Review of: "Bell's theorem is an exercise in the statistical theory of causality"

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After more than 50 years the debate around Bell's work is still fascinating. His theorem makes a very important contribution to our causal understanding of reality. The theorem is derived under certain, let us say, "*philosophical assumptions*", and it provides testable predictions that do not agree with those of quantum theory. Experimental work by Aspect, Clauser and Zeilinger, carrying out such experimental tests, has been awarded with a Nobel prize in 2022.

The experiments refute the joint validity of seemingly reasonable philosophical assumptions, which makes the situation "puzzling" for adherents of a classical worldview, and it is therefore natural that this work is still challenged. This happens mostly from the angle of whether actual experiments fulfill the required philosophical assumptions ("loophole debate"), sometimes from the angle of whether violations of Bell's theorem can be achieved classically as well, and sometimes – as by M. Kupczynski – from the angle of whether Bell's theorem can (or may) be derived under a set of specific assumptions. In the pertinent challenge it is claimed that imperfect correlations between clicks in spin polarization correlation experiments may be explained in a locally causal way, if contextual setting-dependent parameters describing measuring instruments are correctly included in the description (see p. 1 of [1]).

The short note by R. Gill in defense of Bell's theorem deals with the challenges by M. Kupczynski. It uses the modern formalization of causality by DAGs (Directed Acyclic Graphs) as championed by J. Pearl [2], which is currently applied in various disciplines to discuss explanations for macroscopic statistical relationships. The Bell-CHSH causal approach can generally be regarded as a helpful tool in applications beyond physics [3]. Gill's note may also serve as an important reminder about the connection between Bell's work and the work around general questions concerning statistics and causality. Therefore, with some much needed additions and clarifications, the note may be suitable for publication in Qeios and fulfill the dual purpose of being a reply to the publications by Kupczynski while generally clarifying the derivation of Bell's theorem from a causal viewpoint for the interested non-specialist reader.

In my view a derivation of CHSH inequalities based on DAGs is certainly possible. The key issue is a precise understanding of the "philosophical assumptions" that are made in such a derivation because a DAG is not a philosophical position. The required assumptions do certainly go beyond locality. Counterfactual-definiteness is one of them. On this basis a violation of the freedom of choice assumption may also serve as an explanation of Bell violations [4]. Adherents of super-determinism might point out that true independent randomness, as used in the note's DAG argument, does not exist. Other authors might point to retro-causal explanations going beyond a conventional understanding of time.

To serve this dual purpose the note needs more detail and would especially benefit from detailed, clear, explicit statements regarding all "philosophical assumptions" needed for the DAG-based derivation of Bell's theorem. For example:

- The note currently states on p.3 "*The model therefore represents a classical physical model, classical in the sense of pre-quantum theory, and one in which experimental settings can be chosen in a statistically independent manner from the parameters of the physical processes, essentially deterministic, which lead to the actually observed measurement outcomes at the two ends of the long box.*" More clarity is needed, e.g. true randomness in a pre-quantum theory world is a problematic concept because true randomness is not contained in the equations of classical mechanics which are deterministic.
- The general relationship between causal explanations and the requirement for counterfactuals is unclear. Kupczynski claims local causal explanations, but he also writes that "*Therefore, counterfactual expectations … do not exist and Bell and CHSH inequalities may not be derived*" (see p. 4 of [1]). Currently, the note in response to Kupczynski just states on p. 4 that "*each of these four correlations is identically equal to the 'experimentally accessible' correlations*". Based on this short statement a reader cannot form a view about the difference between the position of Kupczynski and Gill and whether "contextuality" as advocated by the former can shed new light on the topic. In particular, the statement "*It is absolutely clear that Kupczynski's notion of a probabilistic contextual local causal is of this form*" (p. 4) has to be taken on faith by a reader who has not spent the time going through all of Kupczynski's work about contextual model. For a publication in Qeios, i.e. in a different journal not used by Kupczynski, there should be at least a quote from specific pages of Kupczynski's publications and a derivation why these models can be put in the form of the DAG shown in Figure 2. Additionally, the sentence in the note "*In fact, we will derive inequalities for the four correlations … for one trial*" (top of p.3) may be expressed differently as experimentally accessible averages and correlations do require a series of trials, so it is unclear what a derivation for one trial can bring.
- The spatio-temporal assumptions also need more clarity. For example, the N x 4 spreadsheet (on p. 2) is a mental construct that can only be created operationally by Charlie once he has received the complete information so that he has (a,b,x,y) for all rounds. Figure 2 shows an arrow from the Experimenter reaching both A and B, while the text in Figure 1 says "a *signal travelling from one side to the other at the speed of light takes longer than the time interval between input and output on each side*". With just one Experimenter generating the measurement setting centrally this can be problematic because Figure 2 has no arrow from Experimenter to Hidden, which needs to be justified. Also, the notion of time is not the same for Alice, Bob and Charlie, so the description "*A complete experiment corresponds to a stack of N copies … ordered by time*" (p. 2) is too short – whose time is it?

[1] Kupczynski, M. (2020). Is the moon there if nobody looks: Bell inequalities and physical reality, Front. Phys. 8:273. See https://www.frontiersin.org/article/10.3389/fphy.2020.00273

[2] Pearl, J. (2009). Causality. Models, Reasoning, and Inference. 2nd Ed. Cambridge University Press.

[3] Gallus, C. et. al. (2023). Bell correlations outside physics. Sci. Rep. 13, 4394 (2023). See https://www.nature.com/articles/s41598-023-31441-x

[4] Blasiak, P. et. al. (2021). Violations of locality and free choice are equivalent resources in Bell experiments. Proc. Natl. Acad. Sci. USA 118.

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