Review of: "Impossibilities, mathematics, and logic"

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I have rated this article as a research article, but would give it a higher rating for a general audience. Fundamentally the intention is sound to explain what methods of disproof are. I would give some actual examples. Several examples are referenced, but I like the proof of the irrationality of the square root of 2. You could say, let p and q be integers such that p/q=sqrt(2). Note that if p and q had any integer factors in common say r, then p/q=rs/rt=s/t for p=rs and q=rt, so you can choose p and q so that they have no factors other than 1 in common. Then squaring both sides of p/q=sqrt(2), p^2/q^2=2, so that p^2=2q^2. But then p is divisible by 2, so that if p=2w, 4w^2=2q^2, and thus 2w^2=q^2. But then q must be divisible by 2, which contradicts the choice of p and q. QED.

An interesting thing to note about this standard proof is that there are hidden assumptions here, which in a rigorous proof must be acknowledged. The main one is the fundamental theorem of arithmetic, that any natural number >1 can be expressed uniquely as a multiplication of natural number exponentials of prime numbers, see https://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic.

In terms of method of disproof, as the author notes, you can either show that an assumption and a set of accepted axioms or inference rules leads to a contradiction, or you can construct a counterexample. The logical principle is known as “ex falso sequitur quodlibet” or “anything follows from the false”. In particular, unless you deny the assumption you will need to deny either an axiom or a rule of inference. Infinite descent for natural numbers is really the use of the principle of mathematical induction reformulated as a method of disproof, so is an axiom or an inference rule.

One final comment. I think something has gone wrong in the author's statement of the disproof of Euler's conjecture that the sum of up to n-th powers of non-zero natural numbers is sufficient to equal any n-th power of a non-zero natural number. I suspect it is a typographical error. See https://en.wikipedia.org/wiki/Euler%27s_sum_of_powers_conjecture.

In conclusion, I think the intent of the paper is worthy as an explanation to a general audience how powerful mathematics is, but I think the article needs more detail on how mathematics uses these methods.