Peer Review

Review of: "The Monty Hall Problem: Does Performance Improve After the Acquisition of Some Probability Knowledge?"

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Unfortunately, this study is fatally flawed; simple logico-mathematical analysis reveals this, as I show below. The 3-door problem has precious little to do with probability; it's really a formal-logic problem, with a tiny component that pertains to calculating odds. The most efficient way to see this is to note that unless the host (i) knows what's behind all doors (a condition the authors do state), and (ii) runs an algorithm that includes opening a non-value door (with a coin flip to pick equivalent doors when that sub-case arises), a SWITCHING policy is illogical. The authors register only (i), not (ii); hence their presentation is logico-mathematically invalid.

Subjects well-versed in probability would, in general, be no better off in solving this problem and its many variants unless they are capable of *deductive reasoning by cases*, and can handle both some epistemic logic (you need both knowledge and belief operators for the agents involved) and the concept of executing an algorithm, as well as an algorithm being known/believed by an agent (which when fully formalized is far from trivial). It's proof by cases that chiefly makes it possible to prove that SWITCHING is logically preferable. All those mathematicians who idiotically got this problem wrong could easily have saved themselves from scandal if they had simply sat down and endeavored to prove the superiority of SWITCHING, or of STAYING, using the machinery of epistemic logic and algorithms as noted. Instead, lazy as can be, and failing to see that this is not a question about probability, they did no such thing; and the rest is history of a most embarrassing sort. The authors of the paper under consideration here are unfortunately sustaining the problem. I'm at a complete and utter loss why anyone would suspect that students of probability would do better, etc. Rather, it's students who are studying *proof discovery* and *formal-proof construction* that, upon being asked to provide a proof in defense of their answer, would produce a performance difference.

Note that variants of the 3-door problem that either negate or ignore (i) and (ii) yield different results. For example, if the host accidentally triggers the opening of a door despite knowing what's behind all doors, the entire situation changes. And if someone P comes upon the game when only two doors are in play, but before the contestant decides whether or not to SWITCH, where P knows nothing about the context (e.g., doesn't know that the host knows the unseen contents, requiring by the way non-trivial iteration of knowledge operators in epistemic logic), then from P's knowledge base, STAY is just logically fine. This is why, by the way, the three-horse analogue of the 3-door problem, in which one horse during the race comes up lame, is so tempting even to mathematicians, to this very day. (They point out that if you bet on Horse 1 among the three initially, and Horse 3 drops out during the race, you suddenly can rejoice because now your odds of winning are 1/2, not ⅓.)

Here, for completeness, is an informal proof that does the trick. Note that (1) and (2) are both employed, and that knowledge of probability has absolutely nothing to do with the skill required to crack this problem.

Proof: We proceed by cases. The three top-level cases, using obvious notation, are MGG (Case 1), GMG (Case 2), GGM (Case 3). The doors go from left to right here, so in Case 1 the money M is behind Door 1. Without loss of generality, we analyze MGG. Suppose, next, in Case 1.1, we have MxGG, indicating that the player selects Door 1. Next, since the host both knows what's behind all three doors, and must follow the algorithm that yields revealing after a coin flip either Door 2 or 3, let's assume he opens Door 3. We then have MxG, with the option to switch now presented. Switching then results in MGx, and the result is LOSE. In Case 1.2, though, we have this progression: MGxG => MGx => MxG => WIN. And in Case 1.3 we have: MGGx => MGx => MxG => WIN. Winning thus happens in 2 out of 3 cases. ⅔ is greater than ⅓, so SWITCHING is superior. **QED**

What might the authors do to improve the situation they find themselves in, given the foregoing? Many responses seem pretty obvious. Minimally, require a proof from subjects. (Proofs, like computer programs, can be invalid; so this demand is not the same as demanding a valid proof. Subjects are simply encouraged to *try* to provide a valid proof.). Next, why not couch the 3-door problem in a way that employs just urns or jars, rather than doors? Doors point to contamination, since many people/subjects may have read about the Monty Hall Problem, but simply not remember that they did.

 doi.org/10.32388/W7GGO5

A final remark: The formal intricacies of n-door problems, suitably camouflaged (after all, training data for today's "foundation models" contains a vast amount of data regarding n-door puzzles), across a non-trivial span of variation, are utterly beyond today's LLMs/LRMs. Why? Because ultimately the only way to solve these problems is via proof or disproof having next to nothing to do with probability, and only external resources of the explicitly logicist variety (e.g., automated deductive reasoners) could possibly remedy things.

Declarations

Potential competing interests: No potential competing interests to declare.