-averages: $\bar{\alpha} = 0.28$, $\bar{\beta} = 0.08$, $\bar{\gamma} = 0.64$. The output elasticities were computed by SSE minimization subject to the constraints of non-negativity, using the Levenberg-Marquardt algorithm ^{[9][14]}. Reunification in 1990 caused structural breaks in the economy of the political entity "Federal Republic of Germany" (FRG), which the Cobb-Douglas function with its constant output elasticities is less able to take into account than the LinEx function with its factor-dependent output elasticities.

The contribution of FRG I to the GDP of the FRG was 51.7% in the year 1970, 39.6% in 1992, and 27.1% in 2009 ^[7]. (The corresponding contributions of the service sector were 44.9%, 59.2%, 72.0%.) After 1990, the capital stock has increased less and less, because of the outsourcing of more and more "simple" energy-intensive industries such as textiles, basic pharmaceuticals, iron, and stone. What is left for "Made in Germany" are novel chemical products and medical instruments, special, high-quality metals, sophisticated machine tools, design and installation of complex industrial plants, transportation systems for people and cargo, etc. The successful export of these energy-intensive products in the course of increasing globalization provided the basis for the expanding German service sector and the associated jobs; for an econometric analysis of this sector, see ^[15]. If the industrial basis breaks down because of a sudden lack of indispensable energy, the service sector will suffer, too. The estimation of such induced losses is a demanding task that would need to take into account subsectoral interdependences, including limited substitution potentials along specific value chains and multiplier effects.

The fundamental importance of the production factor energy is due to its economic role of activating the capital stock. This implies bidirectional causality between energy conversion and value generation¹, and manifests itself for Germany in the four economic recessions and recoveries and the simultaneous downs and ups of the energy input in Fig. 1. Two of them were caused by supply and two by demand, and three were enhanced by feedback between the two. More details are in ^[16].

Worldwide crises of economic growth were triggered a) by wars, which led to the first and the second oil-price shocks 1973-1975 and 1979-1982, and b) by the Lehman Brothers bankruptcy that led to the financial crisis 2008-2010. For German industry, they clearly show in Fig. 1.

4. With industry through the fog of uncertainty

Subsequently, we concentrate on the example of the German industrial sector to exemplify our worry that the cost-share theorem may mislead political advice given by economists who believe in the small economic weight of energy. We think that industry is least affected by the uncertainties that blur the look into the other economic sectors.

The losses of output in the service sector induced by damages in industry caused by a sudden stop of Russian gas are uncertain, as indicated above. In addition, energy data available for the service sector are in end-energy. For instance, in 2019 Germany had a primary energy use of 12.805 PJ. The end-energy obtained from this was 8.973 PJ. Of that, 1.315 PJ went into services, business, and commerce ^[17]; the service sector had a share of roughly 14% in end-energy ^[18]. This share has not changed dramatically during most of the years since 1990. Thus, the direct impact of primary energy variations on the service sector should be much smaller than the indirect impact from variations in the industrial sector (FRG I).

In addition to the uncertainties in the service sector, there are the uncertainties that affect households when an abrupt stop of Russian gas imports leads to a stop of production and unemployment ^[19]. On the other hand, according to the rules established for the “Bundesnetzagentur”, which is the institution entrusted with the distribution of resources like natural gas, households should be minimally affected by gas losses. Therefore, we exclude households from further consideration; see also ^{[3][19]}.

Given this situation, we think that an application of “What if?” to the German industrial sector may best facilitate orientation. We turn back to the not-so-distant, but more peaceful years 2008-2009, when the financial crisis hit hardest. In a sort of thought experiment, we ask: What would have happened to the output of FRG I between the years 2008 and 2009, if, for whatever reason, the primary energy input in the German economy had been reduced from 2008 to 2009 by the same percentage as Bachmann et al. assume for the year 2022, namely 8%? We ask and answer this question for the sole purpose of demonstrating the dramatic difference made by *non*-use and use of the cost-share theorem.

In 2008, the primary energy input in Germany was 14,380 PJ ^[20]. Reducing it by 8% yields a loss of primary energy input of 1150 PJ. (Opposition of environmentalists to more coal and new LNG may render losses larger than the eight percent assumed in ^{[3][4]}. Thus, concerning the consequences of a Russian gas embargo, we may quite underestimate the loss.)

The empirical primary energy input in FRG I was 6797 PJ in 2008 ^[14]. Division of the (hypothetical) loss by this quantity yields $1150/6797 = 17\%$. Multiplying this percentage by the LinEx energy output elasticity $\bar{\gamma} = 64\%$ results in a hypothetical loss of industrial output Y of **11%**.

The Cobb–Douglas function is widely used by mainstream, ecological, and biophysical economists because of its simplicity and despite its shortcomings. We will return to the latter in the next section. Its energy output elasticity, which leads to the theoretical growth curve in the left panel of Figure 1, is 0.53. It would result in an output loss of 9%.

Actually, the loss of industrial output in 2009 during the financial crisis was **13%** ^[14].

Thus, the result of our thought experiment is: If one weights energy not by its cost share but by output elasticities determined econometrically, the computation of the hypothetical output loss of German industry due to an energy loss of 1150 PJ yields a loss that is of the order of magnitude of the empirical loss in 2009, the year of the deepest depression of the financial crisis.

Had we weighted energy by its neoclassical cost share of about 4 percent, we would have obtained an output loss of about 0.7 percent. Then we might have told then-chancellor Scholz and the media that a total embargo of Russian gas would cause economic losses in industry that are much smaller than those during the financial crisis.

5. Substitution and the cost-share theorem

During the second oil-price shock, at an international conference on natural resources, the economist Wolfgang Ströbele pointed out that because of the second law of thermodynamics, capital cannot completely substitute for energy ^[21]. At that, a world-famous mathematical economist rose and shouted with a reddening face: “One must never say that! There is always a way for substitution.”

Substitution is a key element in Bachmann et al.’s study ^{[3][4]}. Hüther’s comments on that are revealing ^[19]. In addition, Cam et al. ^[22] point out that an embargo on Russian gas would affect all of Europe – save the Iberian peninsula – the repercussions of which on Germany are not taken into account by ^{[3][4]}. And neither are they by our study. Thus, the economic damages of an embargo are probably underestimated. But in the following, we can do no more than suggest how to ameliorate the approach of ^{[3][4]} by *not* making use of the cost-share theorem.

We now look into some math in the appendices of ^{[3][4]}. We limit ourselves to a minor and a major point.

The minor point is that there are two typos in the “constant elasticity of substitution” (CES) function, Equation (1) of [3] (Eq.(A1) of [4]): The exponents of α and $(1 - \alpha)$ should be 1.

The major point concerns Figure 1 in the Appendix of [3] (Fig.A1 of [4]). (A check of some numbers indicates that the figure has been computed with the correct CES function.) It shows output Y as a function of energy E for different elasticities of substitution σ : The decline of output with the decline of energy E becomes larger as the magnitude of σ decreases.

If the elasticity of substitution σ is 1, the CES function turns into the Cobb-Douglas function, written down above equation (2) of [3] (eq.A2 of [4]). Below this equation it is said: “... production declines with energy E but with an elasticity of only α . In our calibration (Appendix A.8 in [3] or Subsection A9 in [4]) we choose $\alpha = 0.04$.” Thus, Bachmann et al. weight energy according to the cost-share theorem. This is standard tradition in neoclassical economics. In the highly industrialized countries, the share of energy cost in total factor cost has been 4 to 5 percent, see, e.g., [23].

In the notation of the preceding section, the Cobb-Douglas output elasticity of energy is γ . With $\gamma = 0.53$ the Cobb-Douglas function reproduces the output Y_{CDE} in our Figure 1, upper left panel, with residuals that are small compared to the notorious Solow residuals in neoclassical output computations.

The Cobb-Douglas function allows for asymptotically complete factor substitution. Nevertheless, it reproduces past industrial growth quite well – especially when there are no structural breaks – if one ignores the cost-share theorem: Entrepreneurs know the limits to substitution from experience and acquire capital, labor, and energy accordingly. Things are different when looking into the future.

Remarkable are the estimations of future economic losses due to climate change that have been published by the renowned economists Nordhaus [24], Beckerman [25], and Schelling [26]. They assumed that climate change only affects agriculture. Since this sector contributed less than three percent to the GDP of the USA in 1992, they concluded that even a drastic decline in agricultural production should only result in small losses of welfare. Herman Daly commented on this in his famous article “When smart people make dumb mistakes” [27].

Limits to substitution are taken into account by the LinEx function, which is a “variable elasticity of substitution” type of function.

Now, it would be most interesting to extend the studies of [3][4] to CES functions that *ignore* the cost-share theorem and, following Uzawa [28], are extended to more than two inputs. For the inputs capital k , labor l ,

and energy e (in the notation of our Figure 1), such a CES function is given in [29]. It is

$$y_{CES} = y_{CES}^0 [ak^{-\rho} + bl^{-\rho} + (1 - a - b)e^{-\rho}]^{-1/\rho}, \quad (7)$$

where the distribution parameters a and b must be non-negative. The constant $\rho \equiv 1/\sigma - 1$ (with σ being the elasticity of substitution) must be larger than -1 . We propose to use this CES function and proceed similarly to Bachmann et al. when estimating the losses (in production Y). Conveniently, these losses may be presented as in Fig. 1 in the Appendix of [3] (Fig. A1 in [4]). Thus, the project would be:

- Compute and plot new Cobb-Douglas curves with output elasticities that are close to the ones obtained from the calculation of the Cobb-Douglas curve in Figure 1, i.e., $\alpha = 0.4$ for capital, $\beta = 0.1$ for labor, and $\gamma = 0.5$ for energy.
- Use the three-factor CES function of Eq. (7) and compute the losses of output Y with decreasing energy e for different magnitudes of σ , considering for each σ various combinations of capital k and labor l .
- Plot the resulting curves below the curves of the corresponding new Cobb-Douglas functions.
- Modify the message of [3][4] according to the magnitudes of the losses obtained in this way.

We think that this may help to check the robustness of Fig. 1 in the Appendices of [3][4] and the associated results, especially in view of the econometric results on energy's high output elasticity in industrial production systems, as presented in Section 2 and the literature referenced therein.

6. Why the cost-share theorem is invalid

According to the cost-share theorem (CST), the output elasticity of a production factor equals the factor's cost share in total factor costs. The mathematical proof that the CST is invalid for present industrial economies is given in [6]. Here, we try to summarize in a non-technical way what the validity or non-validity of the CST would require and imply, and what the consequences may be.

To start with, let's quickly recap: The output elasticity of a production factor measures, roughly speaking, the percentage change in output if a production factor changes by one percent (all other factors held constant). At a fixed time t , if all production factors are taken into account, a doubling of all inputs doubles the output, and the factors' output elasticities add up to unity, i.e., constant returns to scale prevail. Hence, a production factor's output elasticity measures its (relative) productive power (relative to the other factors' powers). It indicates the technological importance of the production factor in the sense

of a relative technological weight according to which the factor impacts output through the productive process.

Let's take into account somewhat more specifically the nature of the productive process. Production is the transformation of inputs into output, thus adding economic value. The physical transformation is based on work performance and information processing. In industrial production, raw or pre-processed materials are shaped through work performance and information processing into final products. In modern industrial economies, this is done via the industrial capital stock, activated by energy conversion and handled and supervised by human labor. It is obvious that the physical transformation is subject to the laws of physics, especially thermodynamics, including process-specific technological conditions and constraints. In short, at the heart of production is a physical transformation including specific technological conditions and constraints.

The neoclassical reasoning why the production factors' output elasticities should equal their cost shares is the following: If they were not, the given input factor mix would not be optimal (maximizing profit or minimizing costs).² This would incentivize the substitution of one or several factors by others to level out the disequilibrium. For example, if the output elasticity of energy were higher than its cost share and vice versa for labor, the self-regulating market system would cause a technological shift such that labor would be replaced by energy. So the demand for energy would increase and the demand for labor would decrease. Besides the adjustments in factor quantities, factor prices would also adjust (i.e., wages would fall and energy prices would rise), and all these adjustments would continue up to the point where the output elasticities and cost shares for energy and labor coincide.

However, this line of reasoning has to be put in the context of the following: i) the neoclassical model is timeless, i.e., there is no time-scale specified within which the substitution may actually take place, and ii) the model does not include any binding physical limitations to the substitution process. To what extent the neoclassical production model provides explanatory power is an empirical issue. Does it reflect the realities of modern industrial economies? The answer has to consider the following. First, substitution takes time. For example, the cost of a new machine might be higher than the expected increase in profit, so the investment might be postponed several years until modernization of the capital stock is due anyway. Second, the possibilities for substitution may be limited by binding technological constraints, which may be relaxed only gradually in the course of technological evolution, or sometimes rather revolution. Hence, a disequilibrium between output elasticities and cost shares may actually persist for a long time, possibly for many decades.

In fact, this has characterized the development of technology since the (first) industrial revolution: a perpetual sequence of innovations, including the invention of heat engines, generators, electric motors, transistors, and microprocessors, and manifold technological advances that successively enabled the replacement of relatively expensive routine human labor by combinations of relatively cheap energy and increasingly automated capital, thus increasing automation and boosting wealth creation. Finally, it turns out that the neoclassical model of production and growth, employing the cost-share theorem, fails to explain observed economic growth. Large unexplained residuals remain. Hence, one might expect the neoclassical cost-share theorem to hold for an idealized steady-state economy, but its applicability to the actual dynamically evolving economy remains questionable, at best.

Let us dig a little deeper. The mathematical structure of neoclassical economics was developed by early economists like Jevons, Edgeworth, or Walras, through a one-to-one correspondence from classical physics, especially the mechanics of the point-like mass. The structure is characterized by the assumption of a unique interior equilibrium. In production, this equilibrium is a profit maximum or factor cost minimum, which lies in the interior of the technologically accessible region in factor space. The structure can be visualized as follows. The production system corresponds to a little ball in a salad bowl. The production system moves in factor space like a little ball moves in the three-dimensional space of the salad bowl. Factor prices drive the production system into the point of the cost-minimal factor combination, as gravitational forces drive the ball toward the point of minimal potential energy at the bottom of the salad bowl.³ In this way, neoclassical production economics constitutes a model of technology by a one-to-one correspondence with classical mechanics that is characterized by a unique interior cost-minimizing equilibrium. This equilibrium is located “at the bottom” of (3-dimensional K, L, E) factor space.

The issue with the “salad bowl” structure is that it is static. It does not reflect the process nature of production and technology evolution as they follow from the role of energy in the productive process. This includes at least the following two fundamental aspects: i) In the short run, energy activates the productive capital stock. ii) In the long run, it is the combination of energy and increasingly automated capital that gradually replaces routine human labor, thus driving what is called “productivity.” It is obviously essential to take those two aspects into account when modeling energy-dependent wealth creation and its evolution. In fact, it turns out that the Solow Residual of neoclassical growth theory, based on the “salad bowl structure,” can mostly be resolved by taking into account the role and weight of energy in the productive process.

More specifically, the two mentioned aspects imply the presence of two time-dependent constraints on feasible combinations of K, L, E at any time t : One constraint is that the choice of K, L, E in t must be such that the degree of automation (itself a function of K, L, E) is less than or equal to the maximum degree of automation technologically feasible at time t . Another basic constraint is that the choice of K, L, E at time t must be such that the degree of capacity utilization (also a function of K, L, E) is less than or equal to unity. Hence, at least those two time-dependent technological constraints on automation and utilization that result from the role of energy in the productive process constrain the region in K, L, E factor space that is technologically accessible at any point in time t .

It may be grasped intuitively that when taking into account the role of energy in wealth generation, the geometry of the technology model in factor space is no longer necessarily a salad bowl. Rather, it turns out that the production system is moving on a tilted plane. The motion downhill on this plane is shown in Fig. 4 of [30]. Factor prices and the state of technology define the tilt of the plane, and the production system's motion on the tilted plane in factor space is confined through the mentioned technological constraints on capacity utilization and automation at any point in time t .

Independent of the details of the production model that one may choose, if one takes into account the indicated physical constraints resulting from the physical transformation nature of production via work performance and information processing by K, L, E , this implies limitations to substitutability between the factors, compared to a model ignoring those physical constraints. And obviously, it implies much more complementarity between energy and other factors of production, compared to a model ignoring those physical constraints. It is probably not surprising that models that take into account the specific role of energy as a prime mover in the productive process reveal that energy's output elasticity is typically significantly larger than its relatively low share in total factor cost.

The above qualitative reasoning is based on the optimization mathematics of [6]. The maximization of profit or overall welfare, subject to the constraints on automation and capacity utilization, implies factor shadow prices that drive a wedge between factors' output elasticities and their cost shares, thus invalidating the cost-share theorem. Giving up the cost-share theorem also means giving up the neoclassical duality of factor quantities and factor prices. This duality, which is also employed by Bachmann et al. [3][4], holds only if there are no constraints on factor quantities. Hence, due to the indicated technological constraints on factor quantities, both the duality of factor quantities and prices and the cost-share theorem are, in general, not valid. Instead, the production factor energy plays an essential role both in the utilization of capacity (in the short term) and in the increasing automation of

capital (in the longer term), which is reflected by an output elasticity of energy substantially exceeding its relatively low factor cost share in industrial production systems.

7. Summary and conclusion

In this article, we briefly recall econometric analyses that forgo the neoclassical cost-share theorem (CST) and yield good agreement between computed and observed economic growth. In them, energy's economic weight (output elasticity) is much larger than its cost share. Based on a thought experiment, we suggest that in the Cobb-Douglas and CES functions of Refs. [3] and [4], the output elasticities of energy should be increased to numbers considerably above those that result from the cost-share theorem. We argue that this theorem is generally invalid in industrial production systems. The CST's invalidity is related to energy's crucial role in both the utilization of the industrial capital stock and its increasing automation in the course of technological progress. An algebraic exposition of the CST's disproof is given in [\[6\]](#).

More broadly speaking: Well-established methods in mainstream economics are either blind to essential physical aspects of reality (if energy is neglected as a factor of production altogether) or undervalue their economic impact (if the CST is applied in a production model including energy). Of course, models are just stylized mappings of reality, and they are, in most cases, neither true nor false (except if they are inconsistent), but, depending on the problem at hand, they may come with more or less explanatory and thus predictive power. Our message is this: If the element energy is essential for the problem at hand, e.g., in the case of analyzing the economic impact of an energy embargo or in the context of policies that intend to reduce energy use and associated emissions in view of climate change, then it is essential that an appropriate weight is attributed to the production factor energy – otherwise, the analyses' results may be considerably blurred, and derived energy or environmental policies misguided.

We think that the undervaluation or blindness of economic methods regarding essential aspects of physical reality, especially regarding energy's impact on production and industrial growth, is also shown by the recent (2025) award of the "Nobel Memorial Prize in Economic Sciences" [\[31\]](#): While its emphasis on the role of innovation and technological progress in sustainable economic growth is, of course, valid, it does not refer in any way to the economy's fundamental embeddedness in and dependence on the physical reality, especially of energy's high productive power (its high output elasticity), which is at the very heart of sustainable economic growth. In fact, "technological progress" has been introduced in neoclassical economics to take care of the big difference between the observed economic growth of

industrial countries and the computed growth based on the cost-share theorem. This difference is called the "Solow residual". If one forgoes the CST, as in the Cobb-Douglas and LinEx functions in Fig. 1, or in the LinEx functions in Figs. 2-5 of [6], the remaining residuals are much smaller than any "Solow residuals". In conclusion, it seems all-important to adequately take into account the production factor energy. This is even more crucial since, if energy's impact is underestimated, this may imply that the impact of other factors like capital, labor, or innovation and technological progress is overestimated. To avoid such misspecifications, we believe that the integration of thermodynamics with economics would be a desirable and sound way forward in modeling the impact of energy on sustainable growth and associated energy policies.

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Footnotes

¹ From a technical perspective, value generation is based on work performance and information processing, both driven by energy conversion. In this sense, energy conversion causes value generation. On the other hand, from a behavioral, entrepreneurial planning perspective, the demand for energy, as the demand for capital and labor, is driven by the demand for output, i.e., value generation. In this sense, value generation causes energy conversion. Thus, there is bidirectional causality between the two.

² Maximization of profit or overall welfare is dealt with in [6].

³ From the neoclassical point of view, the factor prices constitute a conservative vector field that can be derived as a gradient from the scalar (negative) output field, i.e., from the production function – just as the gravitational forces constitute a conservative vector field that can be derived as a gradient from the potential energy of the ball in the salad bowl.

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