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Peer Review

Review of: "Autonomous Second-Order ODEs: A Geometric Approach"

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The paper titled: Autonomous second-order ODEs: a geometric approach by A. J. Pan-Collantes and J. A. Alvarez-Garcia is an excellent example of an interdisciplinary approach to analyzing second-order differential equations.

The authors clearly and systematically introduce readers to advanced methods in differential geometry, focusing on the study of the dynamics of autonomous differential equations (Chapters 2 and 3). Moreover, the introduction to jet bundle theory and its applications in analyzing second-order differential equations is another strong point of the paper (Chapter 2.1). The explanation is detailed enough for readers less familiar with the topic to understand.

Particularly noteworthy is the introduction of the concept of energy foliation (Chapter 4). This idea enables a deeper understanding of the geometric structure of differential equations and connects them to classical Lagrangian mechanics innovatively (Theorem 5.1).

Applying the results to practical examples, such as the analysis of the damped harmonic oscillator, is especially commendable (Chapter 5.2). This part of the article effectively demonstrates how theoretical ideas can be applied to analyze real physical systems.

In turn, the sixth and final chapter outlines research plans for the future, indicating that the topic remains open for further exploration.

The style of the paper is clear and accessible, even when dealing with complex topics, showcasing the authors' exceptional ability to explain challenging concepts. Numerous references to classical results in mathematics and physics provide a solid scientific foundation for the work. In particular, in my opinion, all the proofs of the theorems contained in the work are correct.

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Structurally, the paper is well-organized and transparent. Each chapter smoothly transitions to the next, making it easy for readers to follow the flow of arguments.

Declarations

Potential competing interests: No potential competing interests to declare.